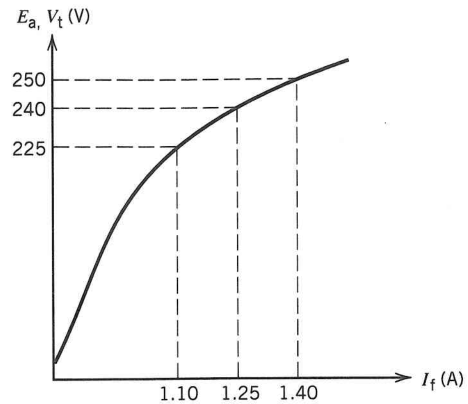

FIGURE 6.19 Magnetization curve of DC machine of Example 6.4.

excited generator, and the field current is adjusted to 1.25 A to obtain a terminal voltage of 240 V at no load. The machine is run at 1200 rpm. The shunt field winding has $N_f = 2500$ turns/pole. The armature and field winding resistances are given as 0.20 Ω and 200 Ω, respectively.

a. Neglecting armature reaction, determine the terminal voltage at rated load current. Calculate voltage regulation.

b. Assume armature reaction at rated load will cause a reduction of 0.15 A in field current. Determine the terminal voltage at rated load current. Calculate voltage regulation.

c. Assume armature reaction at rated load current will cause a demagnetization of 375 A-t, and determine the field current required to produce a terminal voltage of 240 V at rated load current.

Solution

a. From the magnetization curve shown in Fig. 6.19, for a field current $I_f = 1.25$ A, the generated voltage is $E_a = 240$ V. At rated load, the line or terminal current is equal to the armature current and is given by

$$I_t = I_a = 12{,}000/240 = 50 \text{ A}$$

The terminal voltage is obtained as follows:

$$V_t = E_a - I_a R_a = 240 - (50)(0.20) = 230 \text{ V}$$

Therefore, the voltage regulation is given by

$$\text{Voltage regulation} = \frac{V_{nl} - V_{fl}}{V_{fl}} 100\% = \frac{E_a - V_t}{V_t} 100\%$$
$$= \frac{240 - 230}{230} 100\% = 4.3\%$$

b. Due to armature reaction, the effective field current is

$$I_f^{eff} = I_f^{actual} - I_f^{ar} = 1.25 - 0.15 = 1.10 \text{ A}$$

From the magnetization curve, for $I_f = 1.10$ A, the generated voltage is $E_a = 225$ V. Thus,

$$V_t = E_a - I_a R_a = 225 - (50)(0.20) = 215 \text{ V}$$

Therefore,

$$\text{Voltage regulation} = \frac{E_a - V_t}{V_t} 100\%$$
$$= \frac{240 - 215}{215} 100\% = 11.6\%$$

c. At $I_a = 50$ A and $V_t = 240$ V, the generated voltage is calculated as

$$E_a = V_t + I_a R_a = 240 + (50)(0.20) = 250 \text{ V}$$

From the magnetization curve, for a voltage $E_a = 250$ V, the field current is $I_f^{eff} = 1.4$ A.

In the presence of armature reaction, the net mmf is expressed as follows:

$$N_f I_f^{eff} = N_f I_f^{actual} - (N_f I_f)^{ar}$$

Solving for the actual field current yields

$$I_f^{actual} = (1/N_f)[N_f I_f^{eff} + (N_f I_f)^{ar}] = I_f^{eff} + (N_f I_f)^{ar}/N_f$$
$$= 1.4 + 375/2500 = 1.55 \text{ A}$$

EXAMPLE 6.5

The DC generator of Example 6.4 is provided with a series winding. It is operated as a cumulative compound generator, and the terminal voltage is 240 V at both no-load and full-load conditions; that is, there is zero voltage regulation. Assuming a short-shunt connection, determine the number of series turns per pole required. Take the value of the series winding resistance R_s to be 0.02 Ω, and assume that armature reaction causes a demagnetization of 375 A-t.

Solution At rated load conditions, the terminal voltage and current are given by

$$V_t = 240 \text{ V}$$
$$I_s = I_t = 12{,}000/240 = 50 \text{ A}$$

The equivalent circuit of a short-shunt cumulative compound generator is shown in Fig. 6.20.

From the equivalent circuit shown in Fig. 6.20, the generated voltage is found as follows:

$$V_f = V_t + I_s R_s = 240 + (50)(0.02) = 241 \text{ V}$$
$$I_f^{\text{actual}} = V_f / R_f = 241/200 = 1.205 \text{ A}$$
$$I_a = I_s + I_f = 50 + 1.205 = 51.205 \text{ A}$$
$$E_a = V_f + I_a R_a = 241 + (51.205)(0.20) = 251.24 \text{ V}$$

From the magnetization curve, for $E_a = 251.24$ V, $I_f^{\text{eff}} = 1.41$ A. Also, from Example 6.4, $N_f = 2500$ turns, $(NI)^{\text{ar}} = 375$ A-t.

To obtain zero voltage regulation, the total (or net) mmf required at full load must equal the resultant mmf of the shunt and series field windings less the mmf due to armature reaction. Thus,

$$N_f I_f^{\text{eff}} = N_f I_f^{\text{actual}} + N_s I_s - (NI)^{\text{ar}}$$
$$(2500)(1.41) = (2500)(1.205) + N_s(50) - 375$$

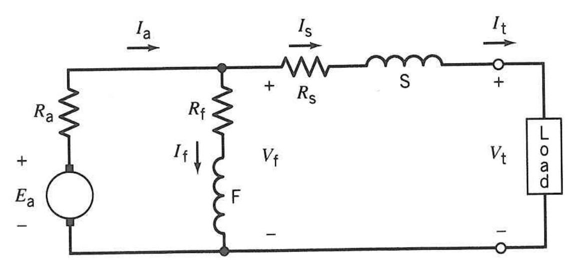

FIGURE 6.20 Equivalent circuit of a short-shunt cumulative compound generator.

Solving for N_s yields

$$N_s = [(2500)(1.41 - 1.205) + 375]/50 = 17.75$$

Say 18 turns/pole.

6.6.5 DC Generator Efficiency

The power flow diagram for a DC shunt generator is shown in Fig. 6.21. As can be seen in this diagram, not all the mechanical power input reaches the load as electrical power output because there are always losses associated with this electromechanical conversion process. Since the generator is assumed to be self-excited, the shunt field winding loss is included with the copper losses in the power flow diagram. If the generator is separately excited, the shunt field copper loss is supplied from a separate electrical source, and it is not included and must be handled separately.

The various losses that occur in the DC shunt generator may be classified as follows:

1. *Electrical or copper losses*. These are the copper losses that occur in the armature and field windings. For a self-excited shunt generator, these losses include

$$\text{Armature copper loss} \quad P_a = I_a^2 R_a$$
$$\text{Shunt field copper loss} \quad P_f = I_f^2 R_f$$

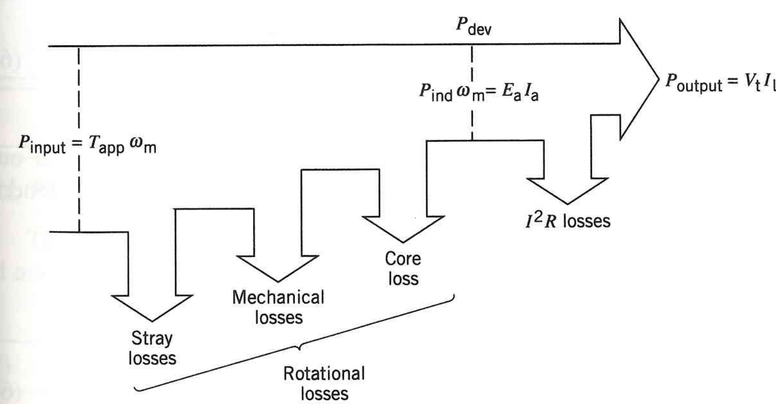

FIGURE 6.21 Power flow diagram for a DC generator.

For a compound generator, another copper loss is present in the series field winding and is added to the armature and shunt field copper losses. The additional copper loss is

$$\text{Series field copper loss} \quad P_s = I_s^2 R_s$$

The armature copper loss depends on load conditions and is typically about 5% when rated current is delivered. The field copper losses are typically from 1% to 2% at rated conditions.

2. *Brush loss*. The electrical loss incurred in the carbon brushes is usually taken equal to $2I_a$ based on the assumption that the total voltage drop across the brushes is about 2 V.
3. *Magnetic or core loss*. These are the hysteresis and eddy current losses occurring in the magnetic circuits of the stator core and poles and the armature core on the rotor.
4. *Mechanical losses*. These are the friction and windage losses. Friction losses include the losses caused by bearing friction and the friction between the brushes and commutator. Windage losses are caused by the friction between rotating parts and air inside the DC machine's casing.
5. *Stray load losses*. These are other losses that cannot be accounted for by the preceding categories.

When the mechanical losses are lumped together with the core loss and stray load loss, they are collectively called rotational loss. Typically, the rotational loss constitutes 3% to 5% of the machine rating, and it is assumed to remain constant for all loading levels.

The DC shunt generator efficiency may be expressed as follows:

$$\eta = \frac{P_{\text{output}}}{P_{\text{input}}} 100\% \tag{6.16}$$

The difference between the mechanical input power and the electrical output power constitutes the various losses incurred within the DC shunt generator. Therefore, efficiency may also be expressed as

$$\begin{aligned} \eta &= \frac{P_{\text{input}} - \Sigma P_{\text{losses}}}{P_{\text{input}}} 100\% \\ &= \frac{P_{\text{output}}}{P_{\text{output}} + \Sigma P_{\text{losses}}} 100\% \end{aligned} \tag{6.17}$$

EXAMPLE 6.6

For the DC compound generator of Example 6.5, the total rotational losses amount to 750 W. Calculate the efficiency when the generator supplies rated current to a load at 240 V.

Solution From Example 6.5, at rated load conditions,

$$V_t = 240 \text{ V}$$

$$I_t = 50 \text{ A} \qquad R_s = 0.02\ \Omega$$

$$I_f = 1.205 \text{ A} \qquad R_f = 200\ \Omega$$

$$I_a = 51.205 \text{ A} \qquad R_a = 0.20\ \Omega$$

The output power of the generator is

$$P_{\text{output}} = V_t I_t = (240)(50) = 12{,}000 \text{ W}$$

The power losses are

$$\begin{aligned} P_{\text{rotational}} &= 750 \text{ W} \\ I_s^2 R_s &= (50)^2(0.02) = 50 \text{ W} \\ I_f^2 R_f &= (1.205)^2(200) = 290 \text{ W} \\ I_a^2 R_a &= (51.205)^2(0.20) = 524 \text{ W} \end{aligned}$$

The total power input is the sum of the power output and the generator losses.

$$P_{\text{input}} = P_{\text{output}} + \Sigma(P_{\text{losses}}) = 13{,}614 \text{ W}$$

Therefore, the efficiency is

$$\eta = (P_{\text{output}}/P_{\text{input}})100\% = (12{,}000/13{,}614)100\% = 88\%$$

DRILL PROBLEMS

D6.6 The open-circuit saturation curve of a DC generator driven at rated speed and separately excited is given by

E_a (V)	10	50	100	200	300	350	400	450	500
I_f (A)	0	0.5	1.0	2.0	3.5	4.4	5.4	6.5	8.0

The field winding has a resistance of 62.5 Ω. The armature winding resistance may be assumed to be negligible.

a. The generator is now operated as a shunt generator by connecting the field winding directly across the armature terminals. Determine the generated voltage.

b. What additional resistance must be added in series with the field winding to obtain a generated voltage of 450 V?

c. What additional resistance must be added in series to the field winding to make the total field-circuit resistance equal to the critical value?

D6.7 A separately excited DC generator has the following open-circuit characteristic when running at 1200 rpm.

E_a (V)	40	80	120	160	200	220	240
I_f (A)	0.15	0.30	0.50	0.75	1.05	1.25	1.50

The effect of the armature winding resistance is negligible.

a. The machine is operated as a shunt generator, and is driven at 1200 rpm. Determine the required field-circuit resistance to obtain an open-circuit voltage of 230 V.

b. The operating speed is reduced to 1000 rpm, and the field-circuit resistance remains unchanged. Find the open-circuit generator voltage.

6.7 DC MOTOR PERFORMANCE

The DC motor differs from a DC generator in that the direction of power flow is reversed. In a motor, electrical energy is converted to mechanical energy. DC motors are used where there is a need for variable-speed drives and for traction-type loads.

The performance of a DC motor can be described by using its equivalent circuit, which is shown in Fig. 6.22. The equivalent circuit looks exactly like that of a DC generator except for the direction of the current that is entering the armature.

The generated voltage E_a across the armature has a polarity opposite to the applied voltage V_t. Thus, it is sometimes referred to as *counter emf*, or *back emf*, and is also denoted as E_b. It is given by Eq. 6.8, which is rewritten here as

$$E_a = E_b = K_a\Phi_p\omega_m \qquad (6.18)$$

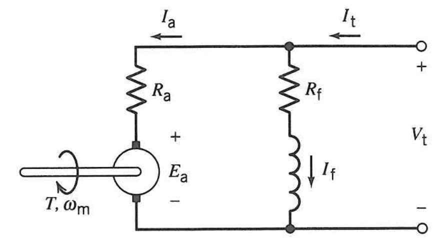

FIGURE 6.22 Equivalent circuit of a DC motor.

The induced or developed torque T_{ind} is given by Eq. 6.9, which is rewritten here as

$$T_{ind} = T_{dev} = K_a\Phi_p I_a \qquad (6.19)$$

This torque is related to the electric power converted to mechanical power P_{conv} as follows:

$$\omega_m T_{dev} = P_{conv} = E_a I_a \qquad (6.20)$$

6.7.1 Speed Regulation

Whereas with a DC generator, the performance measure of interest is its voltage regulation, in the case of a DC motor the performance measure of interest is its speed regulation. The *speed regulation* (*SR*) is similarly defined as follows:

$$\text{Speed regulation (SR)} = \frac{n_{nl} - n_{fl}}{n_{fl}}\ 100\% \qquad (6.21)$$

or

$$\text{Speed regulation (SR)} = \frac{\omega_{nl} - \omega_{fl}}{\omega_{fl}}\ 100\% \qquad (6.22)$$

A motor with zero speed regulation has a full-load speed equal to its no-load speed. Positive speed regulation implies that the motor speed will decrease when the load on its shaft is increased. On the other hand, negative speed regulation implies that the speed will become higher as the load on its shaft becomes higher.

EXAMPLE 6.7

A 220-V shunt motor has an armature resistance of 0.2 Ω and a field resistance of 110 Ω. At no load, the motor runs at 1000 rpm, and it draws a line current of 7 A. At full load, the input to the motor is 11 kW. Consider that the air-gap flux remains fixed at its value at no load; that is, neglect armature reaction.

a. Find the speed, speed regulation, and developed torque at full load.

b. Find the starting torque if the starting armature current is limited to 150% of full-load current.

c. Consider that armature reaction reduces the air-gap flux by 5% when full-load current flows in the armature. Repeat part (a).

Solution

a. Armature reaction is neglected. Referring to Fig. 6.22, the values of the various currents and the generated voltage at no-load conditions are found as follows:

$$\begin{aligned} I_{\mathrm{t,nl}} &= 7\ \mathrm{A} \\ I_{\mathrm{f,nl}} &= V_t/R_f = 220/110 = 2\ \mathrm{A} \\ I_{\mathrm{a,nl}} &= I_{\mathrm{t,nl}} - I_{\mathrm{f,nl}} = 7 - 2 = 5\ \mathrm{A} \\ E_{\mathrm{b,nl}} &= V_t - I_{\mathrm{a,nl}}R_a = 220 - (5)(0.2) = 219\ \mathrm{V} \end{aligned}$$

Similarly, the currents and the generated voltage at full-load conditions are found as follows:

$$\begin{aligned} I_{\mathrm{t,fl}} &= 11{,}000/220 = 50\ \mathrm{A} \\ I_{\mathrm{f,fl}} &= 220/110 = 2\ \mathrm{A} \\ I_{\mathrm{a,fl}} &= 50 - 2 = 48\ \mathrm{A} \\ E_{\mathrm{b,fl}} &= 220 - (48)(0.2) = 210.4\ \mathrm{V} \end{aligned}$$

The no-load speed is $n_{\mathrm{nl}} = 1000$ rpm. Since armature reaction is neglected, $\Phi_{\mathrm{nl}} = \Phi_{\mathrm{fl}}$. Thus, the full-load speed is found as

$$\begin{aligned} n_{\mathrm{fl}} &= (E_{\mathrm{b,fl}}/E_{\mathrm{b,nl}})n_{\mathrm{nl}} = (210.4/219)(1000) = 960.7\ \mathrm{rpm} \\ \omega_{\mathrm{fl}} &= 2\pi n_{\mathrm{fl}}/60 = 2\pi(960.7)/60 = 100.6\ \mathrm{rad/s} \end{aligned}$$

Hence, the speed regulation is computed as

$$\begin{aligned} \text{Speed regulation} &= \frac{n_{\mathrm{nl}} - n_{\mathrm{fl}}}{n_{\mathrm{fl}}} 100\% \\ &= \frac{1000 - 960.7}{960.7} 100\% = 4.1\% \end{aligned}$$

The power developed and torque developed at full load are found as follows:

$$\begin{aligned} P_{\mathrm{dev,fl}} &= E_{\mathrm{b,fl}}I_{\mathrm{a,fl}} = (210.4)(48) = 10{,}099\ \mathrm{W} \\ T_{\mathrm{e,fl}} &= P_{\mathrm{dev,fl}}/\omega_{\mathrm{fl}} = 10{,}099/100.6 = 100.4\ \mathrm{N\text{-}m} \end{aligned}$$

b. At starting, with a 150% limit for the armature current,

$$I_{\mathrm{a,start}} = 1.50 I_{\mathrm{a,fl}} = (1.50)(48) = 72\ \mathrm{A}$$

The electromagnetic torque varies directly with the flux and the armature current; thus,

$$\frac{T_{\mathrm{e,start}}}{T_{\mathrm{e,fl}}} = \frac{K_a \Phi_{\mathrm{fl}} I_{\mathrm{a,start}}}{K_a \Phi_{\mathrm{fl}} I_{\mathrm{a,fl}}}$$

Since armature reaction is neglected, the flux is assumed to remain constant. Solving for the starting torque yields

$$T_{\mathrm{e,start}} = \frac{I_{\mathrm{a,start}}}{I_{\mathrm{a,fl}}} T_{\mathrm{e,fl}} = \frac{72}{48} 100.4 = 150.6\ \mathrm{N\text{-}m}$$

c. The back emf varies directly with the flux and the speed; thus,

$$\frac{E_{\mathrm{b,fl}}}{E_{\mathrm{b,nl}}} = \frac{K_a \Phi_{\mathrm{fl}} n_{\mathrm{fl}}}{K_a \Phi_{\mathrm{nl}} n_{\mathrm{nl}}}$$

At full load, the effect of armature reaction is to reduce the field flux by 5%, that is, $\Phi_{\mathrm{fl}} = 0.95\Phi_{\mathrm{nl}}$. Solving for the full-load speed yields

$$\begin{aligned} n_{\mathrm{fl}} &= \frac{E_{\mathrm{b,fl}}}{E_{\mathrm{b,nl}}} \frac{\Phi_{\mathrm{nl}}}{\Phi_{\mathrm{fl}}} n_{\mathrm{nl}} = \frac{210.4}{219} \frac{1.0}{0.95} 1000 = 1011.3\ \mathrm{rpm} \\ \omega_{\mathrm{fl}} &= 2\pi(1011.3)/60 = 105.9\ \mathrm{rad/s} \end{aligned}$$

Therefore, the speed regulation is computed as

$$\begin{aligned} \text{Speed regulation} &= \frac{n_{\mathrm{nl}} - n_{\mathrm{fl}}}{n_{\mathrm{fl}}} 100\% \\ &= \frac{1000 - 1011.3}{1011.3} 100\% = -1.1\% \end{aligned}$$

The power developed $P_{\mathrm{dev,fl}}$ is the same as in part (a), since the generated voltage and the armature current are unchanged. Because the full-load speed has changed, the torque has also changed; thus,

$$T_{e,fl} = P_{dev,fl}/\omega_{fl} = 10{,}099/105.9 = 95.4 \text{ N-m}$$

DRILL PROBLEMS

D6.8 A DC motor develops a torque of 30 N-m. Determine the electromagnetic torque when the armature winding current is increased by 50% and the flux is reduced by 10%.

D6.9 A shunt motor draws 41 A from a 120-V source when it drives a load at 1750 rpm. The armature and field winding resistances are 0.2 Ω and 120 Ω, respectively. Determine the developed torque.

D6.10 A 240-V shunt motor running at 1750 rpm develops a counter emf of 228 V. It has an armature resistance of 0.15 Ω and a total brush voltage drop of 2 V. Calculate

- **a.** The armature current at 1750 rpm
- **b.** The speed when the armature current is 50 A
- **c.** The speed when the armature current is 25 A

D6.11 The field and armature winding resistances of a 440-V DC shunt machine are 110 Ω and 0.15 Ω, respectively.

- **a.** Calculate the power developed by the DC machine if it absorbs 22 kW while running as a motor.
- **b.** Calculate the power developed by the DC machine if it supplies 22 kW while running as a generator.

6.7.2 DC Motor Efficiency

The power flow diagram for a DC compound motor is shown in Fig. 6.23. The electrical power input P_{input} is equal to $V_t I_t = V_t I_L$. The electrical losses consist of the armature winding loss ($I_a^2 R_a$) and the copper losses ($I_f^2 R_f + I_s^2 R_s$) in the field windings.

The motor efficiency is given by

$$\eta = \frac{P_{output}}{P_{input}} 100\% \tag{6.23}$$

For the case of a shunt motor, the series field winding is not present or is disconnected. Thus, the field copper loss consists only of the shunt field loss

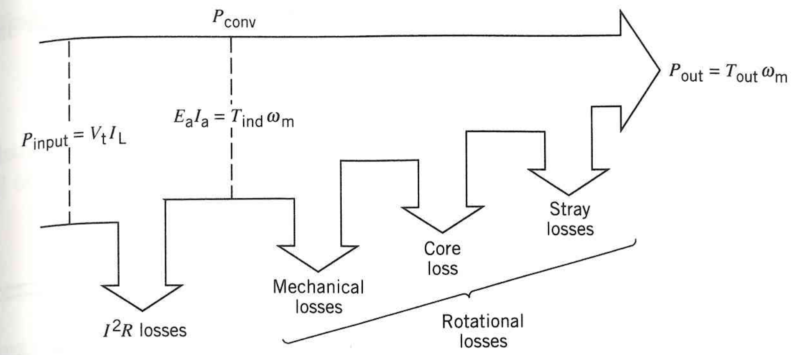

FIGURE 6.23 Power flow diagram for a DC compound motor.

($I_f^2 R_f$). The mechanical losses, core losses, and stray load losses are commonly lumped together under rotational losses and are assumed to remain constant at any loading level.

EXAMPLE 6.8

A 10-hp, 220-V, DC shunt motor has armature and field resistances of 0.25 Ω and 220 Ω, respectively. It is supplied by a 220-V source, and it draws a current of 40 A. The total rotational losses are 450 W. Find the efficiency of the motor.

Solution When the motor is driving a load, the power input P_{input} is

$$P_{input} = V_t I_t = (220)(40) = 8800 \text{ W}$$

The field and armature currents are given by

$$I_f = V_t/R_f = 220/220 = 1 \text{ A}$$
$$I_a = I_t - I_f = 40 - 1 = 39 \text{ A}$$

The copper losses P_{Cu} consist of

$$I_a^2 R_a = (39)^2(0.25) = 380 \text{ W}$$
$$I_f^2 R_f = (1)^2(220) = 220 \text{ W}$$

The total copper losses of the motor are

$$P_{Cu} = 380 + 220 = 600 \text{ W}$$

The power output P_{output} is equal to the difference between the power input and the sum of the total copper losses and the rotational losses P_{rot}.

$$P_{output} = P_{input} - (P_{Cu} + P_{rot}) = 8800 - (600 + 450) = 7750 \text{ W}$$

Therefore,

$$\eta = (P_{output}/P_{input})100\% = (7750/8800)100\% = 88\%$$

DRILL PROBLEMS

D6.12 The series field and armature winding resistances of a 230-V series motor are 0.05 Ω and 0.2 Ω, respectively. The motor draws a current of 20 A while running at 1500 rpm. If the total rotational losses are 400 W, determine the efficiency of the motor.

D6.13 A DC shunt motor is rated at 5 hp, 115 V, 1150 rpm. At rated operating conditions, the efficiency is 85%. The armature circuit resistance is 0.5 Ω, and the field circuit resistance is 115 Ω. Determine the induced voltage at rated operating conditions.

D6.14 A 5-hp, 120-V, 1800-rpm shunt motor is operating at full load and takes a line current of 36 A. Its armature and field resistances are 0.30 Ω and 120 Ω, respectively.

a. What is the efficiency of this motor?

b. What is its rotational loss?

6.7.3 Speed-Torque Characteristics

Consider the DC shunt motor whose equivalent circuit is shown in Fig. 6.24.

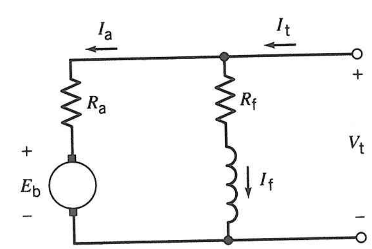

FIGURE 6.24 Equivalent circuit of a DC shunt motor.

From Kirchhoff's voltage law,

$$V_t = E_b + I_a R_a \tag{6.24}$$

Substituting the expression for the back emf given by Eq. 6.18 into Eq. 6.24 and solving for ω_m yields

$$\omega_m = \frac{V_t - I_a R_a}{K_a \Phi_p} \tag{6.25}$$

It may be observed that loss of field excitation results in overspeeding for a shunt motor. Thus, care should be taken to prevent the field circuit from getting open.

