Electromechanical Energy Devices and Power Systems

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Preface

This textbook is intended to serve as a book for a junior level, one-semester course in electric power engineering or energy conversion. Students using this book should have taken an electric circuits analysis course and have been introduced to electromagnetic fields. The student should have gained a working knowledge of complex algebra, sinusoidal analysis, phasor diagrams, phasor analysis, and basic power concepts.

As the curricula of electrical engineering programs became more and more overcrowded, many electrical engineering departments decided to stop requiring all electrical engineering students to take two power engineering courses and began (a) requiring a combined electromechanical energy conversion and power systems course, or (b) to drop the power system analysis portion altogether and require only an electromechanical energy conversion course.

In a recent article prepared by the Power Engineering Education Committee of the Power Engineering Society of IEEE, it was discovered that almost 15% of the electrical engineering programs across the United States and Canada require a combined course in energy conversion and power systems. The objectives in preparing this book were (a) to provide all electrical engineering students with an understanding of the principles of electromechanical energy conversion, as well as an overview of the power system, and (b) to provide students who are interested in power engineering sufficient understanding of machinery and power systems so they can take more advanced courses in the power area. The book has been written to satisfy the requirements of electrical engineering programs that offer a basic core course in electric power or energy conversion to serve all electrical engineering students. Our goal has been to write a book that could be used either for an electromechanical energy conversion course, or a course that combines the electromechanical energy conversion and power systems analysis into one course with primary emphasis on electromechanical energy conversion topics.

Features

This textbook offers a number of special features including:

- *Drill Problems* Drill exercises are presented within the different chapters to enable students to test their understanding of the subject matter just studied.
- *Homework Problems* Several homework problems are given at the end of the chapter with varying levels of difficulty—from simple problems requiring only simple manipulations of the various formulas to more difficult ones requiring in-depth analysis.
- *Illustrations and Diagrams* Several illustrations and circuit diagrams are presented in the text to help demonstrate and describe the theory being studied. The use of circuit diagrams and phasor diagrams proves very helpful in understanding the problem and its solution.
- *Numerical Examples* Illustrative and numerical examples are used extensively throughout the text to help the student fully understand the concepts and how the theory applies to the solution of application type problems.

Text Organization

A chapter-by-chapter overview of the textbook follows:

Chapter 1 This chapter introduces the student to different energy sources and various methods of electric energy conversion.

Chapter 2 The second chapter presents an overview of the electric power system and its components.

Chapter 3 This chapter reviews circuit and power concepts in electrical circuits.

Chapter 4 This chapter introduces the student to magnetic circuits and transformers, both single-phase and three-phase transformers.

Chapter 5 Chapter 5 presents the fundamentals of rotating machines. It provides a unified approach to all the electromechanical energy conversion machines discussed in this book.

Chapter 6 The operational characteristics of various types of DC machines are discussed.

Chapter 7 Chapter 7 presents a thorough coverage of both wound-rotor and salient-pole synchronous machines (generators and motors).

Chapter 8 Chapter 8 presents the theory and application of both three-phase and single-phase induction motors.

Chapter 9 This chapter covers (a) the basic parameters of the conductors that are used in transmission lines, (b) the various formulas for these parameters, and (c) the proper use of tables of conductor characteristics for obtaining the values of the parameters. Models of transmission lines are then presented for use in power system studies.

Chapter 10 This chapter presents the different power flow solution techniques. In other words, the focus of this chapter is the normal steady state operation of the power system.

Chapter 11 Chapter 11 covers the abnormal operating conditions of power systems including fault studies, system protection, and power system stability.

Appendices Six appendices provide supplementary information for material covered in the 11 chapters of this textbook. The final one, Appendix G, contains a glossary of key terms.

In writing this book, it was our goal to provide a degree of flexibility in meeting the needs of various engineering curricula as regards the conduct of the associated power courses. Listed below are suggestions that can serve as guidelines in the use of the text material.

A. One-semester course on electric machines and power systems:
   Chapter 1
   Chapter 2
   Chapter 3 (if needed)
   Chapter 4
   Chapter 5
   Chapter 6 (excluding Section 6.8)
   Chapter 7 (excluding Section 7.5)
   Chapter 8 (excluding Section 8.5)
   Chapter 9
   A brief coverage of Chapters 10 and 11

B. One-semester course on electric machines:
   Chapter 1
   Chapter 2
   Chapter 3 (if needed)
   Chapter 4
   Chapter 5
   Chapter 6
   Chapter 7
   Chapter 8

Supplements

Solutions Manual A solutions manual provides solutions to all drill and end-of-chapter problems. Answers to problems are not provided in the textbook.

Software For those interested, the authors have developed a Power Systems Simulation Package (PSSP), which is available separately from our publisher, John Wiley & Sons, Inc. PSSP has three major functions:

1. Power flow analysis of an electric power system
2. Short circuit analysis of an electric power system
3. Data preparation and editing

In closing, it should be stated that the intent has not been to go in great depth into electromechanical energy conversion or power system analysis, but to provide a thorough and understandable coverage of these two topics to undergraduate electrical engineering students. Our hope has been to expose the student to more than a familiarity with just conventional machines. The authors are of the opinion that although this is a worthwhile objective in itself, it is also important that electrical engineering graduates possess some knowledge of electric power systems. Time for these topics is made available by forgoing some of the in-depth study of conventional machines. The authors would appreciate the comments of the user of this book.

June, 1993

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Electromechanical Energy Devices and Power Systems

One

Energy Resources and Electric Energy Conversion

1.1 INTRODUCTION

The ability of humans to develop sources of energy needed to accomplish useful work has played an essential role in the continual improvement in the standard of living of people around the globe. The use of energy can be seen in everyday devices such as home appliances and machinery. Energy consumption in homes and factories increased beyond the point where useful forms of energy could be produced at the locations at which energy was being used. Hence, centralized energy processing and generating stations, together with elaborate transmission and distribution systems, were developed and the electric power system emerged as a tool for converting and transmitting energy. Energy is converted from its basic source, such as coal or hydro, into electric energy at places usually remote from population centers. Then the electric energy is sent over transmission and distribution systems to various locations, where it is converted to light, heat, and mechanical energy.

The objective of this book is to investigate how energy conversion, particularly electromechanical energy conversion, and transmission and distribution of electric energy take place. The devices used in the operation of a power system are studied. The book is structured so that a power system is described element by element in each chapter.

The first section of this chapter gives a brief description of the various energy resources available to humans. These energy resources are usually not in directly usable form; they have to be converted into more useful forms. The conventional electric energy conversion techniques that are currently in use are described in the next section. Finally, alternative sources or nonconventional energy conversion techniques are presented in the last section.

1.2 ENERGY RESOURCES

Energy resources are the various materials that contain energy in usable quantities. These are present in any of the various energy forms that are transformable to other forms, including electrical, mechanical, chemical, and nuclear energy.