From Eq. 6.19, the armature current may be expressed as follows:

$$I_a = \frac{T_{dev}}{K_a \Phi_p} \tag{6.26}$$

Substituting Eq. 6.26 into Eq. 6.25 yields the speed-torque equation of a DC shunt motor.

$$\omega_m = \frac{1}{K_a \Phi_p} V_t - \frac{R_a}{(K_a \Phi_p)^2} T_{dev} \tag{6.27}$$

If the applied voltage V_t and the flux Φ_p remain constant for any load, the speed will decrease linearly with torque. In an actual machine, however, as the load increases, the flux is reduced because of armature reaction. Since the denominator terms decrease, there is less reduction in speed and speed regulation is improved somewhat. The speed-torque characteristics of DC motors are shown in Fig. 6.25.

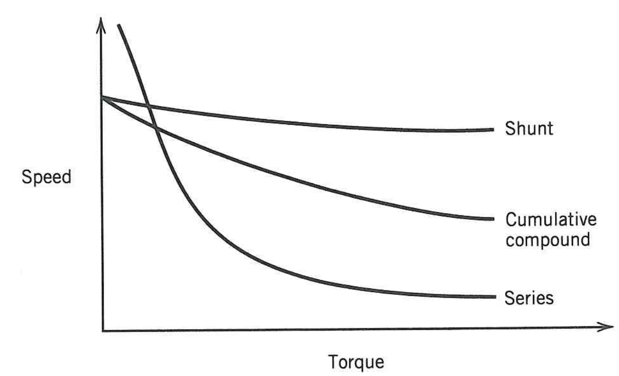

FIGURE 6.25 Speed-torque characteristics of DC motors.

In a compound motor, the magnetic field of the series field winding either aids or opposes the magnetic field of the shunt field winding; thus, the net magnetic flux at the pole is given by

$$\Phi_p = \frac{(\text{mmf})_f \pm (\text{mmf})_s}{R_{\text{path}}} \tag{6.28}$$

where

Φ_p = magnetic pole flux

$(\text{mmf})_f$ = shunt field mmf

$(\text{mmf})_s$ = series field mmf

When the mmfs of the two field windings are additive, the speed-torque characteristic is more drooping than that for a shunt motor. When the mmfs are opposing, the speed-torque characteristic lies above that of a shunt motor.

In a series motor, the excitation is provided solely by the series field winding, which is connected in series with the armature. The flux produced is proportional to the armature current. Thus, the torque developed may be written as follows:

$$T_{\text{dev}} = K_a \Phi_p I_a = K' I_a^2 \tag{6.29}$$

Therefore, the speed may be expressed as

$$\omega_m = \frac{V_t}{K' I_a} - \frac{R_a}{K'} = \frac{V_t}{K'' \sqrt{T_{\text{dev}}}} - \frac{R_a}{K'} \tag{6.30}$$

It is seen that the series motor will run at dangerously high speeds at no load. For this reason, a series motor is never started with no load connected to its shaft.

6.7.4 Motor Starting

Consider the DC shunt motor. At starting, the armature is not rotating. Therefore, the counter emf $E_b = K_a \Phi_p \omega_m = 0$. E_b is also called back emf. Hence, the starting current will be dangerously high and is given by

$$I_{a,\text{start}} = \frac{V_t - E_b}{R_a} = \frac{V_t}{R_a} \tag{6.31}$$

For the motor of Example 6.7, $I_{a,\text{start}}$ will have a value of (220/0.2) = 1100 A, which is approximately 23 times the rated current of 48 A. This starting

current value is obviously too high. Provision must be made to limit the starting current to prevent damage to the motor. Two times rated current is typically allowed to flow during starting so that sufficient torque will be developed.

Two methods of limiting the starting current are as follows:

1. Insert external resistance in the armature circuit.
2. Apply a reduced voltage at starting.

The first method means an additional copper loss, albeit during the starting period only. The second method has the major disadvantage of requiring an expensive variable-voltage supply.

The external resistance R_{ae} is inserted in the armature circuit either manually or automatically. At starting, the armature current is given by

$$I_{a,\text{start}} = \frac{V_t}{R_a + R_{ae}} \tag{6.32}$$

As the motor accelerates to a higher speed, the starting resistance is shunted out in steps. When the full starting resistance is shorted out, the motor accelerates to its base speed.

EXAMPLE 6.9

Design a starter for the DC shunt motor of Example 6.7 using an external resistance to be connected in series with the armature as shown in Fig. 6.26.

This resistance is to be cut out in steps so that the armature current does not exceed 200% of full-load armature current. As the motor speeds up, the armature current will drop. As soon as the armature current falls to its full-load value, sufficient resistance is to be cut out so that the current returns to the 200% level. This process is repeated until the full starting resistance is shorted out. The field winding is to be connected directly across the DC supply.

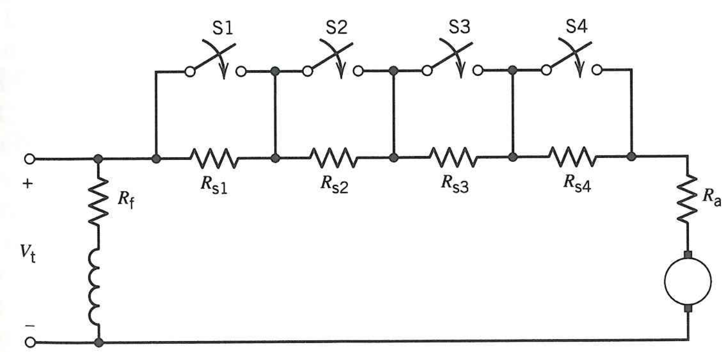

FIGURE 6.26 DC motor starter of Example 6.9.

Solution From Example 6.7, the full-load or rated armature current is

$$I_{a,\text{rated}} = 48 \text{ A}$$

The maximum value of starting current is limited to two times rated armature current; therefore,

$$I_{a,\text{start}} = 2I_{a,\text{rated}} = (2)(48) = 96 \text{ A}$$

With the external resistance R_e inserted in series with the armature, Kirchhoff's voltage law yields

$$V_t = E_b + I_a(R_a + R_e)$$

Therefore,

$$R_e = (V_t - E_b)/I_a - R_a$$
$$E_b = V_t - I_a(R_a + R_e)$$

At starting, the counter emf $E_{b0} = 0$. Thus,

$$\begin{aligned}
R_{e0} &= (220 - 0)/96 - 0.2 = 2.292 - 0.2 = 2.092\ \Omega \\
E_{b1} &= 220 - (48)(2.292) = 110 \text{ V} \\
R_{e1} &= (220 - 110)/96 - 0.2 = 1.146 - 0.2 = 0.946\ \Omega \\
E_{b2} &= 220 - (48)(1.146) = 165 \text{ V} \\
R_{e2} &= (220 - 165)/96 - 0.2 = 0.573 - 0.2 = 0.373\ \Omega \\
E_{b3} &= 220 - (48)(0.573) = 192.5 \text{ V} \\
R_{e3} &= (220 - 192.5)/96 - 0.2 = 0.286 - 0.2 = 0.086\ \Omega \\
E_{b4} &= 220 - (48)(0.286) = 206.5 \text{ V} \\
R_{e4} &= (220 - 206.5)/96 - 0.2 = 0.141 - 0.2 = -0.059\ \Omega
\end{aligned}$$

The negative sign of R_{e4} means that for this step the full value of R_{e3} is shorted out.

The values of resistances that are to be shorted out successively are:

$$\begin{aligned}
R_{s1} &= R_{e0} - R_{e1} = 2.092 - 0.946 = 1.146\ \Omega \\
R_{s2} &= R_{e1} - R_{e2} = 0.946 - 0.373 = 0.573\ \Omega \\
R_{s3} &= R_{e2} - R_{e3} = 0.373 - 0.086 = 0.287\ \Omega \\
R_{s4} &= 0.086\ \Omega
\end{aligned}$$

DRILL PROBLEMS

D6.15 A DC shunt motor is rated 5 kW, 125 V, and 1800 rpm. When the armature is held stationary, a voltage of 5 V applied to the motor terminals will cause a full-load current of 40 A to flow through the armature.

a. Determine the armature current if rated voltage is applied directly across the motor terminals.

b. Determine the value of the external resistance that must be connected in series with the armature in order to limit the starting current to twice the rated armature current.

D6.16 A 240-V DC shunt motor has an armature winding resistance of 0.2 ohm. The full-load armature current is 50 A.

a. Determine the starting current if the motor is connected directly across the 240-V supply. Express this current as a percentage of the full-load value.

b. Calculate the value of resistance that must be connected in series with the armature circuit to limit the starting current to 150% of full-load value.

6.7.5 Applications of DC Motors

The expression for developed torque T_{dev} of a DC motor is given by Eq. 6.19, which shows that the torque is directly proportional to the product of the field flux and the armature current. This relationship is valid not only for the motor operating under load at steady state but also during starting conditions. This torque characteristic is illustrated in Fig. 6.27*a* for the shunt, compound, and series motors.

The expression for the speed ω_m of a DC motor is given by Eq. 6.25, which describes the linear relation between speed and armature current and the inverse variation of speed with respect to field flux. The speed characteristics of the shunt, compound, and series motors are shown in Fig. 6.27*b*.

The DC shunt motor has a relatively constant speed characteristic almost independent of load. It has a speed regulation of about 5% to 10%. Because the field flux changes very little with changes in load levels, the torque developed by the motor is almost directly proportional to the armature current. Therefore, the motor is able to develop a reasonably good starting torque, which is usually limited to less than 250% because of starting current restrictions. DC shunt motors are used primarily for constant-speed applications requiring medium starting torque, such as for driving centrifugal pumps, fans, blowers, conveyors, and machine tools.

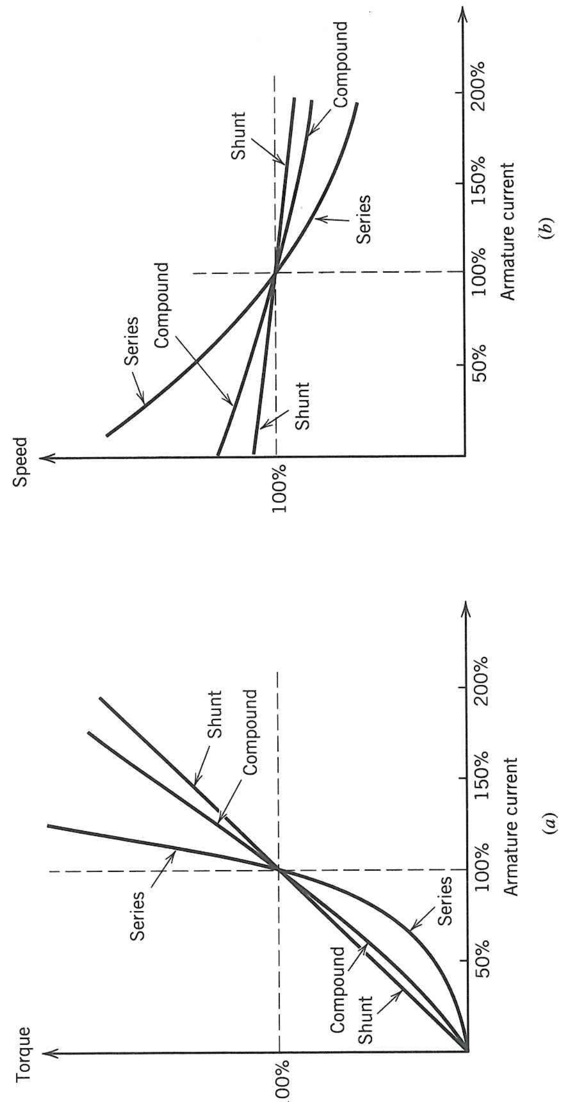

FIGURE 6.27 Speed and torque characteristics of DC motors.

In a DC cumulative compound motor, the flux developed by the series field winding reinforces the flux produced by the shunt field winding. Thus, the torque developed by the compound motor is much higher than that of a DC shunt motor, especially for armature currents above rated value. For the same reason, however, the speed of the compound motor decreases more rapidly with increasing armature current than that of the shunt motor. The speed regulation of the compound motor varies from 15% to 30%, depending on the degree of compounding. Compound motors are used for applications requiring high starting torques and only fairly constant speed and for pulsating loads. They are used to drive conveyors, hoists, compressors, metal-stamping machines, reciprocating pumps, punch presses, crushers, and shears.

The DC series motor derives its flux from its series field, which is connected in series with the armature. Therefore, the torque developed by the motor is directly proportional to the square of the armature current. The speed characteristic of the series motor is described by a large variation in speed from full-load to no-load conditions. This indicates that loads should not be removed completely or reduced to very low levels because of the possibility of the motor "running away." Series motors are used for applications requiring very high starting torques and where varying speed is acceptable. They are used to drive hoists, cranes, and so forth.

*6.8 DC MACHINE DYNAMICS

The previous sections have described the steady-state behavior of DC generators and motors, and the models derived and used are valid only for steady-state conditions. In this section, the dynamic behavior of DC machines is described and transient models are presented. The block diagram representations and the transfer functions of these models are derived. In an introductory course in power engineering, the professor may choose to omit this section.

*6.8.1 Dynamic Equations

A DC machine may be represented by two coupled electrical circuits both containing resistances and inductances as shown in Fig. 6.28. These circuits, the field and the armature, are coupled through the electromagnetic field, which is represented by the generated voltage e_a. The electrical system is coupled to the mechanical system through the electromagnetic torque T_{fld} and an external mechanical torque, which could be an input torque T_S from a prime mover or a load torque T_L. The reference directions of the arrows shown are applicable for modeling the DC generator.

In the following discussion, it is assumed that saturation is negligible and the air-gap flux is directly proportional to the field current i_f. Armature reaction

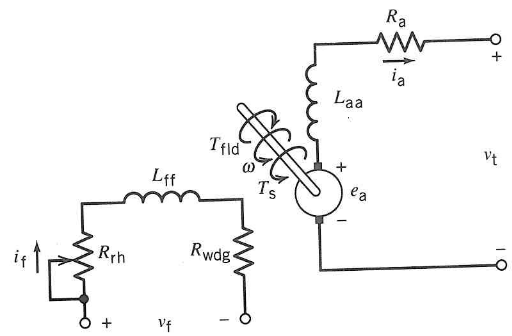

FIGURE 6.28 Schematic representation of a DCgenerator.

is also assumed negligible; however, armature reaction effects may be added on later as additional field excitation requirements. Hence, the expressions for the electromagnetic torque and generated voltage are given by Eqs. 6.33 and 6.34, respectively.

$$\begin{aligned} T_{fld} &= K_a K_f' i_f i_a \\ &= K_f i_f i_a \end{aligned} \qquad (6.33)$$

$$\begin{aligned} e_a &= K_a K_f' i_f \omega_m \\ &= K_f i_f \omega_m \end{aligned} \qquad (6.34)$$

Here $K_f = K_a K_f'$ is a constant.

*6.8.2 Separately Excited DC Generator

For the DC generator shown in Fig. 6.28, the field excitation is supplied by a separate voltage source. If it is assumed that the field circuit is closed at time $t = 0$, the voltage equation may be written as

$$\begin{aligned} L_{ff}\frac{di_f}{dt} + R_f i_f &= V_f u(t) \\ \tau_f \frac{di_f}{dt} + i_f &= \frac{V_f}{R_f} u(t) \end{aligned} \qquad (6.35)$$

where $\tau_f = L_{ff}/R_f$ is the time constant of the field circuit and $u(t)$ is the unit step function.

The generated voltage is given by Eq. 6.34, assuming that the effect of saturation is negligible. For a speed ω_{m0}, this voltage may be written in terms of the generator constant K_g as follows:

$$e_{a0} = (K_f \omega_{m0}) i_f = K_g i_f \qquad (6.36)$$

When the field current is held constant, as when it has reached steady state, the generated voltage becomes directly proportional to the angular velocity; that is,

$$\frac{e_a}{\omega_m} = \frac{e_{a0}}{\omega_{m0}} \qquad (6.37)$$

or

$$e_a = \left(\frac{e_{a0}}{\omega_{m0}}\right)\omega_m \qquad (6.38)$$

For the armature circuit, the voltage equation may be written in terms of the generator terminal voltage as

$$\begin{aligned} L_{aa}\frac{di_a}{dt} + R_a i_a &= e_a - V_t \\ \tau_a \frac{di_a}{dt} + i_a &= \frac{e_a - V_t}{R_a} \end{aligned} \qquad (6.39)$$

where $\tau_a = L_{aa}/R_a$ is the time constant of the armature circuit.

When the separately excited DC generator is supplying an armature current i_a to an electrical load, the electromagnetic torque T_{fld} may be derived from the developed power P_{dev} as follows:

$$\omega_m T_{fld} = P_{dev} = e_a i_a \qquad (6.40)$$

Therefore,

$$\begin{aligned} T_{fld} &= \frac{e_a i_a}{\omega_m} \\ &= \left(\frac{e_{a0}}{\omega_{m0}}\right) i_a \end{aligned} \qquad (6.41)$$

The dynamic equation of motion of the DC machine is dependent on the mechanical torque applied to its shaft; thus,

$$\begin{aligned} J\frac{d\omega_m}{dt} + T_{fld} &= T_{shaft} \\ J\frac{d\omega_m}{dt} &= T_{shaft} - T_{fld} \end{aligned} \qquad (6.42)$$

where

J = moment of inertia of the rotor and the prime mover

T_{shaft} = mechanical applied torque from the prime mover

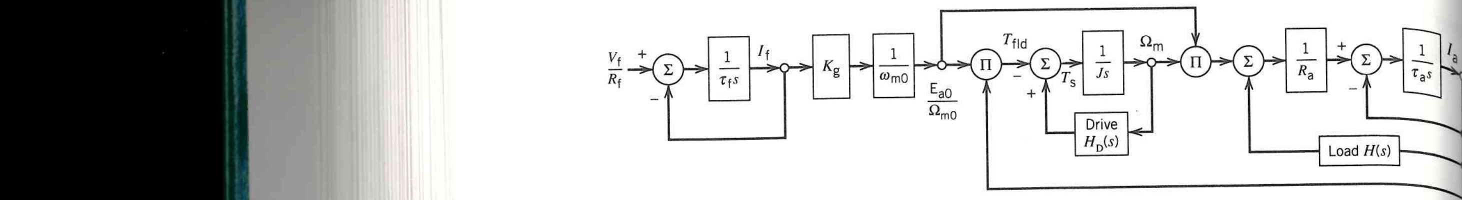

FIGURE 6.29 Block diagram representation of a DC generator.

The differential equations describing the dynamic performance of a separately excited DC generator may be represented with a block diagram. The block diagram representation employing the Laplace-transformed variables is shown in Fig. 6.29.

EXAMPLE 6.10

A 240-V, 50-kW, 1800-rpm, separately excited DC generator has an armature circuit resistance and inductance of 0.10 Ω and 1.0 mH, respectively. The field winding resistance and inductance are 125 Ω and 75 H, respectively. The generator emf constant K_g is 125 V per field ampere at 1800 rpm. The field and armature circuits are initially open, and the prime mover is driving the generator at a constant speed of 1800 rpm.

a. At time $t = 0$, the field circuit is connected to a constant-voltage source of 250 V. Find the expression for the armature terminal voltage as a function of time.

b. In part (a), after the field circuit has reached a steady state, the armature is suddenly connected to a load consisting of a series-connected resistance and inductance of 1.15 Ω and 1.5 mH, respectively. Find the expressions for the armature current, terminal voltage, and electromagnetic torque as functions of time.

Solution

a. The voltage equation for the field circuit is given by Eq. 6.35 as follows

$$\tau_f \frac{di_f}{dt} + i_f = \frac{V_f}{R_f}u(t) = \frac{250}{150}u(t) = 2.0u(t)$$

where $\tau_f = L_{ff}/R_f = 75/125 = 0.6$ s. Thus, the expression for the field current is given by

$$\begin{aligned} i_f(t) &= 2.0\,(1.0 - e^{-t/\tau_f})u(t) \\ &= 2.0\,(1.0 - e^{-t/0.60})u(t) \qquad \text{A} \\ &= 2.0\,(1.0 - e^{-1.667t})u(t) \end{aligned}$$

The generated voltage is given by Eq. 6.36 as

$$\begin{aligned} e_{a0} &= K_g i_f = (125)(2.0)(1.0 - e^{-1.667t})u(t) \qquad \text{V} \\ &= 250(1.0 - e^{-1.667t})u(t) \end{aligned}$$

Since the generator is initially operating at open circuit, the terminal voltage is equal to the generated voltage. Thus,

$$v_t = e_{a0} = 250(1.0 - e^{-1.667t})u(t) \qquad \text{V}$$

b. Since the field circuit has reached a steady-state condition, the generated voltage becomes a constant value $e_a(t) = E_a = 250$ V. The differential equation for the armature circuit takes the form

$$\tau_a \frac{di_a}{dt} + i_a = \frac{E_a}{R_a + R_L}u(t) = \frac{250}{0.10 + 1.15}u(t) = 200u(t)$$

where

$$\tau_a = \frac{L_{aa} + L_L}{R_a + R_L} = \frac{(1.0 + 1.5) \times 10^{-3}}{0.10 + 1.15} = 2.0 \times 10^{-3}\ \text{s}$$

Thus, the expression for the armature current is

$$\begin{aligned} i_a(t) &= 200(1.0 - e^{-t/\tau_a})u(t) \\ &= 200(1.0 - e^{-t/0.002})u(t) \qquad \text{A} \\ &= 200(1.0 - e^{-500t})u(t) \end{aligned}$$

The terminal voltage of the generator can be found as

$$\begin{aligned} v_t &= L_L \frac{di_a}{dt} + R_L i_a \\ &= 1.5 \times 10^{-3}(-500)(200)(-e^{-500t}) \qquad \text{V} \\ &\quad + (1.15)(200)(1.0 - e^{-500t}) \\ &= 230 - 80e^{-500t} \end{aligned}$$

The angular velocity is given by

$$\omega_m = \frac{2\pi n}{60} = \frac{2\pi(1800)}{60} = 60\pi\ \text{rad/s}$$

Therefore, the electromagnetic torque is

$$T_{fld} = \frac{e_a i_a}{\omega_m} = \frac{(250)(200)(1.0 - e^{-500t})}{60\pi} \quad \text{N-m}$$
$$= 265.2(1.0 - e^{-500t})$$

*6.8.3 Separately Excited DC Motor

The schematic representation of a separately excited DC motor is shown in Fig. 6.30. The field circuit is assumed to have been connected to the separate voltage source V_f for a sufficiently long time that the field current I_f has reached a steady value, that is,

$$I_f = \frac{V_f}{R_{wdg} + R_{rh}} = \frac{V_f}{R_f} \tag{6.43}$$

Assuming that the effects of saturation and armature reaction are negligible, and since the field current I_f is constant, the expressions for the developed electromagnetic torque and induced voltage may be expressed as in Eqs. 6.44 and 6.45, respectively.

$$\begin{aligned} T_{fld} &= K_a K_f' I_f i_a \\ &= K_f I_f i_a \\ &= K_m i_a \end{aligned} \tag{6.44}$$

$$\begin{aligned} e_a &= K_a K_f' I_f \omega_m \\ &= K_f I_f \omega_m \\ &= K_m \omega_m \end{aligned} \tag{6.45}$$

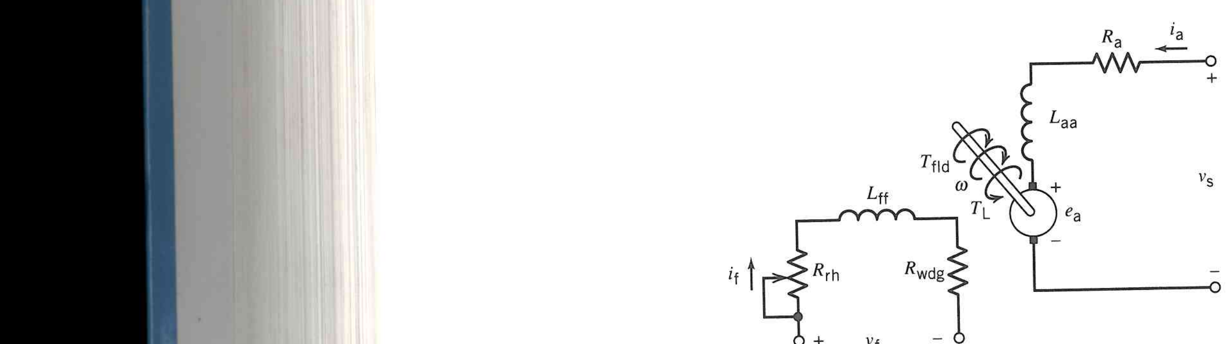

FIGURE 6.30 Schematic representation of a DC motor.

where $K_m = K_f I_f$ = constant and measured in newton-meters per ampere. The motor constant K_m may also be derived from Eq. 6.45 and expressed in volts per radian per second or volt-seconds per radian as follows:

$$K_m = \frac{e_{a0}}{\omega_{m0}} \tag{6.46}$$

The developed torque is available at the motor shaft for driving a mechanical load. The motor develops just enough torque to balance the required torque of the load and its rotational losses. The induced voltage is in opposition to the applied voltage and thus is also called counter emf or back emf. Because of its reverse polarity, the back emf serves to limit the armature current.

For the armature circuit, a loop voltage equation may be written in terms of the voltage applied to the motor terminals as follows:

$$\begin{aligned} L_{aa}\frac{di_a}{dt} + R_a i_a &= V_s - e_a \\ \tau_a \frac{di_a}{dt} + i_a &= \frac{V_s - e_a}{R_a} \end{aligned} \tag{6.47}$$

where $\tau_a = L_{aa}/R_a$ is the time constant of the armature circuit. Substituting the expression for the counter emf given by Eq. 6.45 into Eq. 6.47 and rearranging the terms, Eq. 6.48 is obtained.

$$\tau_a \frac{di_a}{dt} + i_a + \left(\frac{K_m}{R_a}\right)\omega_m = \left(\frac{1}{R_a}\right)V_s \tag{6.48}$$

The dynamic equation of motion of the DC motor is dependent on the mechanical load connected to its shaft. The electromagnetic torque developed by the motor must be equal to the sum of all opposing torques. Thus,

$$T_{fld} = J\frac{d\omega_m}{dt} + T_{load} \tag{6.49}$$

where

J = moment of inertia of the rotor and the prime mover

T_{load} = mechanical load torque connected to the motor shaft

Substituting the expression for the developed electromagnetic torque given in Eq. 6.44 into Eq. 6.49 and rearranging the terms yields

$$-\left(\frac{K_m}{J}\right)i_a + \frac{d\omega_m}{dt} = -\frac{T_{load}}{J} \tag{6.50}$$

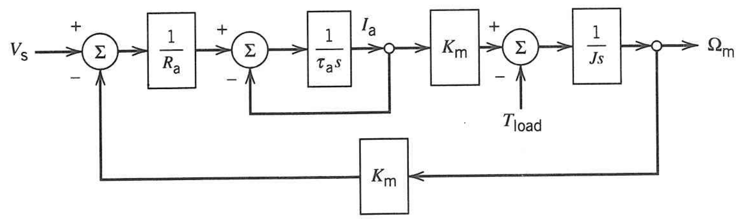

FIGURE 6.31 Block diagram representation of a DC motor.

The block diagram representation of a separately excited DC motor is derived from Eqs. 6.48 and 6.50 and is shown in Fig. 6.31. It may be noted that there are two independent input variables, namely the supply voltage V_s and the load torque T_{load}.

When the response of the motor to changes in the supply voltage is being investigated, the block diagram may be simplified to that of Fig. 6.32. The load torque is typically a function of speed, and it is customary to assume that the load torque is directly proportional to the speed; thus

$$T_{load} = B_{load}\omega_m \tag{6.51}$$

where B_{load} is a proportionality constant.

When the damping torque $T_{load} = B_{load}\omega_m$ is neglected, Fig. 6.32 may be simplified into one simple feedback loop circuit as shown in Fig. 6.33, where $\tau_m = (R_a J)/K_m^2$ is called the inertial time constant.

The overall transfer function may be expressed as follows:

$$\frac{\Omega_m}{V_s/K_m} = \frac{1/(\tau_m\tau_a)}{s(s + 1/\tau_a) + 1/(\tau_m\tau_a)} \tag{6.52}$$

From this overall transfer function, the characteristic equation of the speed response to the voltage input is found as

$$\left(s + \frac{1}{\tau_a}\right)s + \frac{1}{\tau_m\tau_a} = s^2 + s\left(\frac{1}{\tau_a}\right) + \frac{1}{\tau_m\tau_a} = 0 \tag{6.53}$$

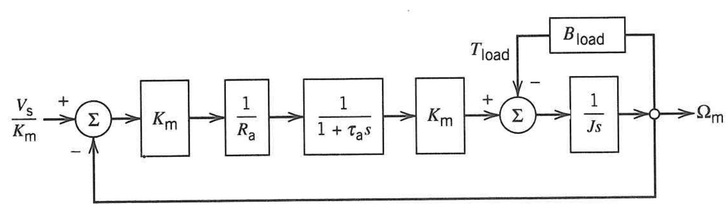

FIGURE 6.32 Block diagram representation of a DC motor.

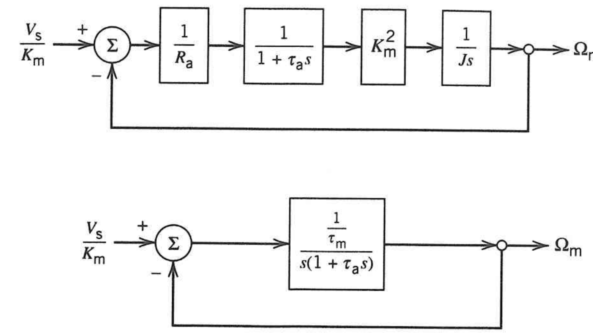

FIGURE 6.33 Simplified block diagrams of a DC motor.

The standard form of the characteristic equation of a second-order system is given by

$$s^2 + 2\alpha s + \omega_n^2 = 0 \tag{6.54}$$

Comparing Eqs. 6.53 and 6.54, the undamped natural frequency ω_n is

$$\omega_n = \sqrt{\frac{1}{\tau_m\tau_a}} \tag{6.55}$$

and the damping factor α is

$$\alpha = \frac{1}{2\tau_a} \tag{6.56}$$

The damping ratio is given by

$$\zeta = \frac{\alpha}{\omega_n} = \frac{1}{2}\sqrt{\frac{\tau_m}{\tau_a}} \tag{6.57}$$

When the response of the motor to load changes is of interest, the block diagram of Fig. 6.31 may be simplified to that shown in Fig. 6.34.

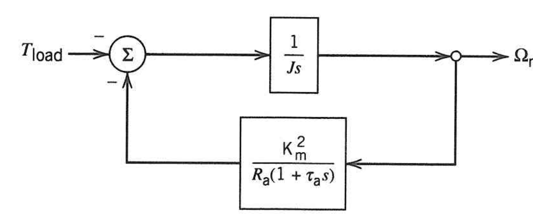

FIGURE 6.34 Simplified block diagram of a DC motor.

The overall transfer function relating the speed response to a change in load torque is

$$\frac{\Omega_m}{T_{load}/J} = -\frac{s + 1/\tau_a}{s(s + 1/\tau_a) + 1/(\tau_m \tau_a)} \tag{6.58}$$

It is seen that the characteristic equation for the speed response with change in load torque has the same undamped natural frequency and damping factor, which are given in Eqs. 6.55 and 6.56, respectively. The negative sign indicates that an additional load torque produces a reduction of speed.

EXAMPLE 6.11

A 400-V, separately excited DC motor has the following parameters:

$$\begin{aligned} \text{Armature resistance} \quad R_a &= 0.025\ \Omega \\ \text{Armature inductance} \quad L_{aa} &= 0.006\ \text{H} \\ \text{Moment of inertia} \quad J &= 25\ \text{kg-m}^2 \\ \text{Motor constant} \quad K_m &= 2.75\ \text{N-m/A} \end{aligned}$$

The motor is connected to a constant 400-V supply. The motor is initially running without load, and the system is at steady state. The no-load armature current is 15 A. The effects of saturation and armature reaction may be neglected.

A constant load torque T_{load} of 1500 N-m is suddenly connected to the shaft of the DC motor. Determine

a. The undamped natural frequency of the speed response
b. The damping factor and damping ratio
c. The initial speed in rpm
d. The initial acceleration
e. The ultimate speed drop

Solution

a. The armature and inertial time constants are given by

$$\tau_a = \frac{L_{aa}}{R_a} = \frac{0.006}{0.025} = 0.24\ \text{s}$$

$$\tau_m = \frac{JR_a}{K_m^2} = \frac{(25)(0.025)}{(2.75)^2} = 0.083\ \text{s}$$

The undamped natural frequency is

$$\omega_n = \sqrt{\frac{1}{\tau_m \tau_a}} = \sqrt{\frac{1}{(0.083)(0.24)}} = 7.10\ \text{rad/s}$$

b. The damping factor and damping ratio are

$$\alpha = \frac{1}{2\tau_a} = \frac{1}{(2)(0.24)} = 2.083$$

$$\zeta = \frac{\alpha}{\omega_n} = \frac{2.08}{7.10} = 0.29$$

c. At time $t = 0$, at the instant the load is suddenly added, the induced emf, or counter emf, is given by

$$E_{a0} = V_s - I_{a0}R_a = 400 - (15)(0.025) = 399.6\ \text{V}$$

Therefore, the initial motor speed is

$$\omega_m = \frac{E_{a0}}{K_m} = \frac{399.6}{2.75} = 145.3\ \text{rad/s}$$

$$= (145.3)\left(\frac{60}{2\pi}\right) = 1388\ \text{rpm}$$

d. The initial acceleration, assuming losses are negligible, is found by using Eq. 6.50. Thus,

$$\alpha_m = \frac{d\omega_m}{dt} = \frac{-T_{load} + K_m I_{a0}}{J}$$

$$= \frac{-1500 + (2.75)(15)}{25} = -58.35\ \text{rad/s}^2$$

e. The ultimate drop in speed is found by application of the final-value theorem of Laplace transforms to Eq. 6.58. Thus,

$$\Delta\omega_m = \lim_{s \to 0}\left[s \frac{-(1/J)(s + 1/\tau_a)}{s(s + 1/\tau_a) + 1/(\tau_m \tau_a)} \frac{\Delta T_{load}}{s}\right]$$

$$= -\frac{\tau_m}{J}\Delta T_{load} = -\frac{0.083}{25}(1500) = -4.98\ \text{rad/s}$$

$$= -4.98\left(\frac{60}{2\pi}\right) = -47.6\ \text{rpm}$$

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PROBLEMS

6.1 A six-pole DC machine has an armature connected as a lap winding. The armature has 48 slots with four conductors per slot. The armature is rotated at 600 rpm, and the flux per pole is 30 mWb. Calculate the induced voltage.

6.2 A four-pole DC generator has a wave-wound armature containing 384 armature conductors. The generator is driven at 1180 rpm and generates a voltage of 480 V. What is the flux per pole?

6.3 A six-pole, 1200-rpm, DC generator has 48 armature slots with four conductors per slot. The flux per pole is 20 mWb, and each armature conductor has a maximum current-carrying capacity of 40 A.

a. Compute the induced voltage, current, and power rating for a lap winding.

b. Repeat part (a) for a wave winding.

6.4 A DC generator has a flux per pole of 125 mWb. The generator has six poles, and it is rotated at 1200 rpm.

a. Determine the induced voltage if the armature has 48 conductors connected as a lap winding.

b. Determine the induced voltage if the armature has 48 conductors connected as a wave winding.

6.5 A shunt-connected generator has four poles, and its lap-connected armature has 576 conductors. The armature and field resistances are 0.10 Ω and 100 Ω, respectively. The flux per pole is 30 mWb. The generator supplies 3.5 kW to a load connected to its terminals at a voltage of 120 V. Determine the generator speed.

6.6 A separately excited generator has six poles with a flux per pole of 25 mWb. The armature is lap wound and has 620 conductors. The generator supplies a certain load at 240 V, and at this load the armature copper loss is 600 W. The generator is driven at a speed of 1120 rpm. Assuming that the total brush contact voltage drop is 2 V, calculate the current and power delivered by the generator.

6.7 A 50-kW, 240-V DC shunt generator has an armature resistance of 0.10 Ω, a field circuit resistance of 120 Ω, and a total brush voltage drop of 2 V. The generator delivers rated current at rated speed and rated voltage. Calculate the following:

a. Load current

b. Field current

c. Armature current

d. Armature induced voltage

6.8 A 10-kW, 120-V DC shunt generator has an armature circuit resistance of 0.2 Ω and a field circuit resistance of 240 Ω. The generator delivers rated current at rated voltage and rated speed. Assume a total brush voltage drop of 2 V. Calculate (a) the armature current and (b) the armature induced voltage.

6.9 A compound generator has armature, shunt field, and series field winding resistances of 0.2 Ω, 200 Ω, 0.1 Ω, respectively. The generator induced voltage is 255 V, and the terminal voltage is 240 V. The generator is connected for long-shunt compound operation.

a. Calculate the power supplied to a load.

b. Repeat part (a) if the generator is reconnected for short-shunt compound operation.

6.10 The shunt field current of a 60-kW, 125-V, DC generator has to be increased from 4.0 A on no load to 5.0 A at full load to produce zero regulation. Each field pole has 1500 turns.

a. Calculate the number of series field turns per pole to produce flat compounding, assuming short-shunt connection.

b. Repeat part (a) assuming long-shunt connection.

6.11 The open-circuit characteristic of a DC shunt generator is given by

E_a (V)	60	120	170	210	240	265	285
I_f (A)	1	2	3	4	5	6	7

The generator operates at a speed of 1200 rpm.

a. Plot the open-circuit characteristic.

b. From the open-circuit characteristic, find the maximum field-circuit resistance in order that the self-excited shunt generator will build up.

c. Determine the value the no-load voltage will build up to for a field-circuit resistance of 48 Ω and a speed of 1200 rpm.

6.12 A separately excited DC generator has the following open-circuit characteristic when driven at 1200 rpm.

E_a (V)	100	200	300	400	500	600	700
I_f (A)	0.50	1.00	1.75	2.75	4.00	5.50	7.50

The field winding resistance is 120 Ω, and the armature winding resistance is 0.2 Ω. Assume a total brush voltage drop of 4 V. Armature reaction effects may be neglected. The machine is driven at 1200 rpm as a self-excited generator.

a. Determine the terminal voltage on no load.

b. At full load when the armature winding current is 80 A, determine the terminal voltage.

6.13 A DC machine is rated 10 kW, 250 V, 1750 rpm and has armature and field winding resistances of 0.2 Ω and 125 Ω, respectively. The machine is self-excited and is driven at 1750 rpm. The data for the magnetization curve are

E_a (V)	15	50	100	150	188	212	250	275	305	330
I_f (A)	0	0.1	0.2	0.3	0.4	0.5	0.75	1.0	1.5	2.0

a. Determine the generated voltage with no field current.

b. Determine the critical field circuit resistance.

c. Determine the resistance of the field rheostat if the no-load terminal voltage is 250 V.

d. Determine the value of the no-load generated voltage if the generator is driven at 1326 rpm and the rheostat is short-circuited.

e. Determine the speed at which the generator is to be driven such that the no-load generated voltage is 200 V with the rheostat short-circuited.

6.14 The self-excited DC machine in Problem 6.13 delivers rated load when driven at 1750 rpm. The rotational loss is 450 W. Neglect the effects of armature reaction. Calculate

a. The generated voltage

b. The developed torque

c. The field current

d. The efficiency of the generator

6.15 A DC shunt generator is rated 20 kW, 220 V, and 1800 rpm. It has armature and field winding resistances of 0.1 Ω and 110 Ω, respectively. The data for the magnetization curve at 1800 rpm are

E_a (V)	5	25	60	120	170	200	215	225	240	253
I_f (A)	0.0	0.1	0.25	0.5	0.75	1.0	1.25	1.5	2.0	2.3

The machine is connected as a self-excited generator.

a. Determine the maximum generated voltage.

b. The generator delivers full-load current at rated voltage with a field current of 2 A. Determine the resistance of the field rheostat.

c. Determine the electromagnetic power and torque developed at full-load condition.

d. Determine the armature reaction effect in equivalent field amperes (I_f^{ar}) at full load.

6.16 The shunt generator of Problem 6.15 is connected as a long-shunt compound generator.

a. Draw a schematic diagram for the generator connection.

b. Determine the number of turns per pole of the series field winding needed to make the no-load and full-load terminal voltages equal to the rated voltage. The series field winding resistance is 0.05 Ω, and the shunt field winding has N_f = 1200 turns/pole. Assume armature reaction effect as in Problem 6.15d.

6.17 A 220-V DC shunt motor has armature and field winding resistances of 0.15 Ω and 110 Ω, respectively. The motor draws a line current of 5 A while running on no load. When driving a load, the motor runs at 1100 rpm and draws 48 A of line current. Calculate the no-load speed.

6.18 A 10-hp, 230-V, 1150-rpm, four-pole, DC shunt motor has a total of 596 conductors arranged in a wave winding having two parallel paths. The armature circuit resistance is 0.15 Ω. When the motor delivers rated output power at rated speed, the motor draws a line current of 38 A and a field current of 2 A. Assume that the effects of armature reaction are negligible. Compute (a) the flux per pole and (b) the developed torque.

6.19 A 10-hp, 230-V, 1200-rpm DC series motor draws a current of 36 A when delivering rated output at its rated speed. The armature and series field winding resistances are 0.20 Ω and 0.1 Ω, respectively. Assume that the magnetization curve is linear and that the effects of armature reaction are negligible.

a. Find the speed of this motor when it is taking a current of 24 A.

b. Determine the developed torque of this motor for the load conditions of part (a).

6.20 A 20-hp, 250-V, 1100-rpm DC shunt motor drives a load that requires a constant torque regardless of the speed of operation. The armature circuit resistance is 0.10 Ω. When this motor delivers rated power, the armature current is 65 A.

- **a.** If the flux is reduced to 75% of its original value, find the new value of armature current.
- **b.** What is the new speed for the conditions of part (a)?

6.21 A shunt motor develops a total torque of 250 N-m at rated load. The field flux decreases by 15%, and at the same time the armature current increases by 40%. Calculate the new value of torque.

6.22 A 220-V DC shunt motor has an armature resistance of 0.2 Ω and a rated armature current of 40 A. Assume a total brush voltage drop of 3 V. Calculate

- **a.** The generated voltage
- **b.** The power developed by the armature in watts
- **c.** The mechanical power developed in horsepower

6.23 A 120-V shunt motor has armature and field winding resistances of 0.10 Ω and 120 Ω, respectively, and a total brush voltage drop of 2 V. The motor operates at rated load and draws a line current of 41 A at a speed of 200 rad/s. Calculate

- **a.** The field current and armature current
- **b.** The counter emf
- **c.** The developed power in kW
- **d.** The developed torque in N-m

6.24 A 25-hp, 240-V shunt motor has an armature resistance of 0.20 Ω and a brush voltage drop of 4 V. The field circuit resistance is 120 Ω. At no load, the motor draws 14 A and has a speed of 1700 rpm. At full load, the motor draws 82 A. Calculate

- **a.** The motor speed at full load
- **b.** The speed regulation
- **c.** The mechanical power developed at full load

6.25 A four-pole DC motor has a flux per pole of 10 mWb. The armature has 600 conductors, which are connected so that there are four parallel paths between the brushes. The armature resistance is 0.50 Ω, including the effect of brush drop. The machine is connected to a source of 120 V, and the machine is loaded so that it takes rated armature current of 50 amperes. Neglect the effect of armature reaction. Calculate

- **a.** The motor speed
- **b.** The torque developed
- **c.** The speed regulation of the motor

6.26 In the DC motor of Problem 6.25, the effect of armature reaction, which was previously ignored, is to reduce the main field flux by 15% under the load of 50 A. All other data remain the same as in Problem 6.25. Determine the speed and horsepower developed by the motor.

6.27 A 10-hp, 230-V series motor has a line current of 37 A and a rated speed of 1200 rpm. The armature and series field resistances are 0.4 Ω and 0.2 Ω, respectively. The total brush voltage drop is 2 V. Calculate the following:

- **a.** Speed at a line current of 20 A
- **b.** No-load speed when the line current is 1 A
- **c.** Speed when the line current is 60 A and the series field flux is 125% of the full-load flux.

6.28 A DC series motor is rated 230 V, 12 hp, and 1200 rpm. It is connected to a 230-V supply, and it draws a current of 40 A while rotating at 1200 rpm. The armature and series field winding resistances are 0.25 Ω and 0.1 Ω, respectively.

- **a.** Determine the power and torque developed by the motor.
- **b.** Determine the speed, torque, and power if the motor draws 20 amperes.

6.29 A 40-hp, 230-V DC shunt motor has armature and field winding resistances of 0.2 Ω and 115 Ω, respectively. At no-load and rated voltage, the speed is 1200 rpm and the motor draws a line current of 5 A. If load is applied to the motor, its speed drops to 1100 rpm. Assume that the effects of armature reaction are negligible. At this load, determine

- **a.** The armature current and the line current
- **b.** The developed torque
- **c.** The horsepower output assuming the rotational losses are constant at 350 W.

6.30 A 250-V shunt motor delivers 15 kW of power at the shaft at 1200 rpm while drawing a line current of 75 A. The field and armature resistances are 250 Ω and 0.10 Ω, respectively. Assuming a contact voltage drop per brush of 1 V, calculate (a) the torque developed by the motor and (b) the motor efficiency.

6.31 A 220-V DC shunt motor has armature and field winding resistances of 0.15 Ω and 110 Ω, respectively. The motor draws a line current of 5 A while running at 1200 rpm on no load. When driving a load, the input to the motor is 12 kW. Calculate

- **a.** The speed of the motor
- **b.** The developed torque
- **c.** The efficiency of the motor at this load

6.32 A 230-V, DC shunt motor has an armature circuit resistance of 0.25 Ω and a field circuit resistance of 115 Ω. At full load the armature draws a current of 38 A and the speed is measured at 1050 rpm. Neglect saturation.

- **a.** Find the developed torque in newton-meters.
- **b.** The field rheostat is adjusted so that the resulting field circuit resistance is 144 Ω. Find the new operating speed assuming the developed torque and armature current remain constant.
- **c.** Assuming that rotational losses amount to 600 W, calculate the efficiency in part (b).

6.33 A 30-hp, 1150-rpm, six-pole shunt motor has an efficiency of 87% when operating at rated load from a 240-V supply. The motor has armature and field resistances of 0.10 Ω and 120 Ω, respectively. Determine

a. The mechanical power developed

b. The developed torque

c. The output shaft torque

6.34 A 20-hp, 240-V, four-pole shunt motor operates at rated load and 1700 rpm and an efficiency of 88%. It has armature and field resistances of 0.2 Ω and 240 Ω, respectively. Determine

a. The power input to the motor

b. The mechanical power developed

c. The shaft torque

6.35 A DC shunt motor is rated 10 hp, 250 V and is connected to a 230-V source. The armature resistance is 0.2 Ω; the field circuit resistance is 125 Ω, and the total rotational losses are 380 W. The motor is driving a load, and it draws an armature current of 30 A while running at a speed of 1700 rpm. Calculate

a. The generated voltage

b. The value of the load torque

c. The efficiency of the motor

6.36 A 220-V shunt motor delivers 20 hp to a load connected to its shaft on full load at 1150 rpm. The motor full-load efficiency is 85%. The armature and field winding resistances are 0.15 Ω and 110 Ω, respectively. Determine

a. The starting resistance such that the starting line current does not exceed twice the full-load current

b. The starting torque with the starting resistance computed in part (a) inserted in the armature circuit

6.37 A 240-V, DC shunt motor has an armature winding resistance of 0.2 ohm. The full-load armature current is 50 A. Find the value of the resistance to be connected in series with the armature circuit so that the speed is 75% of the rated speed if full-load current flows.

6.38 An automatic starter is to be designed for a 10-hp, 230-V shunt motor. The resistance of the armature circuit is 0.20 Ω, and the resistance of the field circuit is 115 Ω. The field winding is connected directly across the 230-V source. When operated at no load and rated voltage, the armature current is 4 A and the motor runs at a speed of 1180 rpm. When the motor is delivering rated output, the armature current is 37 A. The resistance in series with the armature is to be adjusted automatically so that during the starting period the armature current is allowed to rise up to but not exceed twice the rated value. As soon as the current falls to rated value, sufficient series resistance is to be cut out (short-circuited) to allow the current to increase once more. This process is repeated until all the series resistance has been cut out.

a. Determine the total resistance of the starter.

b. Calculate the resistance that should be cut out at each step in the starting process.

6.39 A 20-kW, 220-V, 1800-rpm shunt motor has a total armature circuit resistance of 0.4 ohm. The resistance of the shunt field circuit is 220 ohms. At starting, the maximum allowable armature current is twice the rated value. Determine the steps of the starting resistor and the speed attained by the armature at each step. Neglect the effect of armature reaction.

***6.40** A DC shunt generator is shown in Fig. 6.35. The generator is driven at constant speed. The armature and field winding resistances are 0.2 Ω and 50 Ω, respectively, and the armature and field winding inductances are 10 mH and 25 H, respectively. The generator terminal voltage is passed through a low-pass filter with R = 1 Ω and L = 1 H. Determine the transfer function that relates the output voltage V_o to the input voltage V_i. The generator constant is K_g = 120 V per field ampere.

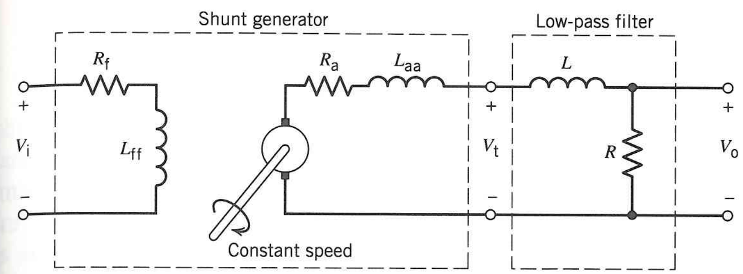

FIGURE 6.35 Shunt generator of Problem 6.40.

***6.41** A separately excited DC motor is shown in Fig. 6.36. The motor drives a mechanical load connected to its shaft. The motor is operated with constant field current. The armature resistance is 0.5 Ω, and the armature inductance is negligible. The other parameters of the motor are K_m = 2 V/(rad/s) and K_t = 1.5 N-m/A. The equivalent inertia of the mechanical load and the motor is J = 10 kg-m^2.

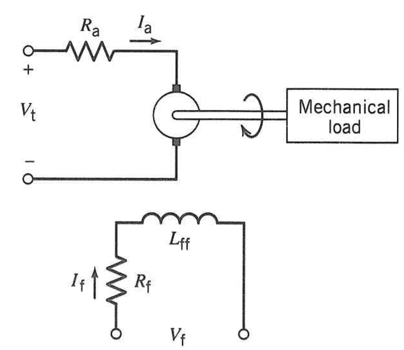

FIGURE 6.36 DC motor of Problem 6.41.

a. Show the block diagram representation of the system.

b. Find the transfer function that relates the speed in rad/s to the voltage applied to the armature circuit.

c. Let the applied voltage be $V_t = 200u(t)$, where $u(t)$ is the unit step function. Find the expression for the motor speed as a function of time t.

d. Find the steady-state speed of the motor.

Seven

Synchronous Machines

7.1 INTRODUCTION

A synchronous machine is an AC machine in which alternating current flows in the armature windings and DC excitation is supplied to the field winding. The armature windings are designed to carry large currents at high voltages; therefore, they are located on the stator. The field winding is excited by smaller currents at a lower voltage; thus, it is placed on the rotor.