The main energy forms include chemical, hydro, nuclear, and geothermal energy. Chemical energy includes such fuels as coal, oil, and natural gas. Energy resources are usually classified in two general categories: renewable resources and expendable resources. Renewable resources, such as water, wind, solar, and tide, are replaced continuously by nature. Expendable resources, such as oil and coal, are expended when used.

Energy may be classified as either primary energy, which is obtained directly from nature, or secondary energy, which is energy derived from primary energy. Primary energy sources include hydro, solar, wind, oil, natural gas, and geothermal. Electrical energy is a form of secondary energy. It is derived from primary energy by using thermal power plants to convert heat energy and hydroelectric plants to transform the energy of water under pressure.

Energy resources are being consumed at a high rate because of the growing requirements of industries and the increasing demands of people. It has been predicted that the world's supply of fossil fuels will be used up within a few hundred years.

The rate of consumption of oil has been rapidly growing. In many countries, oil has been the primary fuel used for electricity generation. It is estimated that 60% of the world's proven oil reserves are in the Middle East and only about 10% are in the United States. The fast growth of oil consumption is due mainly to its use as fuel for automobiles and airplanes and the fact that it is easier to recover and transform to other energy forms than solid fuels.

The distribution of natural gas reserves is not accurately known. Estimates of these reserves are approximate. In the United States, the use of natural gas has risen consistently because of its relatively low cost.

It has been estimated that the world's hydropower resources amount to about twice the current annual generation of all hydroelectric plants in the world. The available hydropower depends on water inflows, and estimates of the available hydro energy could be very approximate.

Geothermal energy is an almost limitless reserve. The temperature of the Earth increases with depth in the Earth's crust. For example, the temperature is about 1200°C at a depth of 30 miles. During earthquakes or volcanic eruptions, molten rocks may be able to escape to the surface in the form of lava. Some of the molten rocks do not reach the surface, but they produce gases and steam by heating water they encounter on their way to the surface. The heated water may escape through the surface as hot springs, and the steam can be used in geothermal plants.

Geothermal energy has been used to supply hot water for the city of Reykjavik, the capital of Iceland. In the northern part of California, geothermal

It has been estimated that the heat content of the world's reserves of uranium is more than 300 times the heat content of the world's reserves of fossil fuels. However, the cost of mining for uranium is extremely high.

Using nuclear fission, current nuclear power plants convert only a small fraction of the energy present in the nuclear fuel. However, the unexpended fuel can be reprocessed for use in breeder reactors.

In nuclear fission, the nucleus of a heavy element such as uranium-235 is split. In nuclear fusion, on the other hand, nuclei of light elements are fused to form heavier nuclei. Either process is accompanied by the release of large amounts of heat.

Nuclear fusion is a more attractive energy source than fission because it is inherently safe, does not produce radioactive waste, does not present any danger of explosion, and cannot result in runaway meltdown, and the fuel (deuterium) used is readily available from the oceans. However, it is still not technically feasible to use nuclear fusion for commercial electric power generation.

The oceans regularly rise and fall in response to the relative positions of the Sun, Earth, and Moon. The water elevation is not the same at different places. The variation of water elevation can be used by electric power plants to produce electric energy. However, such power plants are extremely expensive, and the power output is quite variable because of the continually changing tides.

Wind energy has been used in windmills around the world in countries such as The Netherlands. Wind turbines have also been used for electrical energy production in many countries, including the United States, but these have been in the low power (few megawatt) levels.

The Sun is the ultimate source of energy because it has immense energy reserves. It has been estimated that the total amount of solar energy received by Earth in a year exceeds its total energy requirements. Solar energy is usually used for space heating and water heating. Although the use of solar energy is growing, it is still at a low level. Because the power density is low at the surface of the Earth, large collectors are necessary to accumulate small amounts of power.

Research has been under way to develop other methods of utilizing solar energy for electric power production. However, solar cells have not yet become a feasible alternative for bulk, or large-scale, power production.

1.3 CONVENTIONAL METHODS FOR ELECTRIC ENERGY CONVERSION

At present, two methods are commonly used for electric bulk-power generation. Both methods employ an electric generator that converts the mechanical energy of the prime mover to electrical energy. The main difference between the two methods is the source of the mechanical energy used to rotate the generator.

One method makes use of water under hydraulic pressure to provide the

the generator shaft. The efficiency of conversion of the energy available from the water to electric energy is quite high—up to 80%–90%.

The other method employs a boiler to convert the energy of the fuel (coal, oil, or nuclear fuel) into heat energy, which is used to transform water into high-temperature steam at very high pressure. The steam rotates the steam turbine in the same way as the water turns the hydraulic turbine. The efficiency of conversion of the energy available from the fuel to electric energy is much lower than in the first method, typically ranging from 30% to 40%.

The different types of electric bulk-power generating plants are described in this section. Other power plants employing variations of these methods, such as pumped-storage plants, are also briefly discussed.

1.3.1 Hydroelectric Plants

A schematic diagram of a hydroelectric power plant is shown in Fig. 1.1. The electric power generated by a hydroelectric plant depends on the amount of water flowing through the hydraulic turbine and on the water pressure. The plant capacity, expressed in kilowatts, is given by

$$P = 9.81QH\eta \tag{1.1}$$

where

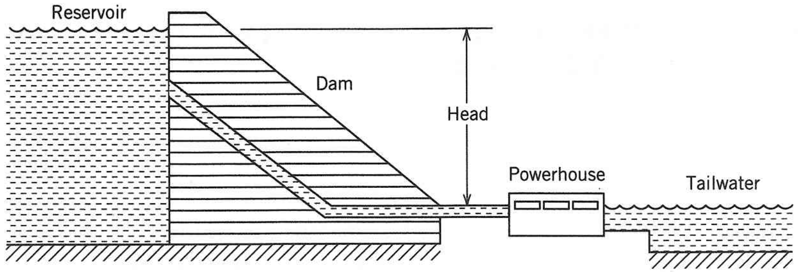
Q = rate of flow in m^3/s

H = pressure head of the water in m

η = efficiency expressed as a fraction of 1

In order to increase the pressure head of the water, river dams or diversion canals are constructed. The hydraulic turbine converts the kinetic energy of the water coming from the reservoir into mechanical energy available at its rotating shaft. A hydraulic turbine may be classified as either an impulse turbine or a reaction turbine.

In a high-pressure impulse turbine, a convergent nozzle transforms the potential energy of the hydrostatic pressure into kinetic energy of moving water.



The pressure at the nozzle outlet is atmospheric. The amount of water entering the turbine is adjusted automatically to obtain the desired power output.

When the water flow rate Q is high and the pressure head H is rather low, the reaction turbine is preferred. In a reaction turbine, water is introduced to the runner blades through a convergent pipe section known as a constrictor, wherein part of the potential energy of the water is converted into kinetic energy. Further energy conversion is carried out at the turbine blades.

The turbine and generator of an electric power plant are located in a common shaft. This results in a common rotational speed n_s, which depends on the number of poles p of the generator rotor and the operating frequency f of the electric power system. Thus,

$$n_s = \frac{120f}{p} \tag{1.2}$$

Hydroelectric plants are usually designed and built for multiple reasons and purposes in addition to electric energy production. These include river flood control, river navigation, irrigation of agricultural lands, water and electric energy supply to power-consuming industries using local raw material resources, and recreational purposes.