A synchronous machine can be operated as a generator or as a motor, just like other rotating machines. However, it is different from the others in that its operating speed is constant. This speed is called *synchronous speed*. The synchronous speed is related to the frequency of the stator currents and the number of magnetic poles of the rotor. This relationship is expressed in the following equation (See Chapter 5, Section 5.3.2):

$$pn_s = 120f \tag{7.1}$$

where

n_s = synchronous speed

p = number of poles

f = frequency

In the United States, the frequency is fixed at 60 Hz. Therefore, the type of rotor and the required number of poles are basically dependent on the particular application, that is, on the speed rating. Thus, for high-speed turbogenerators that operate at either 1800 rpm or 3600 rpm, the number of poles required is four poles or two poles, respectively. Because of their high speeds, these synchronous machines have nonsalient, or round, or cylindrical, rotors. A cylindrical-rotor synchronous machine is illustrated in Fig. 7.1.

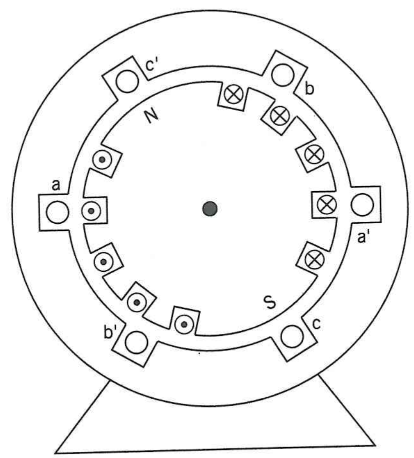

FIGURE 7.1 Cylindrical-rotor synchronous machine.

On the other hand, low-speed synchronous machines like those in hydroelectric power plants have several pairs of poles and allow the use of salient-pole, or projecting-pole, rotors. A salient-pole synchronous machine is illustrated in Fig. 7.2.

DRILL PROBLEMS

D7.1 At what speed must a six-pole synchronous generator be run to generate 50-Hz voltage?

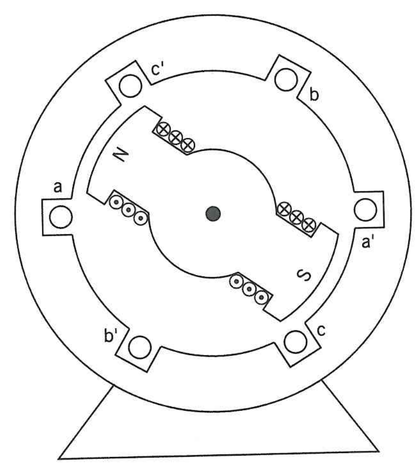

FIGURE 7.2 Salient-pole rotor synchronous machine.

D7.2 Determine the number of poles required for an alternator driven by a prime mover having a speed of 1200 rpm to generate AC at a frequency of 60 Hz.

D7.3 Determine the frequency required to operate a 16-pole, 600-V synchronous motor at 375 rpm.

7.2 ROUND-ROTOR SYNCHRONOUS MACHINES

When the cylindrical-rotor synchronous machine shown in Fig. 7.1 is operated as a generator, the induced voltages expressed in phasor form, with the phase a voltage chosen as reference phasor, are given by Eqs. 7.2, 7.3, and 7.4 for phases a, b, and c, respectively.

$$\mathbf{E}_{an} = E_{rms}\underline{/0^\circ} \quad (7.2)$$

$$\mathbf{E}_{bn} = E_{rms}\underline{/-120^\circ} \quad (7.3)$$

$$\mathbf{E}_{cn} = E_{rms}\underline{/-240^\circ} \quad (7.4)$$

Equations 7.2–7.4 give the voltages that can be measured at the generator terminals when the stator windings are open. These voltages are due to the flux produced by the rotor or field current.

The expression for phase voltage E_{rms} has been derived in Chapter 5 and is rewritten here as Eq. 7.5.

$$E_{rms} = 4.44 K_w f N_a \Phi_p \quad (7.5)$$

where

K_w = machine stator winding factor
f = frequency
N_a = stator winding number of turns per phase
Φ_p = flux per pole

DRILL PROBLEMS

D7.4 A three-phase, 60-Hz synchronous generator operating at no load has a generated voltage of 620 V at rated frequency. If the pole flux is decreased by 15% and the speed is increased by 10%, determine (a) the induced voltage and (b) the frequency.

D7.5 A three-phase, eight-pole, 900-rpm, wye-connected synchronous generator has 120 turns per phase and a stator winding factor of 0.90. A voltage of 2400 V is measured across the machine terminals on no load. Determine the flux per pole.

7.2.1 Equivalent Circuit of a Round-Rotor Machine

The voltage $\mathbf{E}_a$ is the internal generated voltage produced across one phase of the synchronous generator when its terminals are open, or at no-load conditions. However, this voltage $\mathbf{E}_a$ is not the voltage $\mathbf{V}_t$ that is measured at the terminals when the generator is supplying stator current to an electrical load.

When no load is connected to the generator terminals, the rotor magnetic field $\mathbf{F}_r$ induces the internal voltage $\mathbf{E}_a$. Since there is no stator current at no load, the terminal voltage $\mathbf{V}_t$ is equal to $\mathbf{E}_a$.

Consider a lagging power factor load connected to the generator terminals. A stator current $\mathbf{I}_a$ will flow that lags the internal voltage $\mathbf{E}_a$. This current flowing through the stator, or armature, windings produces a synchronously rotating field $\mathbf{F}_s$ at the same angular speed as the rotor magnetic field $\mathbf{F}_r$. The stator magnetic field induces a second voltage $\mathbf{E}_s$ in the stator windings. Because $\mathbf{E}_s$ has been produced by the armature current, it is called armature reaction voltage.

The two magnetic fields $\mathbf{F}_s$ and $\mathbf{F}_r$ are rotating at the same angular velocity. Hence, the respective induced voltages $\mathbf{E}_s$ and $\mathbf{E}_a$ have the same angular frequency. Therefore, they can be combined or added as phasors to give the resultant voltage $\mathbf{E}_t$.

$$\mathbf{E}_t = \mathbf{E}_a + \mathbf{E}_s \tag{7.6}$$

This resultant voltage can also be thought of as the internal voltage induced by the net magnetic field $\mathbf{F}_t$ in the air gap, which is the sum of the stator and rotor magnetic fields:

$$\mathbf{F}_t = \mathbf{F}_r + \mathbf{F}_s \tag{7.7}$$

The phasor voltages and currents and the various magnetic fields are illustrated in Fig. 7.3.

The armature reaction voltage $\mathbf{E}_{ar}$ is directly proportional to the amount of stator current flowing. It is 90° behind the stator current $\mathbf{I}_a$. Thus, $\mathbf{E}_{ar}$ may be expressed as a voltage drop:

$$\mathbf{E}_{ar} = -jX_{ar}\mathbf{I}_a \tag{7.8}$$

where X_{ar} is a proportionality constant. Therefore, the net or resultant voltage may be expressed as follows.

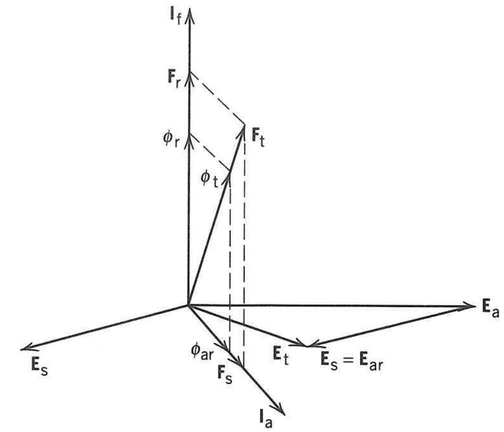

FIGURE 7.3 Synchronous generator voltages and magnetic fields.

$$\mathbf{E}_t = \mathbf{E}_a - jX_{ar}\mathbf{I}_a \tag{7.9}$$

Equation 7.9 may be recognized as Kirchhoff's voltage equation for the circuit of Fig. 7.4.

In an actual physical synchronous generator, the net or resultant magnetic field present in the air gap is realistically not linked completely by the stator windings. The portion of the magnetic flux that does not link the windings is referred to as the leakage flux ϕ_{al}. This leakage flux leads to a voltage drop across what is called leakage reactance X_{al}. In addition, the stator windings inherently contain resistances that give rise to an armature resistance drop. Thus, the overall equivalent circuit of the synchronous generator may be presented as shown in Fig. 7.5.

It is customary to add the leakage reactance to the reactance due to armature reaction to form what is referred to as *synchronous reactance* X_s. Thus,

$$X_s = X_{ar} + X_{al} \tag{7.10}$$

Finally, the equivalent circuit of the synchronous generator is presented in Fig. 7.6. This equivalent circuit is on a per-phase basis; the voltages are given

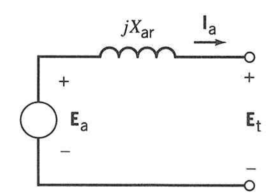

FIGURE 7.4 Internal equivalent circuit.

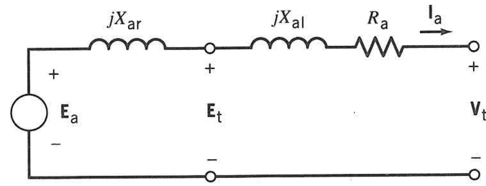

FIGURE 7.5 Overall equivalent circuit.

in line-to-neutral volts, and the resistance and reactance are in ohms per phase. In the per-phase representation, the line current is equal to the phase current.

The phasor diagram illustrating the relationships among the different phasors is shown in Fig. 7.7 for a synchronous generator supplying a lagging power factor load.

DRILL PROBLEMS

D7.6 A three-phase, 10-kVA, 208-V, four-pole, wye-connected synchronous generator has a synchronous reactance of 2 Ω per phase and negligible armature resistance. The generator is connected to a three-phase, 208-V infinite bus. Neglect rotational losses.

a. The field current and the mechanical input power are adjusted so that the synchronous generator delivers 6 kW at 0.85 lagging power factor. Determine the excitation voltage and the angle δ.

b. The mechanical input power is kept constant but the field current is adjusted to make the power factor unity. Determine the percent change in the field current with respect to its value in part (a).

D7.7 A three-phase, 500-kVA, 12-kV, wye-connected synchronous generator has an armature resistance of 1.5 Ω and a synchronous reactance of 36 Ω. At a certain field current, the generator delivers rated kVA to a load at 12 kV and 0.866 lagging power factor. For the same field excitation and the same kVA load but at a leading power factor of 0.9, what will the terminal voltage be?

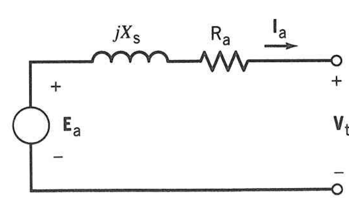

FIGURE 7.6 Equivalent circuit of a synchronous generator.

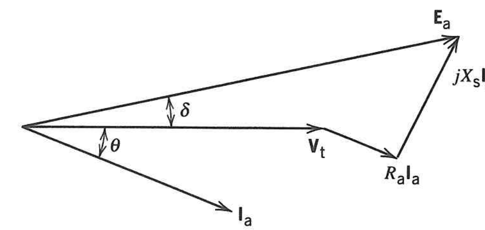

FIGURE 7.7 Phasor diagram for a round-rotor synchronous generator.

7.2.2 Open-Circuit and Short-Circuit Characteristics

Two basic characteristics of the synchronous machine are of interest. These are the open-circuit characteristic and the short-circuit characteristic, and they are discussed ahead.

Open-Circuit Characteristic The relationship described by Eq. 7.5 is plotted in Fig. 7.8. This plot is referred to as the magnetization curve, or saturation curve, or *open-circuit characteristic (OCC)* of the synchronous machine.

The open-circuit characteristic is derived experimentally by driving the synchronous machine at synchronous speed and measuring the terminal voltage (line-to-line) on no load, or at open circuit, for various values of field current. The straight line drawn tangent to the lower portion of the OCC is called the air-gap line; it gives the value of generated voltage if saturation is not present.

Short-Circuit Characteristic Consider a synchronous generator initially operating at steady state and rated voltage at open circuit. Suppose a short circuit is suddenly applied to its terminals. A transient condition would ensue, and the stator current would rise to a high value. After a while the transients would

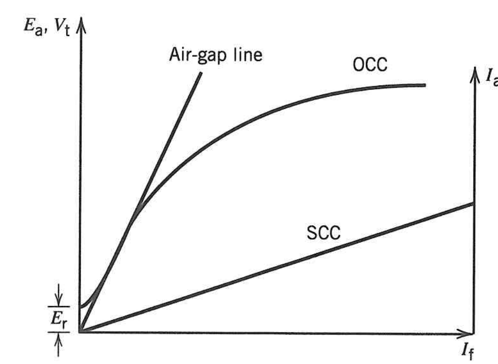

FIGURE 7.8 Open- and short-circuit characteristics.

die down and the stator short-circuit current would settle to a new steady-state value.

If readings of short-circuit current are taken and plotted for different values of field current, the plot described is called the *short-circuit characteristic (SCC)* of the synchronous machine. A typical SCC exhibiting its linear feature is shown in Fig. 7.8 together with the OCC.

Calculation of Synchronous Reactance from OCC and SCC The open-circuit and short-circuit characteristics of the synchronous machine can be used to determine the value of its synchronous reactance. It is seen from Fig. 7.9 that at short-circuit conditions, the stator current is

$$\mathbf{I}_{a,sc} = \frac{\mathbf{E}_a}{R_a + jX_s} \tag{7.11}$$

In most synchronous machines, the armature resistance is negligible compared to the synchronous reactance. If the value of R_a is set equal to zero, the value of X_s may be found from Eq. 7.11. This value of unsaturated synchronous reactance is found from Fig. 7.10 by reading the line-to-line voltage from the air-gap line and dividing by the current read from the SCC corresponding to the field current that produces the air-gap voltage; thus,

$$X_{s,unsat} = \frac{E_{a,ag}/\sqrt{3}}{I_{a,sc}} = \frac{V_{0a}/\sqrt{3}}{I_{0'b}} \tag{7.12}$$

The saturated synchronous reactance may also be found from Fig. 7.10 by taking the rated terminal voltage (line-to-line) measured on the OCC and dividing by the current read from the SCC corresponding to the field current that produces rated terminal voltage. Thus,

$$X_{s,sat} = \frac{V_{t,rated}/\sqrt{3}}{I_{a,sc}} = \frac{V_{0c}/\sqrt{3}}{I_{0'b}} \tag{7.13}$$

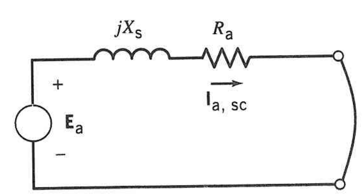

FIGURE 7.9 Short-circuited synchronous generator.

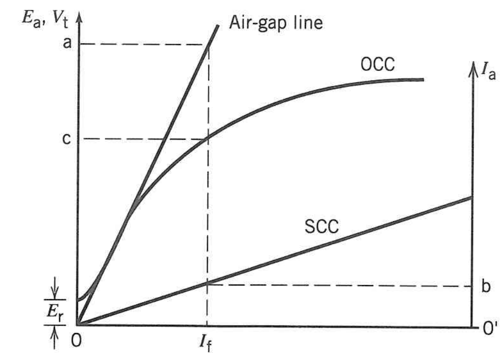

FIGURE 7.10 Calculation of synchronous reactance from open-and short-circuit characteristics.

EXAMPLE 7.1

A three-phase, wye-connected, two-pole synchronous generator is rated at 300 kVA, 480 V, 60 Hz, and 0.8 PF lagging. The open- and short-circuit characteristics are given in Table 7.1.

a. Determine the unsaturated synchronous reactance.

b. Determine the saturated synchronous reactance at the rated conditions.

Solution The open-circuit and short-circuit characteristics of this machine are plotted in Fig. 7.11.

a. The unsaturated synchronous reactance is found by using the line-to-line voltage measured from the air-gap line and the short-circuit current

Table 7.1 Generator Characteristics of Example 7.1

I_f (A)	OCC (V_{LL})	AG line (V_{LL})	SCC (A)
1.0	120		
2.0	240		
3.0	340		
4.0	430		
5.0	480	600	360
6.0	520		
7.0	540		
8.0	550		
9.0	555		
10.0	560		

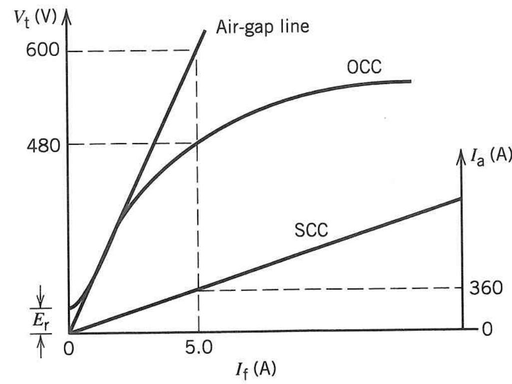

FIGURE 7.11 Open- and short-circuit characteristics of machine of Example 7.1.

corresponding to a field current I_f = 5.0 A; thus,

$$X_{s,\text{unsat}} = (600/\sqrt{3})/360 = 0.962\ \Omega \text{ per phase}$$

b. The saturated synchronous reactance is found by using the rated terminal voltage (line-to-line) as measured from the OCC and the short-circuit current corresponding to a field current I_f = 5.0 A; thus,

$$X_{s,\text{sat}} = (480/\sqrt{3})/360 = 0.770\ \Omega \text{ per phase}$$

7.2.3 Voltage Regulation

Just as in transformers and DC machines, a measure of the performance of a synchronous generator is its voltage regulation, which is defined as

$$\text{Voltage regulation} = \frac{V_{nl} - V_{fl}}{V_{fl}} 100\% \qquad (7.14)$$

where

V_{nl} = voltage at open-circuit, or no-load condition

V_{fl} = voltage at rated, or full-load, condition

The full-load voltage V_{fl} is the same as the terminal voltage V_t, and V_{nl} is equal to the corresponding generated voltage E_a. Thus, voltage regulation may also be expressed as

$$\text{Voltage regulation} = \frac{E_a - V_t}{V_t} 100\% \qquad (7.15)$$

EXAMPLE 7.2

Calculate the percent voltage regulation for a three-phase, wye-connected, 20-MVA, 13.8-kV synchronous generator operating at full-load, or rated, conditions and 0.8 power factor lagging. The synchronous reactance is 8 Ω per phase, and the armature resistance can be neglected.

Solution The rated voltage of 13.8 kV is normally given as a line-to-line voltage. The per-phase terminal voltage is taken as reference phasor; thus,

$$\mathbf{V}_t = (13.8 \times 10^3/\sqrt{3})\angle 0^\circ = 7967\angle 0^\circ \text{ V} \quad \text{(line-to-neutral)}$$

At rated operating conditions and 0.8 power factor lagging, the stator current is found as

$$\mathbf{I}_a = \frac{S_{\text{rated}}}{\sqrt{3}\,V_{\text{rated}}} \angle -\cos^{-1}\text{PF}$$
$$= \frac{20 \times 10^3}{13.8\sqrt{3}} \angle -\cos^{-1} 0.8 = 836.7\angle -36.9^\circ \text{ A}$$

The generated voltage is computed as follows:

$$\mathbf{E}_a = \mathbf{V}_t + \mathbf{I}_a(R_a + jX_s)$$
$$= 7967\angle 0^\circ + (836.7\angle -36.9^\circ)(0 + j8)$$
$$= 13{,}125\angle 24.1^\circ \text{ V} \quad \text{(line-to-neutral)}$$

Since the stator current $\mathbf{I}_a$ = 0 at no load, $|\mathbf{V}_{nl}| = |\mathbf{E}_a|$. Therefore, the percent voltage regulation is computed as follows:

$$\text{Voltage regulation} = \frac{E_a - V_t}{V_t} 100\%$$
$$= \frac{13{,}125 - 7967}{7967} 100\% = 64.7\%$$

DRILL PROBLEMS

D7.8 A three-phase, 1000-kVA, 12-kV, wye-connected synchronous generator supplies 750 kW at 12 kV and 0.8 lagging power factor load. The synchronous reactance is 30 Ω per phase, and the armature resistance is negligible. Calculate the voltage regulation.

D7.9 A three-phase, 1500-kVA, 13.2-kV, wye-connected synchronous generator has an armature resistance of 0.5 Ω and a synchronous reactance of 9.0 Ω. The generator is supplying rated load at rated voltage.

a. Calculate the generated voltage at unity power factor, at 0.8 PF lagging, and at 0.8 leading power factor.

b. Calculate voltage regulation for each of the loads specified for part (a).

7.2.4 Power-Angle Characteristic of a Round-Rotor Machine

The maximum average power that a synchronous machine can deliver is determined by the maximum mechanical torque that can be applied without loss of synchronism with the external system to which the synchronous machine is connected. In this section, an expression for the power supplied by the synchronous generator is derived in terms of the parameters of the machine equivalent circuit and of the system.

For the purpose of this analysis, the external system is represented by an inductive reactance X_e in series with an ideal voltage source $\mathbf{V}_e$. The synchronous machine is represented by its excitation voltage $\mathbf{E}_a$ in series with its synchronous reactance X_s. The armature resistance is assumed to be negligible. Figure 7.12 shows the per-phase circuit representation.

The per-phase terminal voltage is taken as the reference phasor:

$$\mathbf{V}_t = V_t\angle 0° \tag{7.16}$$

The generated voltage is expressed as

$$\mathbf{E}_a = E_a\angle\delta \tag{7.17}$$

The angle δ is called the *power angle*. For a synchronous generator, δ is always a positive angle; that is, the generated voltage leads the terminal voltage by the angle δ.

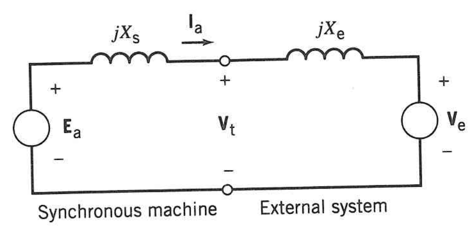

FIGURE 7.12 Synchronous generator connected to external system.

The stator current is found as

$$\mathbf{I}_a = \frac{\mathbf{E}_a - \mathbf{V}_t}{jX_s} \tag{7.18}$$

The complex power delivered by the synchronous generator to the external system is given by

$$\mathbf{S} = P + jQ = 3\mathbf{V}_t\mathbf{I}_a^* \tag{7.19}$$

Substituting Eq. 7.18 into Eq. 7.19, and simplifying, yields

$$\begin{aligned} P + jQ &= 3V_t\angle 0°\left(\frac{E_a\angle -\delta - V_t\angle 0°}{-jX_s}\right) \\ &= 3\left(\frac{E_aV_t}{X_s}\angle -\delta + 90° - j\frac{V_t^2}{X_s}\right) \end{aligned} \tag{7.20}$$

The real part and the imaginary part of Eq. 7.20 represent the three-phase real power P and the three-phase reactive power Q, respectively.

$$P = 3\frac{E_aV_t}{X_s}\sin\delta \tag{7.21}$$

$$Q = 3\left(\frac{E_aV_t}{X_s}\cos\delta - \frac{V_t^2}{X_s}\right) \tag{7.22}$$

Equation 7.21 is called the *power-angle characteristic* for the synchronous machine. Its plot is called the *power-angle curve*, and it is shown in Fig. 7.13.

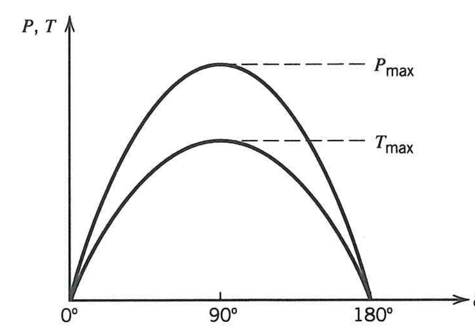

FIGURE 7.13 Power-angle curve and torque-angle curve.

The *maximum power* P_{max} delivered to the external system occurs when $\delta = 90°$; that is,

$$P_{max} = 3\frac{E_a V_t}{X_s} \tag{7.23}$$

The maximum torque T_{max} that can be applied to the shaft of the synchronous generator without stepping out of synchronism, also called the *pull-out torque*, is related to the maximum power P_{max} by

$$T_{max} = \frac{P_{max}}{\omega_m} \tag{7.24}$$

Equations 7.23 and 7.24 give the maximum power and maximum torque, respectively, that a synchronous generator can deliver before it steps out of synchronism. Another important parameter to consider is the synchronous generator MVA rating, which specifies the maximum power that the generator can deliver to an electrical load, continuously without overheating, at a specified voltage and power factor. Thus, the real power (MW) output of the generator is limited to a maximum value equal to its rated MVA at rated voltage and unity power factor.

A synchronous generator is said to be *overexcited* if it delivers, or supplies, reactive power to the load. It is said to be *underexcited* if it receives, or absorbs, reactive power. An overexcited synchronous generator operates at a lagging power factor. The underexcited generator operates at a leading power factor.

EXAMPLE 7.3

A 25-kVA, 230-V, three-phase, four-pole, 60-Hz, wye-connected synchronous generator has a synchronous reactance of 1.5 Ω/phase and negligible stator resistance. The generator is connected to an infinite bus (of constant voltage magnitude and constant frequency) at 230 V and 60 Hz.

a. Determine the excitation voltage $\mathbf{E}_a$ when the machine is delivering rated kVA at 0.8 power factor lagging.

b. The field excitation current I_f is increased by 20% without changing the power input from the prime mover. Find the stator current I_a, power factor, and reactive power Q supplied by the machine.

c. With the field excitation current I_f as in part (a), the input power from the prime mover is increased very slowly. What is the steady-state limit? Determine stator current $\mathbf{I}_a$, power factor, and reactive power Q.

Solution

a. The terminal voltage is taken as reference phasor. Thus,

$$\mathbf{V}_t = (230/\sqrt{3})\angle 0° = 132.8\angle 0°\ \text{V} \quad \text{(line-to-neutral)}$$

The stator current is obtained as follows:

$$\mathbf{I}_a = \frac{25,000}{230\sqrt{3}}\angle -\cos^{-1} 0.8 = 62.8\angle -36.9°\ \text{A}$$

The excitation voltage is calculated as follows:

$$\begin{aligned}\mathbf{E}_a &= \mathbf{V}_t + \mathbf{I}_a(R_a + jX_s)\\ &= 132.8\angle 0° + (62.8\angle -36.9°)(0 + j1.5)\\ &= 203.8\angle 21.7°\ \text{V} \quad \text{(line-to-neutral)}\end{aligned}$$

Therefore, the line-to-line excitation voltage magnitude is

$$E_a = 203.8\sqrt{3} = 353\ \text{V} \quad \text{(line-to-line)}$$

The power angle is the phase angle by which $\mathbf{E}_a$ leads $\mathbf{V}_t$, and it is given by

$$\delta = 21.7°$$

b. The excitation voltage magnitude is increased by 20%; that is,

$$E_a' = 1.20E_a = (1.2)(203.8) = 244.6\text{V} \quad \text{(line-to-neutral)}$$

Since the input power from the prime mover remains unchanged, $P' = P$. Therefore,

$$\begin{aligned}3(E_a'V_t/X_s)\sin\delta' &= 3(E_aV_t/X_s)\sin\delta\\ 244.6\sin\delta' &= 203.8\sin 21.7°\end{aligned}$$

Solving for the new power angle yields

$$\delta' = 17.9°$$

Hence, the stator current is calculated as follows:

$$\mathbf{I}_a = \frac{\mathbf{E}_a - \mathbf{V}_t}{jX_s} = \frac{244.6\angle 17.9^\circ - 132.8\angle 0^\circ}{j1.5} = 83.4\angle -53^\circ \text{ A}$$

The power factor is

$$PF = \cos 53^\circ = 0.60 \text{ lagging}$$

The reactive power is found as follows:

$$Q = 3V_t I_a \sin\theta = 3(132.8)(83.4)\sin 53^\circ = 26.5 \text{ kVAR}$$

Alternatively, the reactive power may be obtained by using Eq.7.22.

$$Q = 3\left[\frac{(244.6)(132.8)}{1.5}\cos 17.9^\circ - \frac{(132.8)^2}{1.5}\right] = 26.5 \text{ kVAR}$$

c. With the field excitation current as in part (a), the excitation voltage magnitude is

$$E_a = 203.8\text{V} \quad \text{(line-to-neutral)}$$

The steady-state limit is the maximum power P_{max} of the generator, and it occurs at $\delta = 90^\circ$. Therefore,

$$P_{max} = 3E_a V_t / X_s = 3(203.8)(132.8)/1.5 = 54.13 \text{ kW}$$

The maximum power condition is depicted in Fig. 7.14.

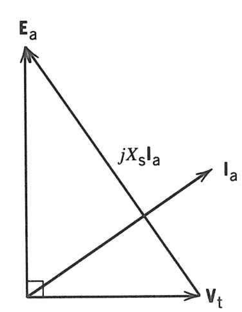

FIGURE 7.14 Phasor diagram at maximum power condition.

Based on the phasor diagram shown in Fig. 7.14, the stator current is computed as follows:

$$\mathbf{I}_a = \frac{\mathbf{E}_a - \mathbf{V}_t}{jX_s} = \frac{203.8\angle 90^\circ - 132.8\angle 0^\circ}{j1.5} = 162.2\angle 33.1^\circ \text{ A}$$

The power factor is given by

$$PF = \cos(\angle\mathbf{V}_t - \angle\mathbf{I}_a) = \cos(0^\circ - 33.1^\circ) = 0.84 \text{ leading}$$

The reactive power is given by

$$Q = 3V_t I_a \sin\theta = 3(132.8)(162.2)\sin(-33.1^\circ) = -35.3 \text{ kVAR}$$

Alternatively, the reactive power may be found by using Eq. 7.22.

$$Q = 3\left[\frac{(203.8)(132.8)}{1.5}\cos 90^\circ - \frac{(132.8)^2}{1.5}\right] = -35.3 \text{ kVAR}$$

DRILL PROBLEMS

D7.10 A three-phase, six-pole, 60-Hz synchronous generator has a synchronous reactance of 4 Ω per phase and a terminal voltage of 2300 V. The field current is adjusted so that the excitation voltage is 2300 V at a power angle of 15°. Find

a. Stator current
b. Power factor
c. Output power
d. Torque required to drive the machine

D7.11 A three-phase, 12-kV, 60-Hz, wye-connected generator has a synchronous reactance of 15 Ω per phase and negligible armature resistance. For a given field current, the open-circuit voltage is 13 kV.

a. Calculate the maximum power developed by the generator.
b. Determine the armature current and power factor for the maximum power condition.

7.2.5 Efficiency

A power flow diagram for a synchronous generator is shown in Fig. 7.15. The input consists of the mechanical power supplied to the machine shaft from the prime mover, and the output is the AC electrical power delivered to the load. The DC electrical power input to the field circuit for field excitation has not been included in Fig. 7.15 because this power is supplied from a separate DC source.

The losses of the synchronous generator are classified in the same manner as in DC machines. Copper losses are present in the three stator windings ($3I_a^2R_a$). Since the field circuit is excited from a separate DC source, the field winding copper losses ($I_f^2R_f$) are not included. Mechanical losses consist of bearing friction and windage losses. Hysteresis and eddy current losses constitute the core losses. Other unaccounted losses are grouped under stray losses and are usually assumed to be equal to 1% of the machine power output.

The efficiency of the synchronous generator is defined as

$$\begin{aligned}\eta &= \frac{P_{\text{output}}}{P_{\text{input}}}100\% \\ &= \frac{P_{\text{output}}}{P_{\text{output}} + \Sigma(\text{losses})}100\% \\ &= \frac{P_{\text{input}} - \Sigma(\text{losses})}{P_{\text{input}}}100\% \end{aligned} \quad (7.25)$$

7.3 SALIENT-POLE SYNCHRONOUS MACHINES

The salient-pole synchronous machine has a nonuniform air gap, as discussed in Chapter 5. Hence, the equivalent circuit and power-angle characteristics derived for nonsalient-pole, or round-rotor, machines have to be modified before they can be applied to salient-pole machines. These modifications are discussed in this section.

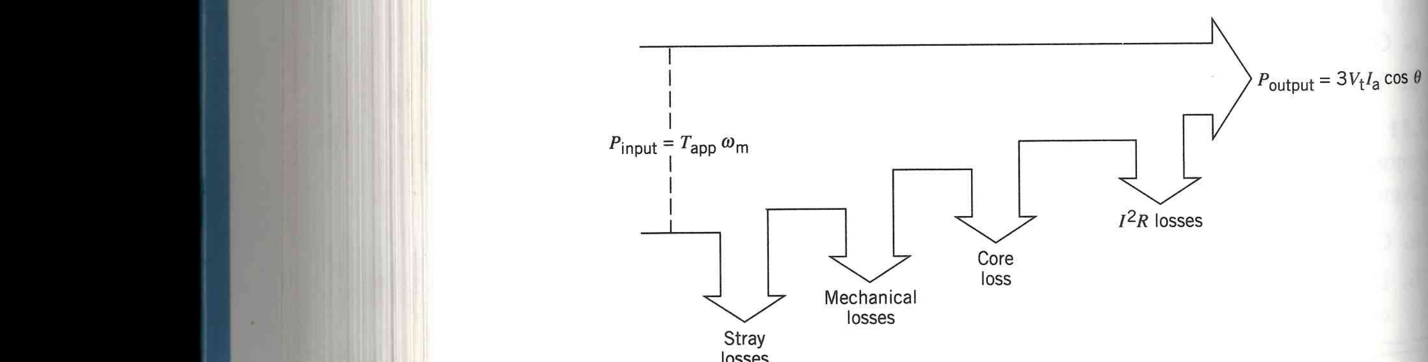

FIGURE 7.15 Power flow diagram for a synchronous generator.

7.3.1 Equivalent Circuit of a Salient-Pole Machine

Consider a salient-pole synchronous machine whose armature resistance can be assumed to be negligible. The flux path through the protruding poles is called the direct axis, and the flux path through the large air gap is called the quadrature axis. Because of the much shorter air gap between the stator and the salient poles, the flux along the direct axis encounters lower reluctance. The flux along the quadrature axis, on the other hand, encounters much higher reluctance. Hence the principles and formulas that have been derived for the round-rotor synchronous machine cannot be used for the salient-pole machine.

The two-reactance theory is used to describe the operation of salient-pole machines. It takes into account the difference in the reluctances in the direct-axis and quadrature-axis flux paths. The stator current $\mathbf{I}_a$ is resolved into two mutually perpendicular components: the direct-axis component $\mathbf{I}_d$ is along the axis of the rotor salient pole, and the quadrature-axis component $\mathbf{I}_q$ is in quadrature to $\mathbf{I}_d$. As shown in the phasor diagram of Fig. 7.16, $\mathbf{I}_a$ is the phasor sum of $\mathbf{I}_d$ and $\mathbf{I}_q$:

$$\mathbf{I}_a = \mathbf{I}_d + \mathbf{I}_q \quad (7.26)$$

Corresponding to the d axis and q axis and associated with each current component, a reactance is defined. These reactances are called *direct-axis reactance* X_d and *quadrature-axis reactance* X_q and are associated with $\mathbf{I}_d$ and $\mathbf{I}_q$, respectively. Values of X_d and X_q are available from the manufacturer of synchronous machines. Hence, the equivalent circuit looks as shown in Fig. 7.17.

Based on the phasor diagram of Fig. 7.16 and the equivalent circuit of Fig. 7.17, the voltage-current relationship for a salient-pole machine is written as

$$\mathbf{E}_a = \mathbf{V}_t + jX_d\mathbf{I}_d + jX_q\mathbf{I}_q \quad (7.27)$$

Equations 7.26 and 7.27 are used to analyze a salient-pole machine.

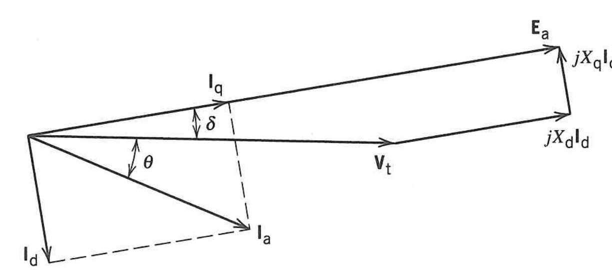

FIGURE 7.16 Phasor diagram for a salient-pole synchronous generator.

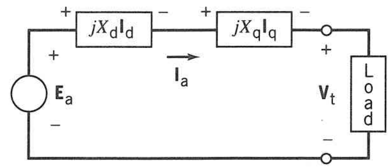

FIGURE 7.17 Equivalent circuit of a salient-pole synchronous generator.

EXAMPLE 7.4

A 75-MVA, 13.8-kV, three-phase, eight-pole, 60-Hz salient-pole synchronous machine has the following d-axis and q-axis reactances: $X_d = 1.0$ pu and $X_q = 0.6$ pu. The synchronous generator is delivering rated MVA at rated voltage and 0.866 power factor lagging. Choose a power base of 75 MVA and a voltage base of 13.8 kV. Compute the excitation voltage E_a.

Solution The following calculations are performed in per unit using a power base of 75 MVA and a voltage base of 13.8 kV. The per-unit terminal voltage $\mathbf{V}_t$ is taken as reference phasor; thus

$$\mathbf{V}_t = 1.0\underline{/0^\circ}$$

At rated conditions and 0.866 PF lagging, the per-unit stator current is given by

$$\mathbf{I}_a = 1.0\underline{/-30^\circ}$$

Refer to the phasor diagram of Fig. 7.18. The expression for $\mathbf{E}_a$ is given by Eq. 7.27 and may be rewritten as follows:

$$\begin{aligned}\mathbf{E}_a &= \mathbf{V}_t + jX_d\mathbf{I}_d + jX_q\mathbf{I}_q + (jX_q\mathbf{I}_d - jX_q\mathbf{I}_d)\\ &= \mathbf{V}_t + jX_q(\mathbf{I}_q + \mathbf{I}_d) + j(X_d - X_q)\mathbf{I}_d\\ &= \mathbf{V}_t + jX_q\mathbf{I}_a + j(X_d - X_q)\mathbf{I}_d\\ &= \mathbf{E}' + j(X_d - X_q)\mathbf{I}_d \end{aligned} \tag{7.28}$$

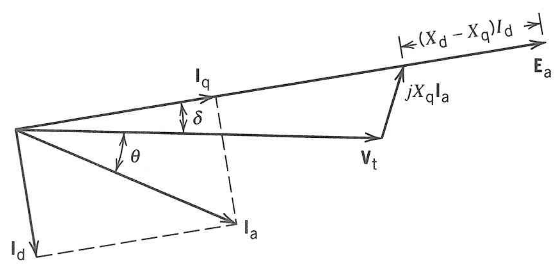

FIGURE 7.18 Phasor diagram for a salient-pole synchronous generator.

where

$$\mathbf{E}' = \mathbf{V}_t + jX_q\mathbf{I}_a \tag{7.29}$$

From Fig. 7.18, it may be seen that the phasor $j(X_d - X_q)\mathbf{I}_d$ lies parallel to $\mathbf{E}_a$. Therefore, $\mathbf{E}' = \mathbf{V}_t + jX_q\mathbf{I}_a$ must be in parallel with $\mathbf{E}_a$. This implies that the phase angle of $\mathbf{E}'$ is equal to the phase angle of $\mathbf{E}_a$, which is δ. Hence,

$$\mathbf{E}' = 1.0\underline{/0^\circ} + (j0.6)(1.0\underline{/-30^\circ}) = 1.40\underline{/21.8^\circ} \text{ pu}$$

Therefore, angle δ is equal to 21.8°.

The angle between $\mathbf{E}_a$ and $\mathbf{I}_a$ is found as follows:

$$(\delta + \theta) = 21.8^\circ + 30^\circ = 51.8^\circ$$

This angle is used to resolve $\mathbf{I}_a$ into its components:

$$\begin{aligned}\mathbf{I}_d &= [I_a \sin(\delta + \theta)]\underline{/\delta - 90^\circ}\\ &= [1.0 \sin 51.8^\circ]\underline{/21.8^\circ - 90^\circ} = 0.786\underline{/-68.2^\circ} \text{ pu}\\ \mathbf{I}_q &= [I_a \cos(\delta + \theta)]\underline{/\delta}\\ &= [1.0 \cos 51.8^\circ]\underline{/21.8^\circ} = 0.618\underline{/21.8^\circ} \text{ pu}\end{aligned}$$

Substituting the values of $\mathbf{I}_d$ and $\mathbf{I}_q$ into Eq. 7.27 yields

$$\begin{aligned}\mathbf{E}_a &= 1.0\underline{/0^\circ} + (j1.0)(0.786\underline{/-68.2^\circ}) + (j0.6)(0.618\underline{/21.8^\circ})\\ &= 1.714\underline{/21.8^\circ} \text{ pu}\end{aligned}$$

Alternatively, $\mathbf{E}_a$ may be calculated using the known value of $\mathbf{E}'$ and the fact that these phasors are in phase with $j(X_d - X_q)\mathbf{I}_d$. Therefore, the magnitudes of the phasors $\mathbf{E}'$ and $j(X_d - X_q)\mathbf{I}_d$ add directly. Thus,

$$\begin{aligned}E_a &= E' + (X_d - X_q)I_d\\ &= 1.40 + (1.0 - 0.6)(0.786) = 1.714 \text{ pu}\end{aligned}$$

Therefore, the excitation voltage phasor is found as follows:

$$\mathbf{E}_a = E_a\underline{/\delta} = 1.714\underline{/21.8^\circ} \text{ pu} = 23.6\underline{/21.8^\circ} \text{ kV}$$

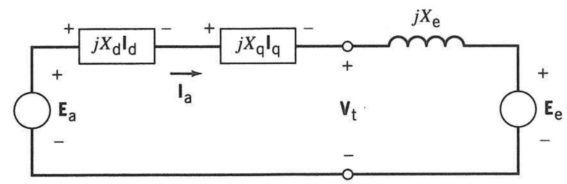

FIGURE 7.19 Equivalent circuit of a synchronous generator connected to an external system.

7.3.2 Power-Angle Characteristic of a Salient-Pole Machine

The derivation of the expression for the power generated by a salient-pole synchronous machine is similar to that of a round-rotor synchronous machine. Consider a salient-pole synchronous generator connected to an external power system. The per-phase equivalent circuit is shown in Fig. 7.19. The relationships among the phasors $\mathbf{V}_t$, $\mathbf{E}_a$, $\mathbf{I}_a$, $\mathbf{I}_d$, and $\mathbf{I}_q$ are shown in the phasor diagram of Fig. 7.20.

The generated voltage is taken as reference phasor; thus,

$$\mathbf{E}_a = E_a\angle 0° \tag{7.30}$$

$$\mathbf{V}_t = V_t\angle -\delta \tag{7.31}$$

From the phasor diagram of Fig. 7.20, the expressions for the magnitudes of the d-axis and q-axis components of the current are found as follows:

$$I_q = \frac{V_t \sin\delta}{X_q} \tag{7.32}$$

$$I_d = \frac{E_a - V_t\cos\delta}{X_d} \tag{7.33}$$

The expression for the complex power delivered by the synchronous generator to the external system is given by

$$\mathbf{S} = P + jQ = 3\mathbf{V}_t\mathbf{I}_a^* = 3V_t\angle -\delta(I_q\angle 0° + I_d\angle -90°)^* \tag{7.34}$$

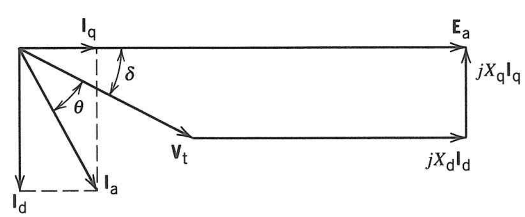

FIGURE 7.20 Phasor diagram for a salient-pole synchronous generator.

Substituting Eqs. 7.32 and 7.33 into Eq. 7.34, and simplifying, yields the expressions for the real power P and reactive power Q.

$$P = 3\left[\frac{E_aV_t}{X_d}\sin\delta + \frac{V_t^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right)\sin 2\delta\right] \tag{7.35}$$

$$Q = 3\left[\frac{E_aV_t}{X_d}\cos\delta - V_t^2\left(\frac{\sin^2\delta}{X_q} + \frac{\cos^2\delta}{X_d}\right)\right] \tag{7.36}$$

On the right-hand side of Eq. 7.35, the first term is identical to the expression for the power delivered by a round-rotor synchronous generator. The second term represents the effects of generator saliency, and it is called the *reluctance power.* The plot of Eq. 7.35 is the power-angle curve for a salient-pole synchronous generator. It is shown in Fig. 7.21.

The direct-axis reactance X_d is larger than the quadrature-axis reactance X_q. When the salient-pole machine approaches a round rotor, the values of X_d and X_q will both approach the value of X_s. When this substitution is made, Eqs. 7.35 and 7.36 will reduce to Eqs.7.21 and 7.22, respectively.

EXAMPLE 7.5

For the 75-MVA, 13.8-kV synchronous generator of Example 7.4, the excitation voltage E_a and the terminal voltage V_t are kept constant at their respective values of 1.714 pu and 1.0 pu. Choose a power base of 75 MVA and a voltage base of 13.8 kV. Find the maximum real power and maximum apparent power that the generator can deliver.

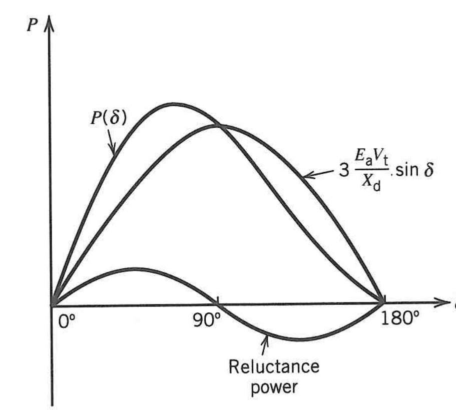

FIGURE 7.21 Power-angle curve of a salient-pole synchronous generator.

Solution

a. The per-unit real power that the generator delivers is found by using Eq. 7.35.

$$P = \frac{(1.714)(1.0)}{1.0}\sin\delta + \frac{(1.0)^2}{2}\left(\frac{1}{0.6} - \frac{1}{1.0}\right)\sin 2\delta$$
$$= 1.714\sin\delta + 0.333\sin 2\delta$$

For maximum power, differentiate P with respect to δ, and set the derivative equal to zero; thus,

$$\frac{dP}{d\delta} = 1.714\cos\delta + 0.666\cos 2\delta$$

By applying a trigonometric identity on $\cos 2\delta$ and simplifying,

$$1.333\cos^2\delta + 1.714\cos\delta - 0.666 = 0.0$$

This equation is a quadratic equation in terms of $\cos\delta$ and can be solved by using the quadratic formula. Thus,

$$\cos\delta = 0.313 \quad \text{or} \quad \cos\delta = -1.60$$

The second solution is obviously extraneous; therefore,

$$\delta = \cos^{-1} 0.313 = 71.8°$$

To check whether $\delta = 71.8°$ yields a maximum, the second derivative is evaluated:

$$\frac{d^2P}{d\delta^2} = -1.714\sin\delta - 1.333\sin 2\delta$$
$$= -1.714\sin 71.8° - 1.333\sin[(2)(71.8°)] = -2.42$$

Since $d^2P/d\delta^2 < 0$, P is maximum at $\delta = 71.8°$, and the maximum power P_{max} is computed as follows:

$$P_{max} = 1.714\sin 71.8° + 0.333\sin[(2)(71.8°)] = 1.826 \text{ pu} = 137 \text{ MW}$$

b. The reactive power delivered at $\delta = 71.8°$ is obtained by using Eq. (7.36).

$$Q = \left[\frac{(1.714)(1.0)}{1.0}\cos 71.8° - (1.0)^2\left(\frac{\sin^2 71.8}{0.6} + \frac{\cos^2 71.8°}{1.0}\right)\right]$$
$$= -1.066 \text{ pu} = -80 \text{ MVAR}$$

The total MVA is computed as follows:

$$S = \sqrt{(1.826)^2 + (-1.066)^2} = 2.114 \text{ pu} = 158 \text{ MVA}$$

DRILL PROBLEMS

D7.12 A three-phase, 100-MVA, 12-kV, 60-Hz, salient-pole synchronous machine has direct-axis and quadrature-axis reactances of 1.0 pu and 0.7 pu, respectively. The stator resistance may be neglected. The machine is connected to an infinite bus and delivers 72 MW at 0.8 power factor lagging.

a. Use $\mathbf{V}_t$ as reference phasor, and draw the phasor diagram. Determine the excitation voltage and the power angle.

b. Determine the maximum power the synchronous generator can supply if the field circuit becomes open. Determine the machine current and power factor for this condition.

D7.13 A 50-kVA, 480-V, three-phase, wye-connected, salient-pole synchronous generator runs at full load at 0.9 leading power factor. The per-phase direct-axis and quadrature-axis reactances are 1.5 Ω and 1.0 Ω, respectively. The armature resistance is negligible. Calculate (a) the excitation voltage and (b) the power angle.

7.4 GENERATOR SYNCHRONIZATION

An individual synchronous generator supplying power to an impedance load acts as a voltage source whose frequency is determined by the speed of rotation of its prime mover and whose voltage is determined by its excitation system. The major disadvantage of such an operating practice is that anytime the generator is out of order, or is under maintenance, the supply of electricity to the load is interrupted.

The electricity supply systems of industrialized countries have hundreds of synchronous generators operating in parallel. These generators are interconnected by a network of transmission lines and substations. The main reasons for interconnection are reliability of service, economy of power system operation, and improved operating efficiency of the individual generators.

The point of connection of the generator to the power system is called an *infinite bus*. The infinite bus can be represented as a voltage source of constant magnitude and constant frequency. When a synchronous generator is connected to a large power system, the frequency and rms voltage at the generator terminals are fixed by the power system.

The process of properly connecting a synchronous generator in parallel with the other generators in the power system, or to the infinite bus, is called *synchronization*. In order to synchronize properly, the following conditions have to be satisfied:

1. The magnitude of the terminal voltage of the incoming generator must be the same as the voltage at the point of interconnection with the power system or infinite bus.
2. The frequency of the incoming generator must be the same as the frequency of the power system or infinite bus.
3. The generator must have the same phase sequence as the infinite bus.
4. The phase angles of corresponding phases of the incoming generator and the power system must be equal.

To verify that these conditions for connecting the incoming generator in parallel with the infinite bus are satisfied, a set of three synchronizing lamps may be used. The schematic diagram for the connection of these lamps for synchronization in a laboratory setup is shown in Fig. 7.22.

The field rheostat of the generator is adjusted to vary the field current until the generator voltage V_2 becomes equal to, or slightly greater than, the infinite bus voltage V_1. If the phase sequences of the generator and the infinite

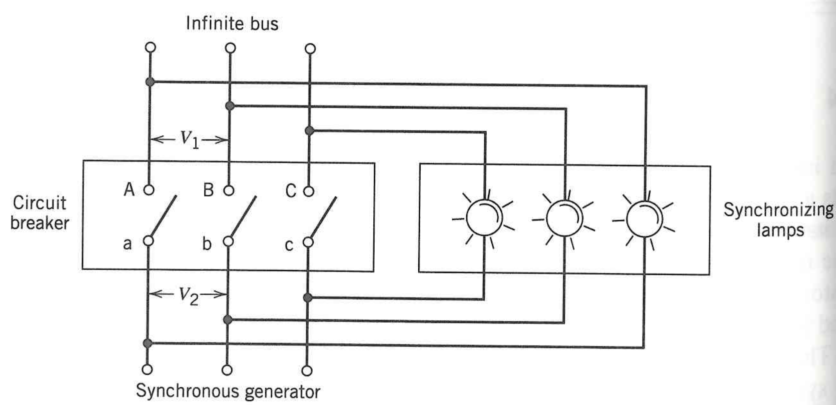

FIGURE 7.22 Schematic diagram for synchronization using synchronizing lamps.

bus are different, the three lamps will brighten up alternately. To correct for this improper condition, any two of the three connections to the synchronous generator are interchanged. If the phase sequence is correct, the lamps are all bright or are all dark at the same time. If the frequencies are slightly different, the three lamps will brighten or darken at the same time. The speed of the prime mover of the synchronous generator is adjusted so that the generator's frequency is the same as that of the infinite bus, at which time all lamps stay dark. When all four conditions are satisfied simultaneously, the circuit breaker is closed, and the generator is now operating in parallel with the rest of the synchronous machines of the system.