1.3.2 Pumped-Storage Plants

The electric power generated by the power plants must be exactly equal to the electric power consumed by various users at all times. The electrical demand of individual consumers may be described as random and irregular, so the total usage of electric power is nonuniform.

The variation of the total power needed by users with the time of day is described by the demand curve. This curve contains a number of peaks and valleys; that is, the total power required is high during certain hours and low during other hours. This indicates that some of the generators are operated below rated capacity, or even shut down, some of the time, which means that much money is invested in power equipment that is not being fully utilized.

Thermal power plants are operated most economically at a constant power level. They are not designed for quick and large changes in generation level. Start-ups and shutdowns of thermal plants take much time and effort and lead unnecessarily to rapid depreciation of the equipment. Therefore, "peak shaving" is and will be done by pumped-storage hydroelectric plants, which can be started up and brought to full power within a few minutes.

During the off-peak periods when the demand for electricity is at a minimum, the pumped-storage hydro plant is used as a pump to transfer water from the lower reservoir to the upper reservoir. At the same time, the electric machine operates as a synchronous condenser, resulting in an improved overall system power factor. During the periods of peak demand, the pumped-storage hydro

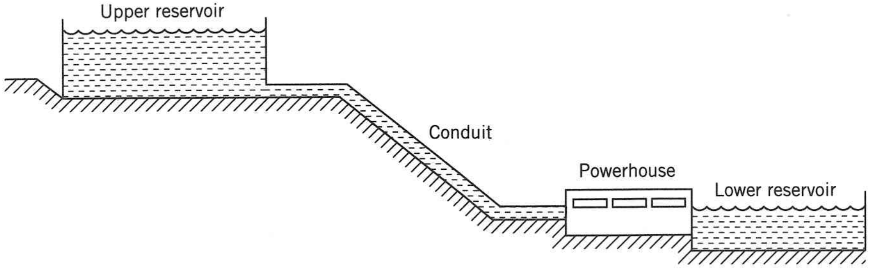


FIGURE 1.2 Schematic diagram of a pumped-storage power plant.

plant uses the water stored in the upper reservoir to drive the electric generator to produce electric power. A schematic diagram of a pumped-storage power plant is shown in Fig. 1.2.

Pumped-storage plants employ a reversible hydraulic turbine that performs the functions of both turbine and pump. This mechanism is illustrated in Fig. 1.3.

The prospects for the application of pumped-storage plants depend largely on their efficiency, that is, the ratio of the energy produced by the generator to the energy expended by the pump. Most modern pumped-storage plants have efficiencies of 70%–75%. Another advantage of pumped-storage plants is their low cost of construction, since there is no need to build a high dam across a river, long tunnels, and so forth.

1.3.3 Condensing Power Plants

A condensing power plant converts the energy of a chemical fuel into mechanical energy, which is then converted into electric energy by a generator. Heat engines are used to convert the energy of the steam or gas into mechanical energy that is available at the rotating shaft.

Heat engines may be classified according to the working substance used, which may be either steam or gas. They may also be classified according to the method used to convert heat energy to mechanical energy as either piston or rotary. The different types of condensing power plants are given in Table 1.1.

The internal combustion engine is not usually used in power plants, except for low-power applications. At modern thermal power plants, steam turbines are generally employed. To increase the efficiency of heat engines, the temperature and pressure of the working substance are raised to very high levels. Modern steam power units operate at a steam temperature of 600°C at a pressure of 30 MPa, where 1 MPa = 10^6 N/m^2. After the working substance (steam) exits from the turbine, it is cooled by water to reduce its temperature down to 30°–40°C, which is followed by a sharp decrease in steam pressure. A schematic diagram of a thermal power plant is shown in Fig. 1.4.

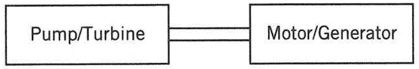


Table 1.1 Types of Condensing Power Plants

Principle of Operation	Work Substance: Steam	Work Substance: Gas
Piston (engine)	Steam engine	Internal combustion
Rotary	Steam turbine	Gas turbine

Steam is produced in the steam generator, or boiler. The water supplied to the boiler is of high purity. Water flows through small-diameter steel pipes designed to withstand high steam pressures. The chemical fuel in the form of pulverized coal, gas, or atomized oil is burned at 1500°–2000°C in the furnace. Large amounts of heated air are introduced by fans in order to improve fuel combustion. The heat produced raises the temperature of water, converts the water into steam, and increases the temperature and pressure to operating levels. Hot gases are removed from the steam generator by induced-draft fans and brought out to the atmosphere through the stack.

Two types of steam generators are employed: drum-type and once-through boilers. The drum-type boiler contains a steel drum that has steam and water in its upper and lower parts, respectively. When the steam leaves the drum, it is heated again in the steam superheater to raise the temperature further.

The once-through boiler has no drum. Water flows through the boiler tubes, is converted into steam, is heated further in a steam superheater, and is then introduced into the turbine. Once-through boilers require fast response and precise control of the feedwater, which must be of high quality. Once-through boilers are cheaper than drum-type boilers and are capable of operating at higher pressures than the drum type.

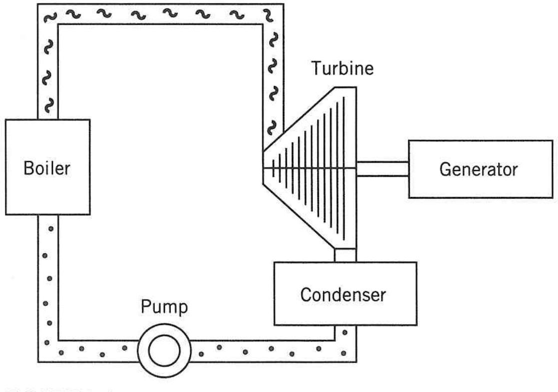


FIGURE 1.4 Schematic diagram of a thermal power plant.

The steam produced by the boiler, at a temperature of about 600°C and a pressure of 30 MPa, is passed to nozzles that transform the internal energy of the steam into kinetic energy. After the steam leaves the nozzles, it enters the turbine rotor blades.

The exhaust steam from the turbine is brought to the condenser, which is used to cool and condense the steam. Cooling water enters the condenser at 10°–15°C and leaves at 20°–25°C. The steam flows over the tubes from the top downward, condenses, exits at the bottom, and is returned as water to the boiler.

1.3.4 Geothermal Power Plants

Geothermal power plants use steam extracted from the Earth. Beneath the Earth's surface, the temperature increases with depth, reaching 1000°–1200°C at a depth of 6–30 miles. The presence of hot springs is a good indication that fairly high temperatures exist right under the surface, and these places are good candidate sites for geothermal power development.