* 7.5 DYNAMICS OF THE PRIME MOVER–GENERATOR SYSTEM

The previous discussions have dealt with modeling and analyzing a synchronous machine under steady-state conditions. In steady state, the electromagnetic torque and mechanical torque balance each other. The machine operates at the synchronous speed determined by the number of poles and the frequency of the power system, which is normally 60 Hz.

When the load on the synchronous generator changes, that is, when the power demand either increases or decreases, the machine will either decelerate or accelerate temporarily. Hence, the dynamics of the rotor come into the picture. There is now a need to apply Newton's second law of motion. Application of Newton's second law results in

$$J\frac{d\omega_m}{dt} = T_m - T_e \tag{7.37}$$

where

J = moment of inertia of the rotor (N-m-s^2)
ω_m = speed of the rotor (rad/s)
$d\omega_m/dt$ = angular acceleration of the rotor
T_m = shaft mechanical torque (N-m)
T_e = electromagnetic torque (N-m)

The angular acceleration may also be expressed in terms of the second derivative of the angular displacement:

$$\frac{d\omega_m}{dt} = \frac{d^2\theta_m}{dt^2} \tag{7.38}$$

The angle θ_m represents the angular position of the rotor with respect to some stationary reference frame. It is convenient to measure the rotor angular position with respect to a synchronously rotating reference frame instead of a stationary frame.

$$\theta_m(t) = \omega_{m,s}t + \delta_m(t) \tag{7.39}$$

where

$\omega_{m,s}$ = synchronous angular velocity of the rotor

δ_m = rotor angular position with respect to the synchronously rotating reference frame

For a synchronous generator with p poles, the electrical radian frequency ω and the rotor angle δ are given by

$$\omega_m(t) = \left(\frac{2}{p}\right)\omega(t)$$

$$\delta_m(t) = \left(\frac{2}{p}\right)\delta(t) \tag{7.40}$$

Substituting Eqs. 7.39 and 7.40 into Eqs. 7.37 and 7.38 and simplifying,

$$\frac{2}{p}J\frac{d^2\delta}{dt^2} = T_m - T_e \tag{7.41}$$

Equation 7.41 is known as the *swing equation* and is used to solve for the electromechanical dynamics of the synchronous machine. It should be noted that Eq. 7.41 is written for generator action. The electromagnetic torque T_e is found as follows:

$$T_e = 3\frac{E_a V_t}{\omega_m X_s}\sin\delta = K_s \sin\delta \tag{7.42}$$

where

$$K_s = 3\frac{E_a V_t}{\omega_m X_s} \tag{7.43}$$

The quantity K_s is called synchronous torque coefficient. For dynamic analysis, K_s can be taken as a constant.

EXAMPLE 7.6

Express Eq. 7.41 in terms of power.

Solution Multiply both sides of Eq. 7.41 by $\omega_m = (2/p)\omega$, where $\omega = 2\pi f$ and $f = 60$ Hz. On the left-hand side of the equation, let the synchronous rotor

velocity ω_s represent the approximate value of ω_m. Thus,

$$\frac{2}{p}\omega_s J\frac{d^2\delta}{dt^2} = \omega_m T_m - \omega_m T_e \tag{7.44}$$

The first term on the right-hand side of Eq. 7.44 is the mechanical power P_m, and the second term is given by Eq. 7.42. Therefore, Eq. 7.44 may also be written as

$$\frac{2}{p}\omega_s J\frac{d^2\delta}{dt^2} = P_m - 3\frac{E_a V_t}{X_s}\sin\delta \tag{7.45}$$

7.6 SYNCHRONOUS MOTOR PERFORMANCE

The equivalent circuit and torque equations derived for a synchronous generator also apply to synchronous motors. Therefore, analyzing the performance of a motor parallels the analysis of generator performance. The per-phase equivalent circuit of the synchronous motor is shown in Fig. 7.23.

For phasor analysis purposes, the terminal voltage $\mathbf{V}_t$ is usually taken as reference, and the positive direction of stator current is into the motor. The phasor diagram of Fig. 7.24 applies to a lagging power factor current.

EXAMPLE 7.7

A three-phase, 208-V synchronous motor has a synchronous reactance of 1.0 Ω per phase and negligible armature resistance. The motor draws 50 kVA at 0.8 power factor leading. Calculate (a) the stator current $\mathbf{I}_a$ and (b) the excitation voltage $\mathbf{E}_a$.

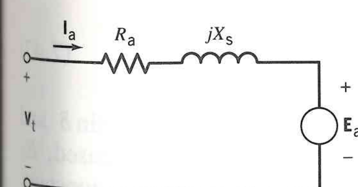

FIGURE 7.23 Equivalent circuit of a synchronous motor.

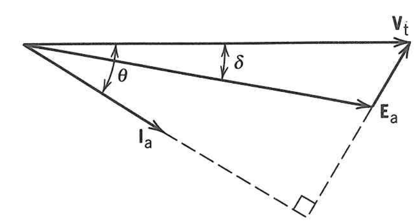

FIGURE 7.24 Phasor diagram of a synchronous motor with $R_a = 0$.

Solution

a. Take the terminal voltage $\mathbf{V}_t$ as reference phasor:

$$\mathbf{V}_t = (208/\sqrt{3})\angle 0^\circ = 120\angle 0^\circ \text{ V} \quad \text{(line-to-neutral)}$$

The stator current is computed as

$$\mathbf{I}_a = \frac{50{,}000}{208\sqrt{3}}\angle\cos^{-1} 0.8 = 138.8\angle 36.9^\circ \text{ A}$$

b. The excitation voltage is found as follows:

$$\begin{aligned}\mathbf{E}_a &= \mathbf{V}_t - (R_a + jX_s)\mathbf{I}_a \\ &= 120\angle 0^\circ - (0 + j1.0)(138.8\angle 36.9^\circ) \\ &= 231.7\angle -28.6^\circ \text{ V} \quad \text{(line-to-neutral)}\end{aligned}$$

Therefore, the line-to-line excitation voltage is

$$E_a = 231.7\sqrt{3} = 401 \text{ V} \quad \text{(line-to-line)}$$

Now suppose that the field current is increased. This increased field excitation increases the generated voltage E_a. However, the real power supplied by the motor remains the same because the mechanical load torque did not change and the rotational speed is not affected by the increased I_f. When the machine losses are neglected, the expression for the real power delivered by the motor is

$$P = 3\frac{V_t E_a'}{X_s}\sin\delta' = 3\frac{V_t E_a}{X_s}\sin\delta \tag{7.46}$$

Alternatively, the real power may be expressed as

$$P = 3V_t I_a' \cos\theta' = 3V_t I_a \cos\theta \tag{7.47}$$

It is evident from Eqs. 7.46 and 7.47 that the expressions $E_a \sin\delta$ and $I_a \cos\theta$ must be constant. Of course, when the field current is increased, E_a increases but only in such a way that its projection $E_a \sin\delta$ remains constant. This is illustrated in Fig. 7.25. Similarly, the stator current I_a is constrained to change so as to have a constant projection $I_a \cos\theta$. It may be seen that as the

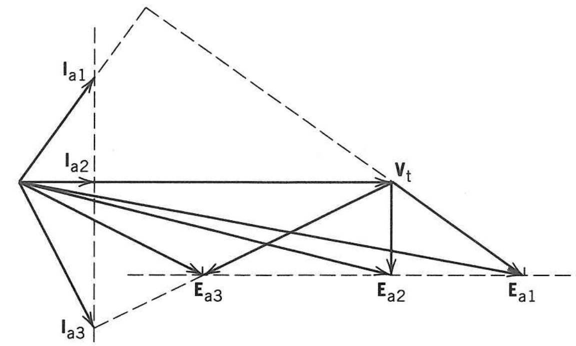

FIGURE 7.25 Effect of field current on synchronous motor.

field current and E_a continue to increase, the magnitude of the stator current initially decreases and then increases again.

At low values of E_a, the stator current lags the terminal voltage. The motor acts as an inductive load and is said to be underexcited. As the field current is increased, the stator current becomes less and less lagging and will later become in phase with the voltage. At this point, the motor is operating at unity power factor and is said to be *normally excited*. As the field current is increased further, the stator current becomes leading. The motor becomes a source of reactive power, and it is said to be overexcited. The variation of the stator current as the field current is changed is plotted in Fig. 7.26. Because of its shape, the plot is called the *V-curve* of the synchronous motor.

In Fig. 7.26, each V-curve corresponds to a different power level. At the vertex of any V-curve, the stator current is at its minimum for that power level and the motor operates at unity power factor. For field currents less than the value corresponding to minimum stator current, the motor operates at a lagging

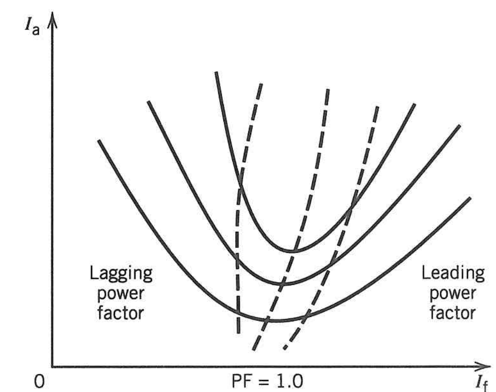

FIGURE 7.26 Synchronous motor V-curves.

power factor and it absorbs reactive power as an underexcited motor. For field currents greater than the value for minimum stator current, the motor operates at a leading power factor, and it supplies reactive power to the external system and is said to be overexcited.

EXAMPLE 7.8

A three-phase, 200-hp, 480-V, six-pole, 60-Hz synchronous motor has a synchronous reactance of 1.5 Ω/phase. Neglect all losses. The motor is connected to an infinite bus, and it delivers its rated horsepower at unity power factor. Determine the pullout torque, that is, the maximum torque the motor can deliver without losing synchronism.

Solution Take the line-to-neutral terminal voltage $\mathbf{V}_t$ as reference phasor:

$$\mathbf{V}_t = (480/\sqrt{3})\angle 0^\circ = 277.1\angle 0^\circ \text{ V}$$

At unity power factor, the stator current is in phase with the applied voltage; thus,

$$\mathbf{I}_a = \frac{(200)(746)}{\sqrt{3}\,480}\angle 0^\circ = 179.46\angle 0^\circ \text{ A}$$

Then the excitation voltage is given by

$$\begin{aligned}\mathbf{E}_a = \mathbf{V}_t - (R_a + jX_s)\mathbf{I}_a &= 277.1\angle 0^\circ - (0 + j1.5)(179.46\angle 0^\circ) \\ &= 386.3\angle -44.2^\circ \text{ V} \quad \text{(line-to-neutral)}\end{aligned}$$

Since the synchronous motor is assumed to be lossless,

$$T_{max}\omega_m = P_{max}$$

The maximum power occurs at a power angle $\delta = 90^\circ$.

$$P = 3\frac{V_t E_a}{X_s}\sin 90^\circ = 3\frac{(277.1)(386.3)}{1.5} = 214 \text{ kW}$$

The mechanical angular speed is found from the radian frequency as

$$\omega_m = (2/p)\omega = (2/6)(2\pi 60) = 125.7 \text{ rad/s}$$

Finally,

$$T_{max} = P_{max}/\omega_m = (214 \times 10^3)/125.7 = 1702.5 \text{ N-m}$$

The power flow diagram for a synchronous motor is shown in Fig. 7.27. The input power to the motor is electrical and is equal to $3V_t I_a \cos\theta$, where θ is the angle between the $\mathbf{V}_t$ and $\mathbf{I}_a$ phasors. The mechanical power developed, P_{dev}, is equal to $T_{ind}\omega_m$. It may be noted that the DC electrical power for field excitation is not included in Fig. 7.27 because it is supplied by a separate DC source.

The copper losses consist of the stator losses of $3I_a^2R_a$. The field winding losses are not included in this diagram because the field winding is excited by a separate DC source. The rotational losses include core losses, consisting of hysteresis and eddy current losses, and mechanical losses, consisting of friction and windage losses. The output power P_{output} is equal to $T_{load}\omega_m$, where T_{load} is the output torque. When the motor is operated at rated conditions, the output mechanical power P_{output} is equal to the motor rating, which is usually given in horsepower.

EXAMPLE 7.9

A 200-hp, 2300-V, three-phase, 60-Hz, cylindrical-rotor motor has a synchronous reactance of 12 Ω per phase and negligible armature resistance. When it is delivering its rated output, the motor's efficiency is 90% and its power angle δ is 17°. Determine

a. The excitation voltage $\mathbf{E}_a$
b. The stator current $\mathbf{I}_a$ and power factor

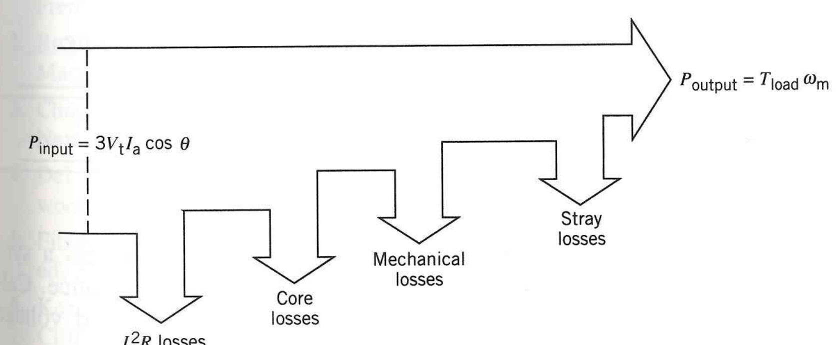

FIGURE 7.27 Power flow diagram for a synchronous motor.

Solution

a. The line-to-neutral terminal voltage is taken as reference phasor; thus,

$$\mathbf{V}_t = (2300/\sqrt{3})\angle 0^\circ = 1328\angle 0^\circ \text{ V}$$

From the power flow diagram, the input power is the same as the mechanical power developed, since the stator copper loss is negligible. Hence,

$$P_{\text{input}} = \frac{P_{\text{output}}}{\eta} = 3\frac{V_t E_a}{X_s}\sin\delta$$

$$= \frac{(200)(746)}{0.90} = 3\frac{1328 E_a}{12}\sin 17^\circ$$

Thus, the line-to-neutral excitation voltage is found as

$$E_a = \frac{(200)(746)(12)}{(0.90)(3)(1328)(\sin 17^\circ)} = 1708 \text{ V}$$

Therefore, the line-to-line excitation voltage is

$$E_a = 1708\sqrt{3} = 2958\text{V} \quad \text{(line-to-line)}$$

b. Since the armature resistance is negligible, the stator current is given by

$$\mathbf{I}_a = \frac{\mathbf{V}_t - \mathbf{E}_a}{jX_s}$$

$$= \frac{1328\angle 0^\circ - 1708\angle -17^\circ}{j12} = 48.8\angle 31.4^\circ \text{ A}$$

The power factor is

$$\text{PF} = \cos(\angle V_t - \angle I_a) = \cos(0^\circ - 31.4^\circ) = 0.85 \text{ leading}$$

DRILL PROBLEMS

D7.14 A three-phase, 6-kV, wye-connected synchronous motor has a synchronous reactance of 12 Ω per phase and negligible armature resistance. Calculate the induced voltage when the motor takes 1000 kVA at rated voltage and

a. 0.8 power factor lagging

b. Unity power factor

c. 0.8 power factor leading

D7.15 A three-phase, 200-hp, 2400-V, 60-Hz, wye-connected, cylindrical-rotor synchronous motor has a synchronous reactance of 12 Ω per phase and negligible armature resistance. The motor draws 150 kW at a power angle of 18 electrical degrees. Determine

a. The excitation voltage

b. The line current

c. The power factor

D7.16 A three-phase, 2400-V, 60-Hz, 8-pole, wye-connected synchronous motor has 5 Ω per phase synchronous reactance and negligible stator resistance. The motor is connected to a 2400-V infinite bus, and it draws 120 amperes at 0.8 power factor lagging. Neglect rotational losses.

a. Determine the output power.

b. Calculate the maximum power.

c. Determine the torque, stator current, and power factor for the maximum power condition.

D7.17 The synchronous reactance of a synchronous motor is 10 Ω per phase, and its armature resistance is negligible. The input power is 1500 kW, and the induced voltage is 4600 V. If the terminal voltage is 4160 V, determine (a) the armature current and (b) the power factor.

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PROBLEMS

7.1 From 2 poles to 10 poles, calculate the prime mover speeds in rpm and rad/s required to generate AC at a frequency of 60 Hz.

7.2 Calculate the frequency produced by a prime mover turning a 10-pole synchronous generator at 800 rpm.

7.3 A three-phase, 50-hp, 2300-V, 60-Hz synchronous motor is operating at 900 rpm. Determine the number of poles in the rotor.

7.4 Determine the speed of an eight-pole synchronous motor operating from a three-phase, 50-Hz, 4160-V system.

7.5 A three-phase, eight-pole, 60-Hz synchronous generator has a 50-mWb flux per pole. The stator winding factor is 0.90. The armature has 104 turns per phase. Calculate the induced voltage.

7.6 A three-phase, 60-Hz synchronous generator operating at no load has an induced voltage of 4300 V at rated frequency. The pole flux is increased by 10% and the rotor speed is increased by 5%. Determine (a) the induced voltage and (b) frequency.

7.7 The open-circuit voltage of a 60-Hz generator is 12 kV at a field current of 8 A. The synchronous generator is operated at 50 Hz and a field current of 3 A. Neglect saturation. Calculate the open-circuit voltage.

7.8 A three-phase, wye-connected synchronous generator is rated at 150 kVA and 2300 V. The generator delivers rated current to a load at rated terminal voltage and 0.8 PF lagging. When the load is removed, the terminal voltage rises to 3500 V. Assume that armature resistance is negligible.

a. Calculate the synchronous reactance of the generator.

b. Draw a phasor diagram.

7.9 A synchronous generator is connected to a 13.8-kV infinite bus. It has a synchronous reactance of 7.5 Ω per phase, and the armature resistance is negligible. The generator delivers a real power output of 50 MW and a reactive power output of 30 MVAR to the infinite bus.

a. Determine the excitation voltage and angle.

b. Draw a phasor diagram indicating the terminal voltage, the excitation voltage, the armature current, and the voltage drop across the synchronous reactance.

7.10 A three-phase, 100-kVA, 240-V, 60-Hz, six-pole, wye-connected synchronous generator is supplying a load of 80 kVA at 230 V and 0.866 power factor lagging. The armature has a synchronous impedance of $0.1 + j0.5\ \Omega$ per phase. Determine the following:

a. Armature current

b. Excitation voltage

c. Power angle

d. Input shaft torque (neglecting rotational losses)

7.11 A cylindrical-rotor synchronous generator has a per-unit synchronous reactance of 1.0 and a negligible armature resistance. The generator supplies rated kVA to a load at a terminal voltage of 1.0 per unit and a leading power factor of 80%. Determine the excitation voltage.

7.12 The following readings are taken from the results of open-circuit and short-circuit tests on a three-phase, 10-MVA, 12-kV, two-pole, 60-Hz cylindrical-rotor generator driven at synchronous speed.

Field current (A)	150	180
Armature current, short-circuit test (A)	400	480
Line voltage, open-circuit test (kV)	11.2	12.0
Line voltage, air-gap line (kV)	13.5	15.0

The armature resistance is measured separately as 0.10 Ω per phase.

a. Determine the unsaturated synchronous reactance in ohms per phase and per unit.

b. Calculate the saturated synchronous reactance in ohms per phase and per unit.

7.13 A three-phase, 50-MVA, 12-kV, wye-connected, 60-Hz synchronous generator has a resistance of 0.10 Ω per phase and a synchronous reactance of 4 Ω per phase. The generator delivers rated load current at rated voltage and 0.9 power factor leading. Calculate the voltage regulation.

7.14 A three-phase, 1000-kVA, 4160-V, wye-connected synchronous generator has an armature resistance and synchronous reactance of $1.5 + j25\ \Omega$ per phase. Determine the voltage regulation if the power factor is

a. 0.8 leading

b. Unity

c. 0.8 lagging

7.15 The synchronous generator described in Problem 7.12 delivers rated kVA to a load at rated voltage and 0.8 power factor lagging.

a. Compute the field current required.

b. Additional points on the open-circuit characteristic are given in the following tabulation. Find the voltage regulation.

Field current (A)	200	250	300	350
Line voltage (kV)	12.5	13.5	14.3	14.8

7.16 A three-phase, 2400-V, 60-Hz, six-pole, wye-connected synchronous generator is connected to an infinite bus. The generator is delivering 500 kW at a power angle of 30°. The stator has a synchronous reactance of 10 Ω per phase and negligible armature resistance. Determine the following:

a. Input torque to the generator, neglecting losses

b. Excitation voltage

c. Armature current and power factor

d. The reactive power delivered

7.17 A three-phase, 2000-kVA, 12-kV, 1800-rpm synchronous generator has a synchronous reactance of 20 Ω per phase and negligible armature resistance.

a. The field current is adjusted to obtain the rated terminal voltage at open circuit. Determine the excitation voltage.

b. A short circuit occurs across the machine terminals. Find the stator current.

c. The synchronous machine is connected to an infinite bus. The generator delivers its rated current at 0.8 power factor lagging. Determine the excitation voltage.

d. Calculate the maximum power the synchronous machine can deliver for the excitation current of part (c).

7.18 Loss data for the synchronous generator of Problem 7.12 are as follows:

Open-circuit core loss at 12 kV = 75 kW
Short-circuit load loss at 480 A = 60 kW
Friction and windage = 65 kW
Field winding resistance = 0.35 Ω

Compute the efficiency at rated load and 0.8 power factor lagging.

7.19 A salient-pole synchronous generator has a direct-axis synchronous reactance of 1.0 per unit and a quadrature-axis reactance of 0.6 per unit. Neglect saturation. The generator delivers full-load current at rated terminal voltage and 0.866 lagging power factor.

a. Draw the phasor diagram.

b. Determine the excitation voltage.

7.20 The salient-pole synchronous machine of Problem 7.12 operates as a generator, and it delivers 0.8 pu of power at 1.0 pu voltage and a power factor of 0.8 leading. Determine (a) the excitation voltage and (b) the power angle.

7.21 A three-phase, 60-MVA, 12-kV, 60-Hz, salient-pole synchronous machine has d-axis and q-axis reactances of 1.2 pu and 0.6 pu, respectively, and negligible armature resistance. The machine is connected to an infinite bus at 12 kV, and the field current is adjusted to make the excitation voltage equal to the terminal voltage.

a. Determine the maximum power that the machine can supply.

b. Find the stator current and power factor at this maximum power condition.

c. Draw the phasor diagram.

7.22 A salient-pole synchronous generator has d-axis and q-axis synchronous reactances of 1.60 pu and 1.20 pu, respectively. The generator is connected to an infinite bus through an external reactance of 0.20 pu. The generator delivers its rated output power at 0.8 power factor lagging to the infinite bus.

a. Draw a phasor diagram showing the bus voltage, the armature current, the generator terminal voltage, the excitation voltage, and the rotor angle.

b. Calculate the rotor angle in degrees.

c. Compute the per-unit terminal and excitation voltages.

7.23 The induced voltage of a synchronous motor is 4160 V. It lags behind the terminal voltage by 30°. If the terminal voltage is 4000 V, determine the operating power factor. The per-phase armature reactance is 6 Ω, and the armature resistance is negligible.

7.24 A three-phase, wye-connected synchronous generator is operating at 80% power factor leading. The synchronous reactance is 2.5 Ω per phase, and the armature resistance is negligible. The armature current is 20 A. The terminal voltage is 440 V.

a. Find the excitation voltage and the power angle.

b. Repeat part (a) when the armature current is increased to 40 A while the PF is maintained at 80% leading.

c. Draw a phasor diagram describing the conditions of both parts (a) and (b).

7.25 A three-phase, 1000-hp, cylindrical-rotor, wye-connected synchronous motor has a negligible armature winding resistance and a synchronous reactance of 38 Ω per phase. The motor receives a constant power of 850 kW at 12 kV. The motor has a full-load current of 48 A. Determine the excitation voltage.

7.26 A three-phase, 2400-V, 60-Hz, four-pole, wye-connected synchronous motor has a synchronous reactance of 16 Ω and negligible armature resistance. The excitation is adjusted so that the induced voltage is 2400 V. The motor drives a load connected to its shaft, and the stator current is 80 A. Calculate

a. The power angle

b. The input power

c. The developed torque

7.27 An overexcited synchronous motor is connected across a 250-kVA inductive load of 0.6 lagging power factor. The motor takes 20 kW while running on no load. Calculate the kVA rating of the motor in order to raise the overall power factor of the motor-inductive load combination to 0.95 lagging.

7.28 A small industrial plant has a total electrical load of 300 kW at 0.6 lagging power factor. A 50-hp pump is to be installed. A synchronous motor operating at 0.8 PF leading is selected to drive the pump. Neglect all losses in the synchronous motor. Calculate (a) the new total load real and reactive powers and (b) the resultant power factor.

7.29 A three-phase, 500-hp, 2400-V, 60-Hz, six-pole synchronous motor has a synchronous reactance of 12 Ω per phase and is assumed to be lossless.

- **a.** Find the motor speed.
- **b.** Determine the maximum possible torque when the motor operates at rated load conditions and unity power factor.
- **c.** Repeat part (b) when the motor operates at 0.8 PF leading.