In geothermal power plants, the hot geothermal steam is used just like the steam produced by the boiler in a condensing power plant. Unlike the steam produced in the boilers of thermal power plants, however, the steam derived from the Earth contains various impurities. This dirty steam can cause corrosion and scaling in the different parts of the power plant, and the exhaust steam and gases contribute to pollution of the surrounding atmosphere. Thus, there is a need to clean the steam before introducing it into the turbine. For this purpose, specially designed corrosion-resistant equipment and steam cleaners, called scrubbers, are employed and a rigorous maintenance program is implemented.

A schematic diagram of a geothermal power plant is shown in Fig. 1.5. The hot geothermal steam is passed through a low-pressure turbine because the steam pressures available are much lower than those at modern coal or nuclear thermal power plants. As the steam comes out of the turbine, it is brought to a condenser. The condensed steam is then taken to a cooling tower, where it is sprayed downward against the flow of air blown at the water spray by a fan. The greater part of the water is evaporated into the atmosphere by the cooling tower, and the remaining water that reaches the bottom of the tower is returned to the ground through dry wells.

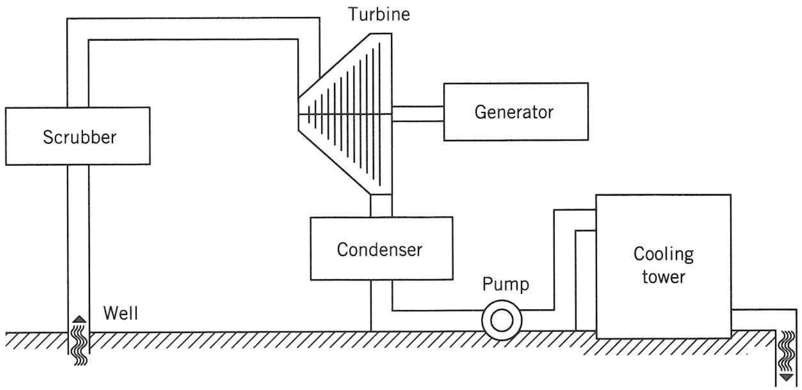


FIGURE 1.5 Schematic diagram of a geothermal power plant.

1.3.5 Gas-Turbine Power Plants

Gas-turbine power plants are normally designed to operate during peak conditions and thus are called peaking power plants. The mixture resulting from the combustion of the fuel and hot air is used to rotate the turbine. The gas turbine converts the heat energy of the gases into mechanical energy for driving the electric generator.

Gas turbines are designed and are operated in much the same way as steam turbines. They have approximately the same efficiency as internal combustion engines. Gas turbines require less space than steam turbines or internal combustion engines.

Modern gas-turbine power plants usually employ liquid fuel, but they may also use gaseous fuel, natural gas, or gas produced artificially by gasification of a solid fuel. In the gas turbine, liquid or gaseous fuel and air are brought into the combustion chamber, where hot combustion gases are produced at high pressure. These gases are discharged against the rotor blades of the turbine. The turbine rotates the electric generator.

1.3.6 Combined Steam- and Gas-Turbine Power Plants

A gas-turbine generating unit may be present together with a steam-turbine unit in a single power plant. In this combination power plant, the gas-turbine unit and the steam unit share the heat produced by burning the fuel. Part of the heat energy is used to produce steam and raise it to the proper temperature and pressure to drive the steam turbine. The hot gases produced during burning of the fuel are used to rotate the gas turbine. Upon leaving the turbine, the exhaust gases are used to heat the feedwater of the steam generator.

The overall efficiency of a combined power plant is significantly higher than that of a single gas-turbine power plant or a conventional steam power plant.

1.3.7 Nuclear Power Plants

In a nuclear power plant, the heat energy released by nuclear fission is used to produce the steam that rotates the turbine that drives the electric generator. The major difference between a nuclear power plant and a conventional thermal power plant is the method of producing heat energy. In the former, heat is produced in a reactor by fission of the nuclear fuel; in the latter, heat is produced in a boiler by the combustion of chemical fuel. In both cases, the heat produced is used to convert water into steam, which is fed into the steam turbine.

The nuclear reactor is the principal component of a nuclear power plant, and it is shown in Fig. 1.6. In the nuclear reactor, nuclei of uranium are split when bombarded by neutrons. The resulting nuclear fragments, neutrons, and other products of the process scatter with extremely high velocities, and they trigger a chain reaction in which more nuclei are split, continuing the process. This is accompanied by the release of enormous amounts of energy in the form of heat.

The nuclear fuel in the form of rods is placed into fuel tubes in the reactor core. Nuclear reaction takes place in the rods, and a large amount of heat is released in the process. The reaction is controlled by raising and lowering the control rods in the reactor core. A coolant flows through the reactor in order to remove the heat produced. The coolant used may be light water, heavy water, steam, or other fluid. The coolant flows over the surface of the fuel rods, where it is heated up. As the coolant exits, it brings out the heat. A large part of the energy released during nuclear reaction is expended in heating the nuclear fuel. Since heat is transferred to the coolant by convection, the velocity of the coolant must be high (3–7 m/s) to increase the output heat.

1.4 ALTERNATIVE METHODS FOR ELECTRIC ENERGY CONVERSION

With the conventional methods used to convert various forms of energy into electrical energy, chemical fuels are burned in furnaces, thereby dissipating their limited reserves. The maximum efficiency of thermal power plants is about 40%. This low conversion efficiency means that a large part of the heat generated is lost to the environment, contributing to the thermal pollution of nearby bodies of water. In addition, the inefficient combustion of fuel results in large amounts of waste by-products being released to the atmosphere. These inherent disadvantages of thermal power plants, however, do not imply that they should be discontinued. In the future, they will continue to be one of the primary types of electric power plants.

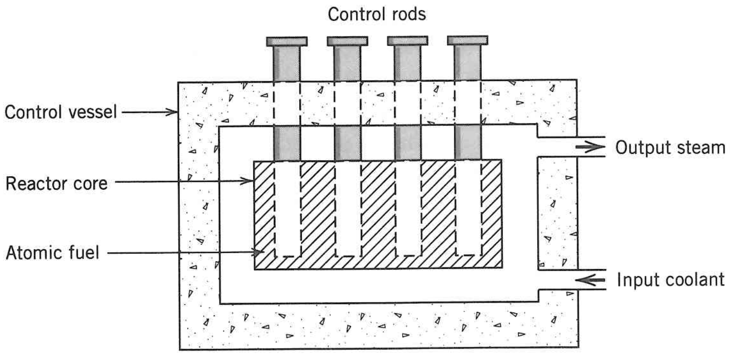


FIGURE 1.6 Schematic diagram of a nuclear reactor.

The development of rivers for hydroelectric power production will continue in the future because it is an advantageous, although cost-intensive, method of converting a renewable form of energy.

The conventional methods of producing electric power involve the construction of huge dams and the inefficient use of chemical fuels. In the future, the continually rising demand for cheap energy and the increasing requirements of other industries for the raw material resources will necessitate the replacement of existing traditional methods of energy conversion by alternative techniques. These alternatives will be energy conversion methods that can directly convert heat, nuclear, and chemical energy into electricity.

The methods for converting various forms of energy directly into electricity are based on known physical phenomena. However, these methods of direct energy conversion are still not able to compete with the conventional energy conversion techniques used in the power industry on a large scale.

Direct conversion techniques have been used for electricity generation as low-capacity, self-contained power sources. They are used in inaccessible and remote areas not serviced by the electric utility. Galvanic cells and storage batteries produce electric energy from the chemical reaction between different materials. The principles of operation of thermal converters, photoelectric batteries, and thermionic converters are based on various physical and natural phenomena. At present, these direct-conversion plants operate at a lower efficiency than existing commercial electric power plants.

1.4.1 Magnetohydrodynamic Plants

Magnetohydrodynamic (MHD) generators are devices that convert heat directly into electricity. This direct conversion makes use of fuel resources more efficiently. The theoretical efficiency of heat engines depends on the maximum and minimum temperatures of the working substance. Whereas the operating temperature of a steam turbine is about 750°C, which results in an efficiency of energy conversion of about 40%, the operating temperatures of an MHD generator can be as high as 2700°–3000°C, which could result in much higher efficiency.

The theory of operation of MHD generators is based on Faraday's law of electromagnetic induction, which states that an electromotive force (emf) is induced in a conductor moving in a magnetic field. The emf is induced in any conductor regardless of its physical state, that is, whether it is solid, liquid, or gaseous. The interaction of an electrically conducting liquid or gas with a magnetic field is called magnetohydrodynamics.

A schematic diagram of an MHD generator is shown in Fig. 1.7. The ionized gas is passed between metal plates located in a strong magnetic field. An emf is induced, which causes an electric current to flow between electrodes inside the generator duct and in an external circuit. The flow of ionized gas, or what is more commonly called plasma, is opposed by electromagnetic forces resulting from the interaction of the current and the magnetic flux. The work done against the retarding forces provides the mechanism by which energy is converted from one form to another.

Traditionally, the three known states of matter have been solid, liquid, and gas. Gas has been considered electrically neutral. When a gas is heated, however, the outer electrons are knocked out as a result of multiple head-on collisions between the atoms. With its nuclei stripped of all the electrons, the gas is in a fourth state of matter referred to as high-temperature plasma. This state of matter is not normally found on Earth because very high temperatures and pressures are required to bring a substance to this state. At 3000°C, some gases turn to low-temperature plasmas composed of free atoms, ions, and electrons. A low-temperature plasma is highly electrically conducting.

1.4.2 Thermal Converters

The thermal converter is the device most often used for direct heat-to-electricity conversion. However, thermal converters are characterized by a relatively low power output.

The main advantages of thermal converters are that they have no moving parts, do not require high pressures, can use any heat source, and can operate for a long time. They are used extensively as power sources on spacecraft, missiles, beacons, and so forth.

Present thermal converters have outputs varying from several watts to a few kilowatts. The efficiency of these converters is up to about 10%. Although higher efficiencies will probably be achieved in the future, present thermal converters are still not competitive in cost and efficiency with conventional high-capacity electric power plants.

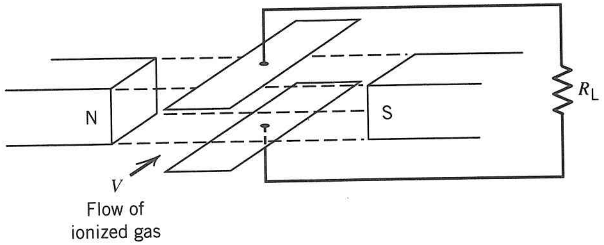


FIGURE 1.7 Schematic diagram of an MHD generator.

1.4.3 Thermionic Converters

A schematic representation of a simple thermionic converter is shown in Fig. 1.8. Electrons are emitted from the cathode when it is heated. Part of the energy supplied to the cathode is transferred by electrons to the anode and is delivered to the external circuit in the form of electric current.

In thermionic converters used for power generation, the cathode can be heated by nuclear reaction. The efficiency of the first converters of this type is about 15%, and current estimates are that this figure can be raised to 40%.

1.4.4 Electrochemical Cells

In an electrochemical cell, or fuel cell, chemical energy is converted directly to electricity. The fuel and oxidizer are stored externally and are supplied as needed. The electrodes and the electrolyte are not consumed in the energy conversion process. However, commercially attractive and cheap fuel cells using natural fuels and oxygen are still unavailable at present.

In a galvanic cell, an emf is produced by metal ions passing into a solution on interaction with the molecules and ions of the latter. When a zinc electrode is immersed in a solution of zinc sulfate ($ZnSO_4$), as shown in Fig. 1.9, the molecules of water tend to settle around the positive ions of metallic zinc, which then go into the solution of zinc sulfate.

The passage of positive ions into solution raises the electrode to a higher negative potential, which inhibits the ions from going into solution. At a certain point, the two oppositely directed flows of ions—from the electrode to the solution and back—become equal, that is, come to dynamic equilibrium.

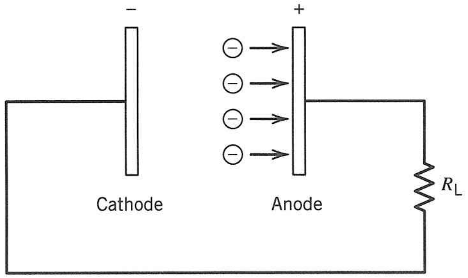


FIGURE 1.8 Schematic diagram of a thermionic converter.

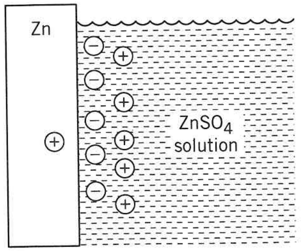


FIGURE 1.9 Zinc electrode in $ZnSO_4$ solution.

An important application of galvanic cells is in storage batteries. A current from an external source is passed through the electrodes of a storage battery for some time during the charging cycle so that at a later time, current can be delivered to an external load. However, the power industry has limited use of storage batteries because of the scarcity of reactive chemical fuels and their low power capabilities.

Fuel cells are quiet and efficient in operation, and they do not release any harmful air pollutants. With progress in fuel cell design and electrochemistry, the applications of fuel cells in the future may include electric power generation and power supply for automobiles.

1.4.5 Solar Power Plants

The principle of operation of a solar cell is based on the phenomenon of photoelectricity; that is, when light is incident on a body, electrons are liberated. Although photoelectricity has long been known, it has been used only lately, primarily because of advances in semiconductor technology.

When an *n*-type semiconductor is brought into contact with a *p*-type semiconductor, a contact potential difference is established at the interface by the diffusion of electrons. When the *p*-type semiconductor is exposed to light, its electrons absorb photons of light and pass into the *n*-type semiconductor. Thus electric current is produced in a closed circuit.

At present, the most advanced devices of this type are silicon solar cells. Silicon solar cells may be up to 15% efficient. However, the difficulty of manufacturing semiconductors and their high cost restrict the use of silicon solar cells to special applications, such as in satellites.