7.30 A three-phase, 2300-V, wye-connected synchronous motor has a synchronous impedance of $0.05 + j1.25\ \Omega$ per phase. The motor draws its full-load current of 500 A at unity power factor and a field current of 6 A. Find the field current when the motor takes 400 A at 0.8 power factor leading.

7.31 A three-phase, 2300-V, 60-Hz, cylindrical-rotor synchronous motor has a synchronous reactance of 10 Ω per phase and negligible armature resistance. The motor delivers 250 hp at a power angle of 20 electrical degrees, and the efficiency is 90%. Determine

- **a.** The excitation voltage
- **b.** The stator current
- **c.** The power factor

7.32 A three-phase, 13.2-kV, 60-Hz, wye-connected, synchronous motor has an armature resistance of 2 Ω per phase and a synchronous reactance of 30 Ω per phase. When the motor delivers 1500 hp, it takes a current of 80 A at a leading power factor and the efficiency is 90%.

- **a.** Determine the power factor.
- **b.** Calculate the excitation voltage.

7.33 A three-phase, 20-kVA, 480-V, wye-connected synchronous machine operates as a motor, and it draws rated current at rated voltage and 0.8 lagging power factor. The total losses are 1500 W, and the armature resistance is 2.5 Ω per phase. Determine the efficiency of the motor.

7.34 A three-phase, 4160-V, wye-connected, cylindrical-rotor synchronous motor has a synchronous reactance of 8 Ω per phase and negligible armature resistance. The combined rotational losses (friction and windage plus core loss) amount to 5 kW. The highest excitation voltage possible is 4350 V. The motor delivers an output of 400 hp to a mechanical load connected to its shaft.

- **a.** Calculate the stator current at maximum excitation.
- **b.** Compute the smallest excitation voltage for which the motor will remain in synchronism.

7.35 A three-phase, 1000-hp, 2300-V, wye-connected synchronous motor operates at rated voltage and rated frequency. The motor delivers rated power at 0.8 power factor leading, an efficiency of 92%, and a power angle of 25°. The synchronous reactance is 8 Ω per phase, and the armature resistance is negligible. Determine (a) the line current and (b) the excitation voltage.

7.36 A three-phase, 2000-hp, 13.2-kV, 60-Hz, six-pole, wye-connected, cylindrical-rotor synchronous motor operates at rated load, 0.85 power factor leading, and an efficiency of 94%. The synchronous reactance per phase is 32 Ω, and the armature resistance is negligible. Determine the following:

- **a.** Rated torque
- **b.** Armature current
- **c.** Excitation voltage and power angle
- **d.** Pullout torque

7.37 A synchronous motor delivers rated kVA at rated voltage and leading power factor. Let the power angle between the excitation voltage phasor and the terminal voltage phasor be denoted by δ and the power factor angle denoted by θ. Draw the phasor diagram corresponding to this load condition, and show the following relationship.

$$\tan\delta = \frac{I_a X_q \cos\theta + I_a R_a \sin\theta}{V_t + I_a X_q \sin\theta - I_a R_a \cos\theta}$$

Consider θ to be negative when the stator current $\mathbf{I}_a$ lags the terminal voltage $\mathbf{V}_t$.

7.38 A three-phase, salient-pole synchronous machine has d-axis and q-axis reactances of 1.4 pu and 0.6 pu, respectively. The armature resistance is negligible. The machine operates as a synchronous motor and draws 0.8 pu of power at 1.0 per-unit voltage and 0.866 power factor leading.

- **a.** Draw the phasor diagram.
- **b.** Determine the excitation voltage and power angle δ.
- **c.** Determine the power due to field excitation and the power due to the saliency of the machine.

Eight

Induction Motors

8.1 INTRODUCTION

Just like DC machines and synchronous machines, the induction machine may be used as a generator or as a motor. Because their performance cannot compare with that of synchronous machines, induction generators have not been very popular. In recent years, however, induction generators have found use in wind power plants. Because of its wide use and popularity, the induction motor is called the workhorse of the power industry. This chapter will describe the principles of operation and performance analysis of three-phase and single-phase induction motors.

An induction motor is an AC machine in which alternating current is supplied to the stator armature windings directly and to the rotor windings by induction or transformer action from the stator. Hence, it has also been called a rotating transformer. Its stator windings are similar to the stator windings of synchronous machines. However, the rotor of the induction motor may be either of two types:

a. A *wound rotor* carries three windings similar to the stator windings. The terminals of the rotor windings are connected to insulated slip rings mounted on the rotor shaft. Carbon brushes bearing on these rings make the rotor terminals available to the user of the machine. For steady-state operation, these terminals are shorted.

b. A *squirrel-cage rotor* consists of conducting bars embedded in slots in the rotor magnetic core, and these bars are short-circuited at each end by conducting end rings. The rotor bars and the rings are shaped like a squirrel cage, hence the name squirrel-cage rotor.

Most induction motors have squirrel-cage rotors. From a modeling point of view, however, the two types of rotors are similar.

The three balanced alternating voltages applied to the stator cause balanced stator currents to flow. As shown in Chapter 5, these currents produce a rotating mmf that can be represented as a rotating magnetic field. This rotating magnetic field induces voltages in the rotor windings, by Faraday's law. These induced voltages, in turn, cause balanced currents to flow in the short-circuited rotor. These rotor currents then produce a rotor mmf, which can also be represented as a rotating magnetic field. The interaction of these two rotating magnetic fields produces an electromagnetic torque T_e, which is used to turn a mechanical load T_m. At steady state, when the motor losses are neglected, T_m and T_e are equal.

The speed of rotation of the rotor magnetic field, when viewed from a stationary position on the stator, is equal to the speed of rotation of the stator magnetic field, which is the synchronous speed n_s. However, the rotor speed n_r is different from the synchronous speed. If n_r were equal to n_s, there would be no variation of flux linkage in the rotor, or there would be no net flux cutting; hence, no voltage would be induced in the rotor. Therefore, rotor speed has to be less than synchronous speed. The difference in speed is represented by the *slip* s, which is defined as follows.

$$s = \frac{n_s - n_r}{n_s} \tag{8.1}$$

where

n_r = rotor speed (rpm)

n_s = $120f/p$ = synchronous speed (rpm)

f = frequency

p = number of poles

The difference between the speed of the rotor magnetic field and the speed of the rotor expressed in revolutions per minute is called *slip rpm* and is equal to $(n_s - n_r)$. Therefore, the frequency of the rotor currents is given by

$$f_r = (n_s - n_r)\frac{p}{120} = s\left(\frac{pn_s}{120}\right) \tag{8.2}$$

Since $(pn_s/120)$ is equal to the frequency of the stator currents, Eq. 8.2 can also be written as

$$f_r = sf \tag{8.3}$$

For steady-state operation, the slip has a normal range of values between 1% and 5%.

EXAMPLE 8.1

A three-phase, 10-hp, 208-V, 60-Hz, four-pole, wye-connected induction motor delivers rated output power at a slip of 5%. Determine the following:

a. Synchronous speed
b. Rotor speed and slip rpm at the rated load
c. Rotor frequency at the rated load
d. Speed of stator rotating magnetic field
e. Speed of the rotor rotating magnetic field
(i) relative to the rotor
(ii) relative to the stator
(iii) relative to the stator magnetic field

Solution

a. The synchronous speed of this motor is

$$n_s = 120f/p = (120)(60)/4 = 1800 \text{ revolutions/minute (rpm)}$$

b. At rated output power, the slip $s = 0.05$. Thus, the rotor speed, which is also the actual speed of the motor, is given by

$$n = n_r = (1 - s)n_s = (1 - 0.05)(1800) = 1710 \text{ rpm}$$

Thus, the slip rpm is found as follows.

$$\text{Slip rpm} = n_s - n_r = 1800 - 1710 = 90 \text{ rpm}$$

Alternatively,

$$\text{Slip rpm} = sn_s = (0.05)(1800) = 90 \text{ rpm}$$

c. The rotor frequency of this motor is given by

$$f_r = sf = (0.05)(60) = 3 \text{ Hz}$$

d. The speed of rotation of the stator magnetic field is equal to the synchronous speed $n_s = 1800$ rpm.

e. The speed of rotation of the rotor magnetic field is
(i) Relative to the rotor:

$$n_2 = sn_s = 90 \text{ rpm}$$

(ii) Relative to the stator:

$$n + n_2 = n + sn_s = n_s = 1800 \text{ rpm}$$

(iii) Relative to the stator magnetic field = 0 rpm

DRILL PROBLEMS

D8.1 A six-pole induction motor runs at 1158 rpm when it is connected to a 60-Hz source. Determine the synchronous speed and the percent slip.

D8.2 A three-phase, 60-Hz induction motor runs at 1192 rpm at no load and at 1120 rpm at full load. Determine (a) the number of poles and (b) the slip at rated load.

D8.3 A three-phase, 60-Hz, 12-pole, wye-connected induction motor has a full load slip of 5%. Calculate

a. Full-load speed
b. Synchronous speed
c. Slip rpm

D8.4 A three-phase, 208-V, eight-pole, 60-Hz induction motor operates at a slip of 5%. Determine the following:

a. Speed of the stator and rotor magnetic fields
b. Speed of the rotor
c. Slip speed of the rotor
d. Rotor frequency

D8.5 A four-pole, 60-Hz induction motor drives a load at 1740 rpm. Determine the following:

a. Slip
b. Speed of the stator field with respect to the stator
c. Speed of the stator field with respect to the rotor
d. Speed of the rotor field with respect to the stator
e. Speed of the rotor field with respect to the rotor

8.2 EQUIVALENT CIRCUIT OF A THREE-PHASE INDUCTION MOTOR

An equivalent circuit is invaluable in the performance analysis of the three-phase induction motor. Therefore, the equivalent circuit, as well as approximate equivalent circuits, are developed in this section. When the parameters of the equivalent circuits are not available, standard tests for determining these parameters are performed; these tests are described in the following section.

8.2.1 Development of Equivalent Circuit

The general form of the equivalent circuit for a three-phase induction motor can be derived from the equivalent circuit of a three-phase transformer. The induction motor can be thought of as a three-phase transformer whose secondary, or the rotor, is short-circuited and is revolving at the motor speed. Because the motor normally operates at balanced conditions, only a single-phase equivalent circuit is necessary.

When balanced three-phase currents flow in both stator and rotor windings, the resultant synchronously rotating air-gap flux wave induces balanced three-phase voltages in both stator windings and rotor windings. The stator induced voltage has a frequency equal to the frequency f of the applied voltage, while the rotor induced voltage has a frequency f_r given by Eq. 8.3.

Consider the stator first. The applied voltage per phase across the stator terminals is equal to the sum of the stator induced voltage per phase, plus the voltage drop across the stator winding resistance, plus the voltage drop across the stator leakage reactance due to the leakage flux, which links only the stator winding. Mathematically, in phasor form, the relationship may be expressed as

$$\begin{aligned} \mathbf{V}_1 &= \mathbf{E}_1 + R_1\mathbf{I}_1 + jX_1\mathbf{I}_1 \\ &= \mathbf{E}_1 + (R_1 + jX_1)\mathbf{I}_1 \end{aligned} \tag{8.4}$$

where

$\mathbf{V}_1$ = stator terminal voltage per phase
$\mathbf{E}_1$ = stator induced voltage per phase
$\mathbf{I}_1$ = stator current
R_1 = stator winding resistance
X_1 = stator leakage reactance

The magnetic core can be modeled as a parallel combination of a resistance R_c, to account for hysteresis and eddy current losses, and a reactance X_m, to account for the magnetizing current required to produce the air-gap magnetic flux. The magnetizing current in an induction motor is much larger than that in a transformer because of the presence of the air gap in a motor.

Next, a model of the rotor is developed. Let $\mathbf{E}_2$ denote the rotor induced voltage at standstill, that is, $s = 1.0$. At standstill, the induction motor may be viewed as a transformer with an air gap, and the stator per-phase induced voltage $\mathbf{E}_1$ is related to the rotor per-phase induced voltage $\mathbf{E}_2$ by the turns ratio (N_1/N_2); that is,

$$\mathbf{E}_1 = \left(\frac{N_1}{N_2}\right)\mathbf{E}_2 \tag{8.5}$$

where

N_1 = number of turns in the stator winding
N_2 = number of turns in the rotor winding

The voltage induced in the rotor of the induction motor is directly proportional to the relative motion of the rotor and the synchronously rotating air-gap magnetic field. When the induction motor is rotating at a speed n, or a slip s, the rotor induced voltage $\mathbf{E}_{2s}$ is equal to the induced voltage at standstill $\mathbf{E}_2$ multiplied by the slip. In the short-circuited rotor circuit, the induced voltage $\mathbf{E}_{2s}$ appears as a voltage drop across the rotor resistance and leakage reactance. The rotor resistance does not depend on the slip. However, the rotor leakage reactance does, and is equal to $X_r = 2\pi f_r L_r = s2\pi f L_r$, where L_r is the leakage inductance of the rotor winding due to flux linking the rotor winding only. Thus, the rotor induced voltage at slip s may be expressed mathematically as follows:

$$\begin{aligned} \mathbf{E}_{2s} = s\mathbf{E}_2 &= R_r\mathbf{I}_r + j(2\pi f_r L_r)\mathbf{I}_r \\ &= R_r\mathbf{I}_r + js(2\pi f L_r)\mathbf{I}_r \\ &= R_r\mathbf{I}_r + jsX_2'\mathbf{I}_r \end{aligned} \tag{8.6}$$

where

$\mathbf{E}_{2s}$ = rotor induced voltage at slip s
$\mathbf{E}_2$ = rotor induced voltage at standstill ($s = 1.0$)
$\mathbf{I}_r$ = rotor phase current
R_r = rotor resistance per phase
$X_2' = 2\pi f L_r$ = rotor leakage reactance per phase at standstill

Dividing both sides of Eq. 8.6 by the slip s and referring rotor quantities to the stator side, as in a transformer, yields

$$\mathbf{E}_2 = \left(\frac{R_r}{s} + jX_2'\right)\mathbf{I}_r$$

$$\left(\frac{N_1}{N_2}\right)\mathbf{E}_2 = \left(\frac{N_1}{N_2}\right)^2\left(\frac{R_r}{s} + jX_2'\right)\left(\frac{N_2}{N_1}\right)\mathbf{I}_r \quad (8.7)$$

$$\mathbf{E}_1 = \left(\frac{R_2}{s} + jX_2\right)\mathbf{I}_2$$

where

$\mathbf{E}_1 = (N_1/N_2)\mathbf{E}_2$ = rotor induced voltage referred to the stator

$\mathbf{I}_2 = (N_2/N_1)\mathbf{I}_r$ = rotor current referred to the stator

$R_2 = (N_1/N_2)^2 R_r$ = rotor resistance referred to the stator

$X_2 = (N_1/N_2)^2 X_2'$ = rotor leakage reactance referred to the stator

The stator circuit represented by Eq. 8.4 and the rotor circuit represented by Eq. 8.7 are at the same frequency f of the applied voltage. Therefore, these stator and rotor circuits can be joined together and combined with the model of the magnetic core into the per-phase equivalent circuit of the induction motor, which is shown in Fig. 8.1.

EXAMPLE 8.2

A three-phase, 25-hp, 440-V, 60-Hz, four-pole, induction motor has the following impedances referred to the stator in Ω/phase.

$$R_1 = 0.50 \qquad R_2 = 0.35$$
$$X_1 = 1.20 \qquad X_2 = 1.20 \qquad X_m = 25$$

The combined rotational losses (mechanical and core losses) amount to 1250 W, and they are assumed to remain constant. For a rotor slip of 2.5% at rated voltage and rated frequency, find

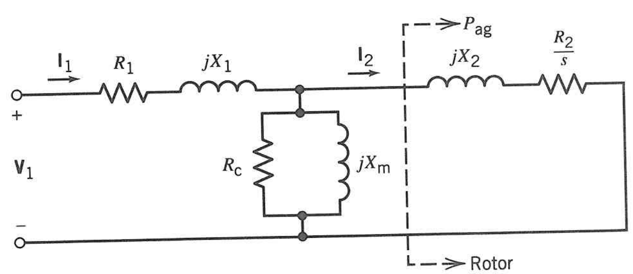

FIGURE 8.1 Per-phase equivalent circuit of a three-phase induction motor.

a. The motor speed
b. The stator current
c. The power factor
d. The efficiency of the motor

Solution Since the core loss is lumped with the rotational losses, the resistance R_c is neglected. Thus, the per-phase equivalent circuit of the induction motor appears as shown in Fig. 8.2.

a. The synchronous speed is

$$n_s = 120f/p = (120)(60)/4 = 1800 \text{ rpm}$$
$$\omega_s = 2\pi n_s/60 = 2\pi(1800)/60 = 188.5 \text{ rad/s}$$

The motor speed is found from the given slip.

$$n = (1 - s)n_s = (1 - 0.025)1800 = 1755 \text{ rpm}$$
$$\omega = 2\pi n/60 = 2\pi(1755)/60 = 183.8 \text{ rad/s}$$

b. The impedance of the rotor referred to the stator is

$$Z_2 = R_2/s + jX_2 = 0.35/0.025 + j1.20 = 14.0 + j1.20\ \Omega$$

Therefore, the input impedance is

$$\begin{aligned} Z_{in} &= R_1 + jX_1 + (Z_2)(jX_m)/(Z_2 + jX_m) \\ &= 0.50 + j1.20 + (14.0 + j1.20)(j25)/[14.0 + j(1.20 + 25)] \\ &= 10.42 + j7.64 = 12.92\angle 36.3^\circ\ \Omega \end{aligned}$$

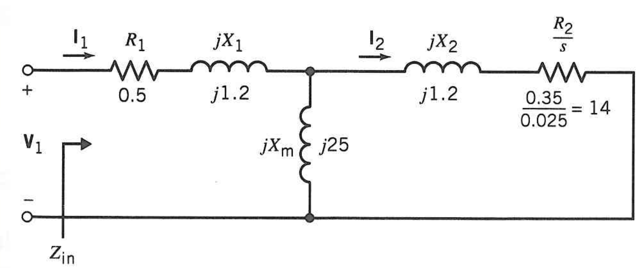

FIGURE 8.2 Equivalent circuit for motor of Example 8.2.

The terminal voltage per phase is taken as reference phasor; thus,

$$\mathbf{V}_1 = (440/\sqrt{3})\angle 0^\circ = 254.0\angle 0^\circ \text{ V} \quad \text{(line-to-neutral)}$$

The stator current is found as follows.

$$\mathbf{I}_1 = \mathbf{V}_1/Z_{\text{in}} = (254.0\angle 0^\circ)/(12.92\angle 36.3^\circ) = 19.66\angle -36.3^\circ$$

c. The power factor PF is found as follows.

$$\text{PF} = \cos 36.3^\circ = 0.806 \text{ lagging}$$

d. The power input to the motor is given by

$$P_{\text{input}} = 3V_1 I_1 \text{PF} = 3(254.0)(19.66)(0.806) = 12,075 \text{ W}$$

The stator copper loss SCL is given by

$$\text{SCL} = 3I_1^2 R_1 = 3(19.66)^2(0.50) = 580 \text{ W}$$

The rotor current is computed as follows:

$$\mathbf{I}_2 = \{j25/[14.0 + j(25 + 1.2)]\}\mathbf{I}_1$$
$$= [j25/(14.0 + j26.2)]\left[19.66\angle -36.3^\circ\right] = 16.54\angle -8.2^\circ \text{ A}$$

Hence, the rotor copper loss RCL is

$$\text{RCL} = 3I_2^2 R_2 = 3(16.54)^2(0.35) = 287 \text{ W}$$

The output power is found as follows.

$$P_{\text{out}} = P_{\text{input}} - \text{SCL} - \text{RCL} - P_{\text{rot}}$$
$$= 12,075 - 580 - 287 - 1250 = 9958 \text{ W}$$

Therefore, the efficiency is

$$\eta = P_{\text{out}}/P_{\text{input}} = (9958/12,075)100\% = 82.5\%$$

An approximate equivalent circuit for an induction motor is derived by moving the shunt elements, R_c and X_m in parallel, representing the core to the motor terminals. This simplification introduces little error but greatly reduces

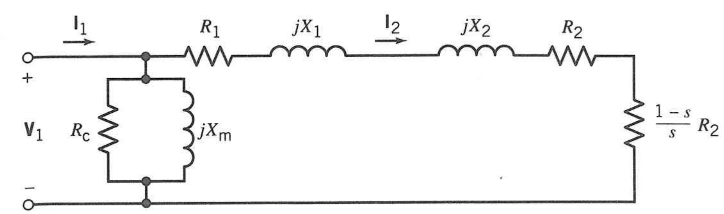

FIGURE 8.3 Approximate equivalent circuit of an induction motor.

the computational effort. This approximate equivalent circuit is shown in Fig. 8.3. Also shown in the figure is the equivalent rotor resistance R_2/s, which has been decomposed into R_2 and $R_2[(1-s)/s]$. The first resistance component, R_2, represents the rotor copper loss, and the second component represents the power developed by the motor.

EXAMPLE 8.3

A three-phase, 220-V induction motor has the following data:

$$R_1 = 0.20\ \Omega \qquad R_2 = 0.15\ \Omega$$
$$X_1 = 0.50\ \Omega \qquad X_2 = 0.30\ \Omega$$

The core effect can be neglected. The motor operates at 3% slip. If the total losses are 1000 W, determine the following:

a. Stator current

b. Power factor

c. Efficiency of the motor

Solution

a. With the effects of the core neglected, the approximate per-phase equivalent circuit of the three-phase induction motor reduces to that shown in Fig. 8.4.

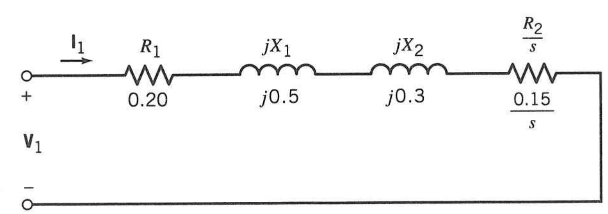

FIGURE 8.4 Equivalent circuit of motor of Example 8.3.

The terminal voltage per phase is taken as reference phasor; thus,

$$\mathbf{V}_1 = (220/\sqrt{3})\angle 0^\circ = 127\angle 0^\circ \text{ V} \quad \text{(line-to-neutral)}$$

The stator current is found as follows:

$$\begin{aligned}\mathbf{I}_1 &= \mathbf{V}_1/[R_1 + R_2/s + j(X_1 + X_2)] \\ &= (127\angle 0^\circ)/[0.2 + 0.15/0.03 + j(0.5 + 0.3)] \\ &= 24.14\angle -8.75^\circ \text{ A}\end{aligned}$$

b. The input power factor is given by

$$\text{PF} = \cos(\angle \mathbf{V}_1 - \angle \mathbf{I}_1) = \cos 8.75^\circ = 0.988 \text{ lagging}$$

c. The input power is given by

$$P_{\text{input}} = 3V_1 I_1 \text{PF} = 3(127)(24.14)(0.988) = 9090 \text{ W}$$

The output power is computed as follows.

$$P_{\text{output}} = P_{\text{input}} - \text{losses} = 9090 - 1000 = 8090 \text{ W}$$

Therefore, the efficiency is

$$\eta = P_{\text{output}}/P_{\text{input}} = (8090/9090)100\% = 89\%$$

DRILL PROBLEMS

D8.6 A three-phase, 440-V, 60-Hz, four-pole, wye-connected induction motor has the following per-phase parameters, which are referred to the stator:

$$R_1 = 0.10\ \Omega \qquad X_1 = 0.4\ \Omega$$
$$R_2 = 0.15\ \Omega \qquad X_2 = 0.4\ \Omega \qquad X_m = 12\ \Omega$$

The motor core loss is 2000 W, and the friction and windage losses amount to 1500 W. At a slip of 5%, determine the following:

a. Input current and power factor

b. Power input

c. Power output

d. Efficiency of the motor

D8.7 A three-phase, 30-hp, 480-V, four-pole, 60-Hz induction motor has the following equivalent circuit parameters in Ω per phase referred to the stator:

$$R_1 = 0.25 \qquad R_2 = 0.20$$
$$X_1 = 1.30 \qquad X_2 = 1.20 \qquad X_m = 35$$

The total core, friction, and windage losses may be assumed constant at 1250 W. The motor is connected directly to a 440-V source, and it runs at a slip of 3.5%. Compute the following:

a. Motor speed

b. Input current and power factor

c. Shaft output torque

d. Efficiency of the motor

8.2.2 Determination of Parameters from Tests

To determine the parameters of the equivalent circuit of the three-phase induction motor, it is subjected to tests similar to the open-circuit and short-circuit tests for three-phase transformers.

No-Load Test Like the open-circuit test on a transformer, this test is performed to obtain the shunt parameters of the motor, which represent the magnetizing current and its core loss. The *no-load test* is taken at rated frequency, and the voltage applied to the motor is rated voltage.

When the motor is running at no load, the slip is close to zero; therefore, $n \approx n_s$. Hence, the equivalent rotor resistance (R_2/s) is large. Referring to the approximate equivalent circuit of Fig. 8.2, the rotor impedance containing this large resistance is in parallel with the shunt magnetizing reactance X_m; the parallel combination is approximately equal to jX_m. Thus, the equivalent circuit of Fig. 8.2 reduces to the simple series equivalent circuit shown in Fig. 8.5.

When the motor operates at rated voltage and rated frequency, the combined rotational losses including friction and windage loss, hysteresis and eddy current loss, and stray load loss are assumed to remain constant at any load. This constant value is the value of the rotational loss at no load and is found as follows.

$$P_{\text{rot}} = P_{\text{nl}} - 3I_{\text{nl}}^2 R_1 \tag{8.8}$$

FIGURE 8.5 Approximate equivalent circuit for no-load test.