1.4.6 Tidal Power Plants

Tidal energy has been used for a long time in various devices, particularly mills. An advantage of tidal power plants over run-of-river hydroelectric plants with no pondage or reservoir is that the performance of the former is determined by cosmic phenomena rather than by weather conditions. The main disadvantage of tidal power plants is their irregular operation. Daily variations in the available tidal energy prevent the tidal power from being used regularly in power systems during periods of peak demand.

The turbine can be run either for power generation or for pumping. When the generator is shut down, the seawater is allowed to flow directly to or from a tidal basin. The turbine operates as a pump to transfer the seawater to the basin, thus raising the operating head of water for power generation.

Use of a reversible variable-pitch turbine allows the operation of a tidal power plant to be varied and adjusted in accordance with the load demand curve. During off-peak hours, when the load demand is low, the turbine pumps water from the sea, increasing the water level in the basin. During the peak demand hours, the turbine drives the generator, converting the stored energy into electricity.

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Two

Power System Components and Analysis

2.1 INTRODUCTION

One of the primary contributors to the advances and improvements in human life-style over the centuries has been the ability to use and control energy. To discover new sources of energy, to obtain an essentially inexhaustible supply of energy for the future, to make energy available wherever needed, and to convert energy from one form to another and use it without creating pollution are among the greatest challenges facing our world. The electric power system is one of the tools for converting and transporting energy that is playing an important role in meeting this challenge. Highly trained engineers are needed to develop and implement the advances in science and technology to solve the problems of the electric power industry and to ensure a high degree of system reliability along with the utmost regard for the protection of our ecology.

An electric power system consists of three principal divisions: the generating system, the transmission system, and the distribution system. Transmission lines are the connecting links between the generating stations and the distribution systems and lead to other power systems over interconnections. A distribution system connects all the individual loads to the transmission lines at substations that perform voltage transformation and switching functions.

Energy is converted at the generating station from one of its basic forms, such as fossil fuels, hydro, and nuclear, into electric energy. This electric energy is then sent through a transmission system to loads at various places, where it is usually converted into other useful forms of energy. Thus, electric energy is used primarily to transmit the energy from one source, such as heat from burning coal, at one location to another location to do work, such as running a compressor on a refrigerator. This book discusses how these energy conversions and electric power transmissions take place and describes the various devices used in the operation of the electric power system.

2.2 POWER SYSTEM STRUCTURE

The physical plant associated with a power system can be divided into generation (G), transmission (T), and distribution (D) facilities as shown in Fig. 2.1. The generating system provides the system with electric energy.

The transmission and subtransmission systems are meshed networks; that is, there is more than one path from one point to another. This multiple-path structure increases the reliability of the transmission system. The transmission network is a high-voltage network designed to carry power over long distances from generators to load points. The subtransmission network is a low-voltage network whose purpose is to transport power over shorter distances from bulk power substations to distribution substations. The transmission system, which is usually 138 to 765 kilovolts (kV), and the subtransmission system, which is usually 34 to 138 kV, consist of

1. Insulated wires or cables for transmission of power
2. Transformers for converting from one voltage level to another
3. Protective devices, such as circuit breakers, relays, communication and control systems
4. Physical structures for containing the foregoing, such as transmission towers and substations

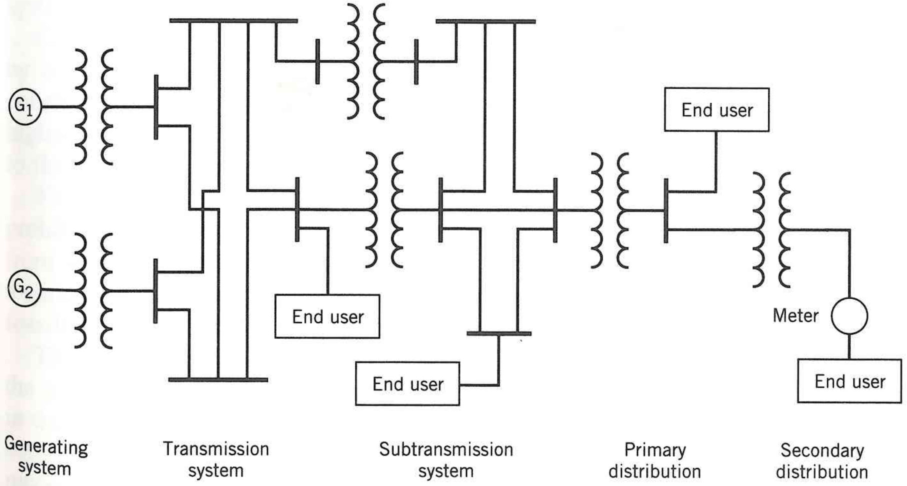

FIGURE 2.1 Sample power system structure.

The distribution of electric power includes that part of an electric power system below the subtransmission level, that is, the distribution substation, primary distribution lines or feeders, distribution transformers, secondary distribution circuits, and customers' connections and meters. The substation contains a transformer to bring the voltage down from subtransmission to distribution levels. Distribution voltages are typically 4 to 34 kV. Each substation transformer serves one or more feeders.

The primary distribution system extends from the distribution substations to the distribution transformers. A distinction is made between main feeders, which are connected to the substation, and lateral feeders, which are connected to the main feeders. These main feeders and laterals are illustrated in Fig. 2.2.

Each feeder is equipped with a circuit breaker or recloser to protect itself and the substation transformer against damage by short-circuit currents. Beyond the substation, the primary feeder consists of underground cables or overhead lines to transport the power and associated devices to control and protect the feeder. These include switches, capacitors, fuses, voltage regulators, sectionalizers, reclosers, and step-down distribution transformers. Many transformers are connected to the primary feeder for stepping voltage levels down to customer levels, which are 240, 208, or 120 V. The distribution transformers serve the secondary distribution system, which has small conductors connecting 1 to 10 residential customers to each distribution transformer.

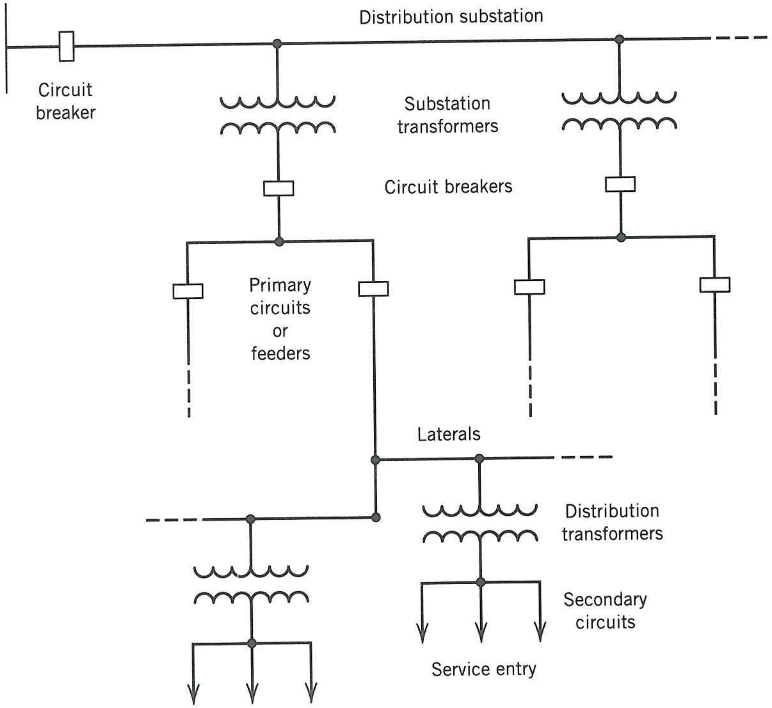

FIGURE 2.2 Electric distribution system components.

Depending on the size of their power demand, customers may be connected to the transmission system, subtransmission system, primary distribution, or secondary distribution. Each customer is connected to the power system through a meter.

The primary distribution circuits in central business districts of large urban areas consist of underground cables that are used to interconnect the distribution transformers in an electric network. With this exception, the primary system is most often radial; that is, it constitutes a tree. For additional security, it is quite frequently loop-radial: the main feeder loops through the load area and comes back to the substation. The two ends of the loop are usually connected to the substation by two separate circuit breakers. A loop-radial configuration provides a backup capability. Opening selected sectionalizing switches results in a radial configuration that is used for normal operation. When a failure occurs in any section served by a radialized feeder, that section is isolated and the other part of the loop can be used to supply the unaffected customers downstream of the faulted section.

2.2.1 Electric Energy Production

Most of the electric power in the United States is generated in steam-turbine plants. Water power accounts for less than 20% of the total, and that percentage will drop because most of the available sources of water power have been developed. Gas turbines are used to a minor extent for short periods when a system is carrying peak load.

Coal is the most widely used fuel for the steam plants. Nuclear plants fueled by uranium account for a continually increasing share of the load, but their construction is slow and uncertain because of the difficulty of financing the higher costs of construction, increasing safety requirements, public opposition to the operation of nuclear plants, and delays in licensing.

The supply of uranium is limited, but the fast breeder reactor, which is now prohibited in the United States, has greatly extended the total energy available from uranium in Europe. Nuclear fusion is the great hope for the future, but a controllable fusion process on a commercial scale is not expected to become feasible until well after the year 2000.

There is some use of geothermal energy in the form of steam derived from the ground in the United States and other countries. Solar energy, still mainly in the form of direct heating of water for residential use, should eventually become practical through research on photovoltaic cells. Great progress has been made in increasing the efficiency and reducing the cost of these cells, but there is still a long way to go. Electric generators driven by windmills are operating in a number of places, and they supply small amounts of energy to power systems. Efforts to extract power from the changing tides and from waves are under way. An indirect form of solar energy is alcohol obtained from grain and mixed with gasoline to make an acceptable fuel for automobiles. Synthetic gas made from garbage and sewage is another form of indirect solar energy.

2.2.2 Transmission and Distribution

The voltage of large generators is usually in the range 13.8 to 24 kV. Large modern generators, however, are built for voltages ranging from 18 to 24 kV. No standard for generator voltages has been adopted.

Generator voltage is stepped up to transmission levels in the range 115 to 765 kV. The standard high voltages (HVs) are 115, 138, and 230 kV. Extra-high voltages (EHVs) are 345, 500, and 765 kV. Research is being conducted on lines in the ultra-high-voltage (UHV) levels of 1000 to 1500 kV. The advantage of higher levels of transmission voltage is apparent when consideration is given to the transmission capability in megavolt-amperes (MVAs) of a line. The capability of transmission lines varies with the square of the voltage. Capability is also dependent on the thermal limits of the conductor, on the allowable voltage drop, on reliability, and on the stability requirements for maintaining synchronism among the machines of the system. Most of these factors are dependent on line length.

High-voltage transmission usually employs overhead lines supported by steel, cement, or wood structures. At present, the application of underground transmission is negligible in terms of the total length of transmission lines. This is primarily because of the much higher investment, as well as repair and maintenance, costs of underground compared to overhead transmission. The use of underground cables is mostly confined to heavily populated urban areas or wide bodies of water.

The first step down of voltage from transmission levels is at the bulk-power substation, where the reduction is to a range of 34.5 to 138 kV, depending on the transmission voltage. Some industrial customers may be supplied at these voltage levels. The next step down in voltage is at the distribution substation, where the voltage on the transmission lines leaving the substation ranges from 4 to 34.5 kV and is commonly between 11 and 15 kV. This is the primary distribution system. A very popular voltage at this level is 12 kV (line-to-line). This voltage is usually described as 12-kV Y/7.2-kV Δ. A lower primary system voltage that is less widely used is 4160-V Y/2400-V Δ. Most industrial loads are fed from the primary system, which also supplies the distribution transformers providing secondary voltages over single-phase, three-wire circuits for residential use. Here the voltage is 240 V between two wires and 120 V between each of these and the third wire, which is grounded. Other secondary circuits are three-phase, four-wire systems rated 208-V Y/120-V Δ or 480-V

2.2.3 Electrical Load Characteristics

The ultimate objective of any power system is to deliver electrical energy to the consumer safely, reliably, economically, and with good quality. Operation of the power system requires that proper attention be given to the safety not only of the utility personnel but also of the general public.

At the consumer load centers, electrical energy is converted to other more desirable and useful forms of energy. This implies that the supply of electricity should be available where, when, and in whatever amount the consumer requires.

The quality of the supplied electrical energy is partially dependent on energy usage by the consumers. When there is high demand on the limited capabilities of the power system, voltage may deviate from its acceptable levels. Switching of large machinery could cause fluctuation of the voltage as well as the frequency. Unusually high and prolonged demands may lead to overloaded equipment, which may cause tripping of protective devices to prevent further damage to the equipment.

Finally, the power system must be operated such that the overall costs of producing electricity, including all attendant losses in the generation and delivery systems, are minimized. The most economical conditions derive not only from proper operating procedures but also from efficient system planning and design.

Electrical loads are commonly grouped into four categories: residential, commercial, industrial, and other. These are sometimes subdivided into subgroups depending on their usage levels, for example, residential A, B, or C. The residential loads are private homes and apartments. They include lighting, cooking, comfort heating and cooling, refrigerators, water heaters, washers and dryers, and many more different appliances.

Commercial loads include office buildings, department stores, grocery stores, and shops. Industrial loads consist of factories, manufacturing plants, steel, lumber, paper, mining, textile, and other industrial factories. In both commercial and industrial groups, loads include lighting, comfort heating and cooling, and various types of office equipment. In addition, industrial loads contain various types and sizes of motors, fans, presses, furnaces, and so on.

Electrical load refers to the amount of electrical energy or electrical power consumed by a particular device or by a whole community. It is also referred to as electrical demand. At the individual consumer level, the electrical demand is quite unpredictable. However, as the demands of the various users are accumulated and added at a feeder or a substation, they begin to exhibit a definite pattern.