The three-phase induction motor is considered to be wye connected. Thus, the per-phase no-load resistance R_{nl} is found as

$$R_{nl} = \frac{P_{nl}}{3I_{nl}^2} \tag{8.9}$$

where

P_{nl} = total power input at no load

I_{nl} = stator current per phase at no load

It may be noted that R_{nl} is different from R_1 because the former includes the effects of the rotational losses and the no-load copper loss.

The per-phase no-load impedance is computed as

$$Z_{nl} = \frac{V_{nl}}{\sqrt{3}I_{nl}} = R_{nl} + jX_{nl} \tag{8.10}$$

where V_{nl} is the line-to-line terminal voltage at no load. Therefore, the per-phase no-load reactance is computed as follows:

$$X_{nl} = \sqrt{Z_{nl}^2 - R_{nl}^2} \tag{8.11}$$

From the approximate equivalent circuit of Fig. 8.5, the apparent reactance is seen to be

$$X_{nl} = X_1 + X_m \tag{8.12}$$

DC Test The stator resistance R_1 may be assumed to be equal to its DC value. To find this value, a DC voltage is applied to two stator terminals of the motor and the current and applied voltage are measured. The stator resistance is computed as

$$R_1 = \frac{1}{2}\frac{V_{DC}}{I_{DC}} \tag{8.13}$$

Blocked-Rotor Test In this test, the rotor of the induction motor is blocked so that it cannot rotate; therefore, $s = 1$. Thus, the three-phase motor appears like a short-circuited three-phase transformer. Similarly, a reduced three-phase voltage is applied to the stator such that rated current will flow through the windings.

During normal running conditions of the induction motor, the rotor frequency is proportional to the slip. Therefore, when the performance of the motor is being investigated at, or near, rated loads (at low values of slip), the *blocked-rotor test* should be taken at a lower frequency. A test frequency of 25% of rated frequency is recommended by the Institute of Electrical and Electronics Engineers (IEEE).

The blocked-rotor resistance is found as follows:

$$R_{bl} = \frac{P_{bl}}{3I_{bl}^2} \tag{8.14}$$

where

P_{bl} = total power input at blocked rotor

I_{bl} = stator current per phase at blocked rotor

The blocked-rotor impedance at the test frequency f_{test} is given by

$$Z_{bl} = \frac{V_{bl}}{\sqrt{3}I_{bl}} \tag{8.15}$$

The blocked-rotor reactance at the test frequency f_{test} is computed as

$$X_{bl,test} = \sqrt{Z_{bl}^2 - R_{bl}^2} \tag{8.16}$$

The blocked-rotor reactance computed at the test frequency is corrected to rated frequency by multiplying $X_{bl,test}$ by the ratio (f_{rated}/f_{test}):

$$X_{bl} = \left(\frac{f_{rated}}{f_{test}}\right)X_{bl,test} \tag{8.17}$$

If the exciting current component of the stator current is neglected, the equivalent circuit of the induction motor shown in Fig. 8.2 reduces to that shown in Fig. 8.6 for the blocked-rotor conditions.

With the shunt magnetizing reactance neglected in the approximate equivalent circuit of Fig. 8.6, it is seen that the blocked-rotor resistance and frequency-corrected reactance are related to the series parameters as follows.

$$R_{bl} = R_1 + R_2 \tag{8.18}$$

$$X_{bl} = X_1 + X_2 \tag{8.19}$$

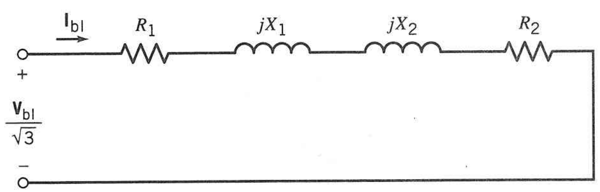

FIGURE 8.6 Approximate equivalent circuit for blocked-rotor test.

The value of the stator resistance R_1 is determined from the DC test. Thus, the rotor resistance R_2 referred to the stator is computed as

$$R_2 = R_{bl} - R_1 \tag{8.20}$$

There is no simple way to apportion the blocked-rotor reactance between the stator leakage reactance X_1 and the rotor leakage reactance X_2 referred to the stator. However, the performance of the induction motor is only slightly affected by the relative distribution of X_{bl} between X_1 and X_2. Thus, it may be assumed that

$$X_1 = X_2 = \tfrac{1}{2}X_{bl} \tag{8.21}$$

Hence, the value of the shunt magnetizing reactance is computed as

$$X_m = X_{nl} - X_1 \tag{8.22}$$

EXAMPLE 8.4

A three-phase, 5-hp, 208-V, four-pole, 60-Hz induction motor is subjected to a no-load test at 60 Hz, a blocked-rotor test at 15 Hz, and a DC test. The following data are obtained.

	No Load	Blocked Rotor	DC
Voltage (V)	208	35	20
Current (A)	4	12	25
Power (W)	250	450	

Determine the parameters of the equivalent circuit and the combined rotational losses of the motor.

Solution From the DC test, the stator resistance is found as

$$R_1 = \tfrac{1}{2}(20/25) = 0.40\ \Omega$$

From the no-load test, the combined rotational loss is computed as

$$P_{rot} = 250 - 3(4)^2(0.4) = 230.8\ \text{W}$$

The no-load impedance parameters are

$$R_{nl} = 250/[3(4)^2] = 5.2\ \Omega$$
$$Z_{nl} = 208/[\sqrt{3}(4)] = 30.0\ \Omega$$
$$X_{nl} = [(30.0)^2 - (5.2)^2]^{1/2} = 29.5\ \Omega$$

From the blocked-rotor test, the reduced-frequency parameters are computed as follows:

$$R_{bl} = 450/[3(12)^2] = 1.04\ \Omega$$
$$Z_{bl} = 35/[\sqrt{3}(12)] = 1.68\ \Omega$$
$$X_{bl,test} = [(1.68)^2 - (1.04)^2]^{1/2} = 1.32\ \Omega$$

The rotor resistance referred to the stator side is computed as

$$R_2 = 1.04 - 0.40 = 0.64\ \Omega$$

The blocked-rotor reactance referred to rated frequency is

$$X_{bl} = (60/15)(1.32) = 5.28\ \Omega$$

This value of the blocked-rotor reactance is equally divided between stator and rotor leakage reactances; thus,

$$X_1 = X_2 = \tfrac{1}{2}(5.28) = 2.64\ \Omega$$

Finally, the magnetizing reactance is found by subtracting X_1 from X_{nl}:

$$X_m = 29.5 - 2.64 = 26.9\ \Omega$$

DRILL PROBLEMS

D8.8 A three-phase, 25-hp, 208-V, six-pole, 60-Hz, wye-connected induction motor is tested with the following results:

	No Load (at 60 Hz)	Blocked Rotor (at 15 Hz)	DC
Voltage (V)	208	25	15
Current (A)	24	66	70
Power (W)	1400	2300	

Determine the parameters of the equivalent circuit of the motor.

D8.9 The following data were obtained from no-load and blocked-rotor tests at rated frequency and a DC test on a three-phase, 30-hp, 460-V, 60-Hz, four-pole, wye-connected induction motor:

	No Load	Blocked Rotor	DC
Voltage (V)	440	95	20
Current (A)	15	52	55
Power (W)	4000	6200	

Determine the parameters of the equivalent circuit and the rotational losses of the motor.

8.3 PERFORMANCE ANALYSIS OF AN INDUCTION MOTOR

When an induction machine is running at no load, the slip is close to zero. Hence, the equivalent rotor resistance R_2/s is infinitely large. This large resistance value results in a very small flow of current I_2. Thus, the electromagnetic torque assumes a small value, just enough to overcome the combined rotational losses consisting of the friction and windage and core losses.

When a mechanical load is connected to the motor shaft, the initial reaction of the motor is for its speed to decrease. Thus, the slip s increases, the equivalent rotor resistance (R_2/s) decreases, and the rotor current I_2 increases. Therefore, the electromagnetic torque increases and, when it becomes equal to the sum of the load torque and rotational losses, the motor will continue to run at a steady-state speed whose value will be less than the no-load speed.

The equivalent circuit of Fig. 8.1 and the power flow diagram of Fig. 8.7 can be used to analyze the steady-state performance of induction motors. In Fig. 8.7, $\omega_s = (2/p)\omega$, where $\omega = 2\pi f = 120\pi$ rad/s.

The power input to the induction motor is expressed as

$$P_{\text{input}} = 3P_1 = 3V_1 I_1 \cos\theta_1 \qquad (8.23)$$

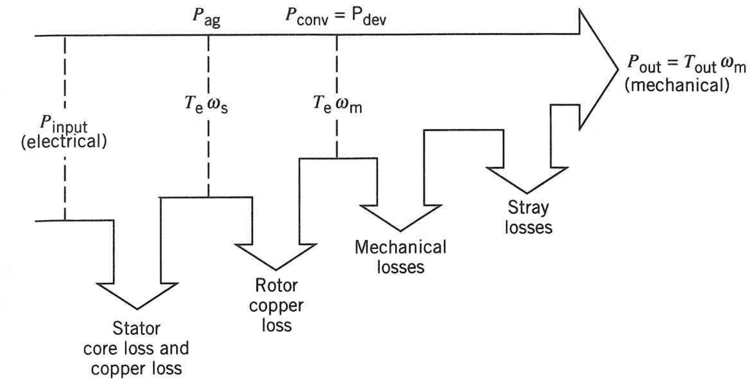

FIGURE 8.7 Power flow diagram for an induction motor.

The stator copper loss SCL is given by

$$\text{SCL} = P_{\text{cu1}} = 3I_1^2 R_1 \qquad (8.24)$$

The equivalent rotor resistance R_2/s represents the power transferred across the air gap from the stator to the rotor. The expression for the *air-gap power* is given by

$$P_{ag} = P_{\text{input}} - \text{SCL} - P_{\text{core}}$$
$$= 3I_2^2\left(\frac{R_2}{s}\right) \qquad (8.25)$$

This air-gap power may be decomposed into two components. The first is the rotor copper loss RCL, and it is expressed as

$$\text{RCL} = P_{\text{cu2}} = 3I_2^2 R_2 \qquad (8.26)$$

The other component of the air-gap power is P_{conv}, the power converted from electrical to mechanical form. It is also called the *developed power* P_{dev}, and it is given by

$$P_{\text{conv}} = P_{\text{dev}} = P_{ag} - \text{RCL}$$
$$= 3I_2^2 R_2\left(\frac{1-s}{s}\right) \qquad (8.27)$$
$$= P_{ag}(1-s)$$

The power output available at the shaft of the motor is found by subtracting the mechanical loss from the power developed:

$$P_{\text{output}} = P_{\text{dev}} - P_{\text{mech}} \qquad (8.28)$$

Finally, the efficiency of the induction motor is calculated as

$$\eta = \frac{P_{\text{output}}}{P_{\text{input}}}100\% \tag{8.29}$$

EXAMPLE 8.5

A three-phase, 25-hp, 230-V, 60-Hz induction motor draws 60 A from the source at 0.866 lagging power factor. The motor losses include the following:

Stator copper loss $P_{\text{cu1}} = 850$ W
Magnetic core loss $P_{\text{core}} = 450$ W
Rotor copper loss $P_{\text{cu2}} = 1050$ W
Mechanical loss $P_{\text{mech}} = 500$ W

Find the following:

a. Air-gap power P_{ag}
b. Slip s
c. Mechanical power developed P_{dev}
d. Output power
e. Efficiency of the motor

Solution

a. The terminal voltage per phase is taken as reference phasor; thus,

$$\mathbf{V}_t = (230/\sqrt{3})\angle 0^\circ = 132.8\angle 0^\circ \text{ V} \quad \text{(line-to-neutral)}$$

The power input to the motor is found as

$$P_{\text{input}} = 3V_tI_1\text{PF} = 3(132.8)(60)(0.866) = 20{,}700 \text{ W}$$

Therefore, the power transferred across the air gap is calculated as

$$P_{\text{ag}} = P_{\text{input}} - P_{\text{core}} - P_{\text{cu1}} = 20{,}700 - 450 - 850 = 19{,}400 \text{ W}$$

b. The power transferred across the air gap P_{ag} may also be expressed, in terms of the rotor copper loss, as

$$P_{\text{ag}} = 3I_2^2R_2/s = P_{\text{cu2}}/s$$

Thus, the slip is

$$s = P_{\text{cu2}}/P_{\text{ag}} = 1050/19{,}400 = 0.054$$

c. The power developed is given by

$$P_{\text{dev}} = P_{\text{ag}} - P_{\text{cu2}} = 19{,}400 - 1050 = 18{,}350 \text{ W}$$

d. The power output of the motor is given by

$$P_{\text{output}} = P_{\text{dev}} - P_{\text{mech}} = 18{,}350 - 500 = 17{,}850 \text{ W}$$

e. Therefore, the efficiency of the motor is found as

$$\eta = (P_{\text{output}}/P_{\text{input}})100\% = (17{,}850/20{,}700)100\% = 86.2\%$$

EXAMPLE 8.6

A three-phase, 50-hp, 480-V, 60-Hz, four-pole, induction motor has the following data:

$$R_1 = 0.100\ \Omega/\text{phase} \qquad X_1 = 0.35\ \Omega/\text{phase}$$
$$R_2 = 0.125\ \Omega/\text{phase} \qquad X_2 = 0.40\ \Omega/\text{phase}$$

Stator core losses and mechanical (friction and windage) losses are 1200 W and 900 W, respectively. At no load, the motor draws 21 A at 0.0 power factor. When the motor is operated at $s = 0.025$, determine the following:

a. Line current and power factor
b. Electromagnetic torque
c. Output power
d. Efficiency of the motor

Solution

a. The approximate per-phase equivalent circuit is shown in Fig. 8.8. The terminal voltage per phase is taken as reference phasor; thus,

$$\mathbf{V}_1 = (480/\sqrt{3})\angle 0^\circ = 277.1\angle 0^\circ \text{ V} \quad \text{(line-to-neutral)}$$

At no load, the rotor current is approximately zero; $\mathbf{I}_2 \approx 0.0$. Therefore, the magnetizing current $\mathbf{I}_m$ is equal to the no-load stator current $\mathbf{I}_{1,\text{nl}}$; that is,

$$\mathbf{I}_m = \mathbf{I}_{1,\text{nl}} = 21\angle -\cos^{-1} 0.0 = 21\angle -90^\circ \text{ A}$$

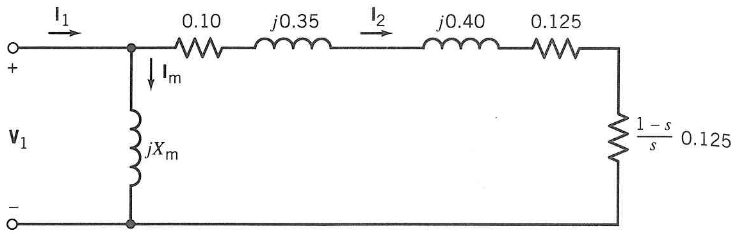

FIGURE 8.8 Equivalent circuit for motor of Example 8.6.

At a slip $s = 0.025$, the rotor current is found as

$$\mathbf{I}_2 = \mathbf{V}_1/[R_1 + (R_2/s) + j(X_1 + X_2)]$$
$$= (277.1\angle 0°)/[0.100 + (0.125/0.025) + j(0.35 + 0.40)]$$
$$= 53.8\angle -8.4° \text{ A}$$

Therefore, the stator current is computed as

$$\mathbf{I}_1 = \mathbf{I}_2 + \mathbf{I}_m = 53.8\angle -8.4° + 21\angle -90° = 60.5\angle -28.5° \text{ A}$$

The power factor is found as

$$\text{PF} = \cos(\angle \mathbf{V}_1 - \angle \mathbf{I}_1) = \cos 28.5° = 0.88 \text{ lagging}$$

b. The synchronous speed is given by

$$n_s = 120f/p = (120)(60)/4 = 1800 \text{ rpm}$$
$$\omega_s = 2\pi n_s/60 = 2\pi(1800)/60 = 188.5 \text{ rad/s}$$

The air-gap power is computed as

$$P_{ag} = 3I_2^2R_2/s = 3(53.8)^2(0.125)/0.025 = 43{,}417 \text{ W}$$

Hence, the electromagnetic torque developed is determined from the air-gap power; that is,

$$T_e = P_{ag}/\omega_s = 43{,}417/188.5 = 230 \text{ N-m}$$

c. The power developed by the motor is given by

$$P_{dev} = (1 - s)P_{ag} = (1 - 0.025)43{,}417 = 42{,}332 \text{ W}$$

Therefore, the output power is found as

$$P_{output} = P_{dev} - P_{mech} = 42{,}332 - 900 = 41{,}432 \text{ W}$$

d. The power input to the motor is given by

$$P_{input} = P_{ag} + P_{cu1} + P_{core} = P_{ag} + 3I_1^2R_1 + P_{core}$$
$$= 43{,}417 + 3(60.5)^2(0.10) + 1200 = 45{,}715 \text{ W}$$

Therefore, the efficiency is

$$\eta = P_{output}/P_{input} = (41{,}432/45{,}715)100\% = 90.6\%$$

DRILL PROBLEMS

D8.10 A three-phase, six-pole, 60-Hz induction motor is operating at a speed of 1152 rpm. The power input to the motor is 44 kW, the rotational losses are 500 W, and the stator copper loss is 1600 W. Find the following:

a. Slip
b. Air-gap power
c. Rotor copper loss
d. Developed torque and developed horsepower
e. Output torque and output horsepower

D8.11 A three-phase, four-pole, 60-Hz, wye-connected wound-rotor induction motor is rated at 15 hp and 208 V. Its equivalent circuit parameters are

$$R_1 = 0.25\ \Omega \qquad R_2 = 0.15\ \Omega$$
$$X_1 = 0.50\ \Omega \qquad X_2 = 0.50\ \Omega \qquad X_m = 18\ \Omega$$

The combined rotational losses consisting of the friction and windage plus core losses amount to 300 W. For a slip of 4%, find the following:

a. Line current
b. Air-gap power
c. Power converted from electrical to mechanical form
d. Efficiency of the motor

D8.12 A four-pole, 60-Hz induction motor is rated at 40 hp and 440 V. The motor drives a load at 1710 rpm. The core loss and friction and windage loss are 450 W and 250 W, respectively. The motor parameters are given in Ω/phase as follows:

$$R_1 = 0.15 \qquad R_2 = 1.20$$
$$X_1 = 0.75 \qquad X_2 = 0.75 \qquad X_m = 25$$

Determine the following:

a. Line current and power factor
b. Real and reactive power input
c. Air-gap power
d. Mechanical power and torque developed
e. Shaft horsepower and torque
f. Efficiency of the motor

8.4 TORQUE-SPEED CHARACTERISTICS

The torque-speed characteristics of three-phase induction motors can be analyzed from the approximate equivalent circuit of Fig. 8.3 and the power flow diagram of Fig. 8.7.

8.4.1 Starting Torque

At starting, the slip is unity [$s = (n_s - n_r)/n_s = n_s/n_s = 1$]. The equivalent circuit of Fig. 8.3 reduces to that of Fig. 8.9.

The starting electromagnetic torque $T_{e,\text{start}}$ is given by

$$T_{e,\text{start}} = P_{ag,\text{start}}/\omega_s \tag{8.30}$$

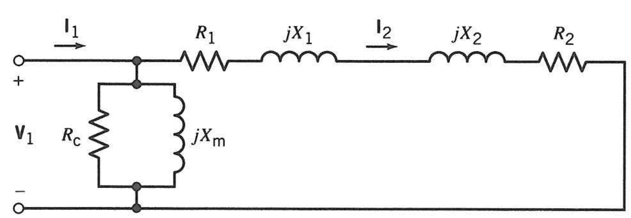

FIGURE 8.9 Equivalent circuit for starting conditions.

The power transferred across the air gap at starting is

$$P_{ag,\text{start}} = 3(I_{2,\text{start}})^2\left(\frac{R_2}{s}\right) = 3(I_{2,\text{start}})^2 R_2 \tag{8.31}$$

where the rotor current $\mathbf{I}_{2,\text{start}}$ is given by

$$\mathbf{I}_{2,\text{start}} = \frac{\mathbf{V}_1}{R_1 + R_2 + j(X_1 + X_2)} \tag{8.32}$$

Taking the magnitude of $\mathbf{I}_{2,\text{start}}$ and substituting it in Eqs. 8.30 and 8.31, the starting torque is obtained.

$$T_{e,\text{start}} = \frac{3V_1^2 R_2}{\omega_s[(R_1 + R_2)^2 + (X_1 + X_2)^2]} \tag{8.33}$$

As may be seen from Eq. 8.32, the starting current is large compared to rated load current. This is because R_1, R_2, X_1, and X_2 are small and V_1 is the rated terminal voltage. In order to limit this starting current, for larger motors such as those rated above 5 hp, an applied voltage smaller than rated voltage is used to start the induction motor.

EXAMPLE 8.7

The three-phase, 25-hp, 440-V induction motor of Example 8.2 has the following impedances in Ω per phase referred to the stator.

$$R_1 = 0.50 \qquad R_2 = 0.35$$
$$X_1 = 1.20 \qquad X_2 = 1.20 \qquad X_m = 25$$

The motor is operated at rated voltage and rated frequency.

a. What is the starting torque?
b. When the rotor resistance is doubled, what is the new value of starting torque?

Solution The approximate equivalent circuit at starting is shown in Fig. 8.10.

a. The terminal voltage per phase is taken as reference phasor; thus,

$$\mathbf{V}_1 = (440/\sqrt{3})\angle 0^\circ = 254.0\angle 0^\circ \text{ V} \quad \text{(line-to-neutral)}$$

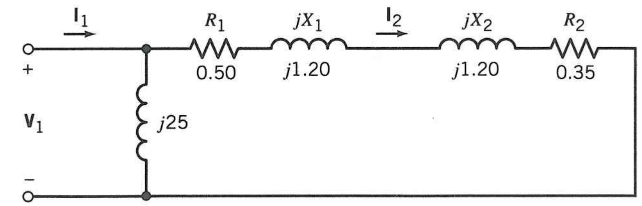

FIGURE 8.10 Approximate equivalent circuit for motor of Example 8.7.

By using Eq. 8.33, the starting torque is calculated as follows.

$$T_{e,\text{start}} = \frac{3(254.0)^2(0.35)}{188.5[(0.50 + 0.35)^2 + (1.20 + 1.20)^2]} = 55.4 \text{ N-m}$$

b. When rotor resistance is doubled, $R_2 = 2(0.35) = 0.70\ \Omega$, the new value of starting torque is found as follows:

$$T_{e,\text{start}} = \frac{3(254.0)^2(0.70)}{188.5[(0.50 + 0.70)^2 + (1.20 + 1.20)^2]} = 99.8 \text{ N-m}$$

8.4.2 Torque Versus Speed

The torque-speed characteristic of an induction motor is studied in terms of its torque-versus-slip relationship. The slip and speed are related through Eq. 8.1.

From the equivalent circuit of Fig. 8.3, the expression for the electromagnetic torque T_e is derived as

$$T_e = \frac{P_{ag}}{\omega_s} \tag{8.34}$$

where

$$P_{ag} = 3I_2^2\left(\frac{R_2}{s}\right) \tag{8.35}$$

$$I_2 = \frac{V_1}{|(R_1 + R_2/s) + j(X_1 + X_2)|} = \frac{V_1}{\sqrt{(R_1 + R_2/s)^2 + (X_1 + X_2)^2}} \tag{8.36}$$

Substituting Eqs. 8.35 and 8.36 into Eq. 8.34, the expression for the torque is obtained.

$$T_e = \frac{3V_1^2(R_2/s)}{\omega_s[(R_1 + R_2/s)^2 + (X_1 + X_2)^2]} \tag{8.37}$$

Equation 8.37 can be further simplified as follows:

$$T_e = \frac{3V_1^2 R_2 s}{\omega_s[(R_1 s + R_2)^2 + (X_1 + X_2)^2 s^2]} \tag{8.38}$$

In order to plot Eq. 8.38, the maximum torque $T_{e,\max}$ and the slip $s_{\max}$ at which it occurs have to be determined. The maximization theorem from calculus is applied; that is, take the derivative of T_e with respect to s, set the derivative to zero, and solve for s. The solution yields $s = s_{\max}$, where

$$s_{\max} = \frac{R_2}{\sqrt{R_1^2 + (X_1 + X_2)^2}} \tag{8.39}$$

It can be shown that the second derivative of T_e with respect to s is negative. Therefore, T_e is at its maximum value at $s_{\max}$. Substituting the value of $s_{\max}$ given by Eq. 8.39 into Eq. 8.38 and simplifying, the maximum torque $T_{e,\max}$ is obtained as follows:

$$T_{e,\max} = \frac{3V_1^2}{2\omega_s[R_1 + \sqrt{R_1^2 + (X_1 + X_2)^2}]} \tag{8.40}$$

The synchronous speed is given by

$$\omega_s = \left(\frac{2}{p}\right)2\pi f = \frac{4\pi f}{p} \tag{8.41}$$

With the starting torque $T_{e,\text{start}}$, the maximum torque $T_{e,\max}$, and $s_{\max}$ known, the torque-speed curve can now be plotted, and it is shown in Fig. 8.11.

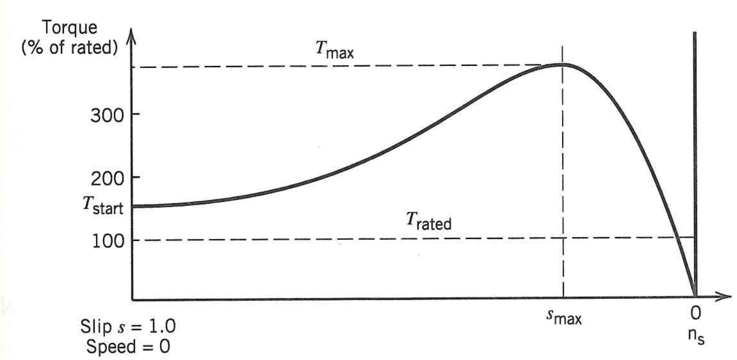

FIGURE 8.11 Torque-speed curve of an induction motor.