A typical plot of the electrical load is shown in Fig. 2.3, and it is referred to as a *load curve*. The period of interest is normally 1 day; thus, it is called a daily load curve. The daily load curve shows the kilowatt demand from

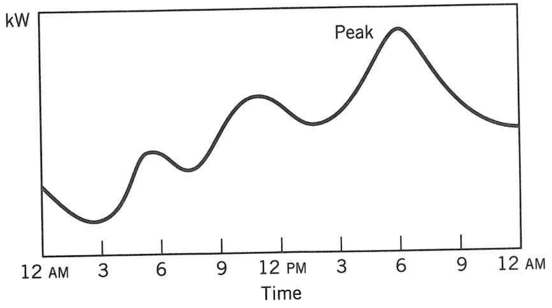

FIGURE 2.3 A typical load curve.

The daily load curves for Monday through Friday are similar in shape and maximum values. Weekend load curves are generally different, particularly for industrial and commercial customers because of shutdown of operations on Saturdays and Sundays.

Load curves may be constructed for feeders, substations, generating plants, or for the whole system. Load curves are also drawn for different periods or seasons of the year. Thus, the system peak load for summer, or winter, may be read as the highest ordinate from the corresponding load curve.

The daily load curves may be accumulated for a whole year and presented in another curve, called the annual *load duration curve* (*LDC*). A typical load duration curve is shown in Fig. 2.4. The LDC shows the 8760 hourly loads during the whole year, although not in the order in which they occurred. Rather, the LDC shows the number of hours during which the load exceeded a certain kilowatt demand. If the area under the LDC curve is calculated and divided by the total number of hours, the average demand is determined.

2.3 POWER SYSTEM ANALYSIS PROBLEMS

Power system studies can be grouped into two types: steady-state analysis for normal system conditions and transient analysis for abnormal conditions.

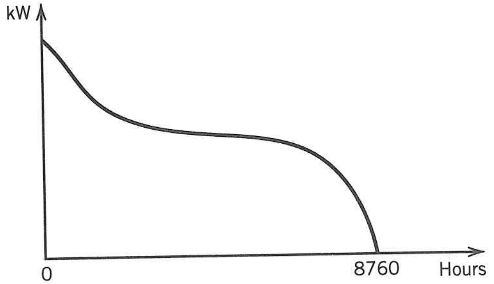

FIGURE 2.4 A typical load duration curve.

Economic dispatch and power (load) flow fall in the steady category. Transients include fault analysis for symmetrical and unsymmetrical faults, electrical transients, and stability analysis.

2.3.1 Economic Dispatch

The power industry may seem to lack competition. This idea arises because each power company operates in a geographic area not served by other companies. Competition is present, however, in attracting new industries to an area. Favorable electric rates are a compelling factor in the location of an industry, although this factor is much less important in times when costs are rising rapidly and rates charged for power are uncertain than in periods of stable economic conditions. Regulation of rates by state public utility commissions places constant pressure on companies to achieve maximum economy and earn a reasonable profit in the face of advancing costs of production.

Economic dispatch is the process of apportioning the total load on a system among the various generating plants to achieve the greatest economy of operation. The power plants are controlled by a computer as the load changes so that the total power demand is allocated for the most economical operation.

2.3.2 Power Flow

The electric power flow problem is perhaps the most studied and documented problem in power engineering. A power flow study is the determination of the voltage, current, power, and power factor or reactive power at various points in an electric network under existing or contemplated conditions of normal operation. These studies are essential in planning the future development of the system because its satisfactory operation depends on knowing the effects of interconnections with other power systems, new loads, new generating stations, and new transmission lines even before they are installed.

Digital computers provide the solutions of power flow studies of complex systems. For instance, some computer programs handle more than 1500 buses or nodes, 2500 transmission lines, 500 transformers, and so forth. Complete analysis results are printed quickly and economically.

2.3.3 Fault Calculations

A *fault* in a circuit is any failure that interferes with the normal flow of current. Most faults on transmission lines of 115 kV and higher are caused by lightning, which results in the flashover of insulators. The high voltage between a phase conductor and the grounded supporting tower causes ionization, which provides a path to ground for the charge induced by the lightning stroke. Once an ionized path to ground is established, the resultant low impedance to ground allows flow of current from the conductor to ground and through the ground to the grounded neutral of a transformer or generator, thus completing the circuit. Line-to-line faults not involving ground are less common.

Opening circuit breakers to isolate the faulted portion of the line from the rest of the system interrupts the flow of current in the ionized path and allows deionization to take place. After an interval of about 20 cycles, circuit breakers can usually be reclosed without reestablishing the arc. Experience in the operation of transmission lines has shown that ultra-high-speed reclosing breakers successfully reclose after most faults. Of the cases in which reclosure is not successful, an appreciable number are caused by permanent faults where reclosure would be impossible regardless of the interval between opening and reclosing. Permanent faults are caused by lines being on the ground, by insulator strings breaking because of ice loads, by permanent damage to towers, and by lightning arrester failures.

Experience has shown that between 70% and 80% of transmission line faults are single line-to-ground faults, which arise from the flashover of only one line to the tower and ground. The smallest number of faults, roughly 5%, involve all three phases and are called three-phase faults. Other types of transmission line faults are line-to-line faults, which do not involve ground, and double line-to-ground faults. Except for the three-phase fault, all other faults are unsymmetrical, which cause an imbalance between the voltages and currents in the three phases.

The short-circuit current that flows in different parts of a power system immediately after the occurrence of a fault differs from the current that flows a few cycles later, just before the circuit breakers are called on to open the line on both sides of the fault. Both of these currents differ from the current that would flow under steady-state conditions, if the fault were to remain and was not isolated from the rest of the system by the operation of the circuit breakers. Two of the factors on which the proper selection of circuit breakers depends are the current flowing immediately after the fault occurs and the current that the breaker must interrupt to isolate the fault.

Fault calculations involve determining these currents for various types of faults at various locations in the system. The data obtained from fault calculations also serve to determine the settings of relays that control the circuit breakers.

2.3.4 System Protection

Two possible abnormal conditions may occur while operating a power system. One is equipment or transmission line overloading, in which the current rating of the system element is exceeded. The second abnormal operating condition

is the occurrence of a short circuit or fault, which could be due to insulation breakdown or phase conductors coming into contact with other phase conductors or with ground.

The amount of short-circuit current that would flow depends on the type of fault, its location, and how long the fault persists before it is cleared or removed. It is also affected by the size and configuration of the power system, the presence of fault-current limiters, and the speed and effectiveness of protective and switching devices.

The value of the fault current and the speed at which the fault or the faulted element is isolated determine the extent of damage to equipment and undesirable voltage and frequency dips. Such deviations from normal conditions cause additional system losses and could lead to loss of synchronism and complete system breakdown.

Therefore, there is a need for an automatic protection scheme that can quickly identify the faulted element and disconnect it from the system. In the case of overloads where there is no imminent danger, such protection gives an alarm signal to alert utility personnel for corrective action to remedy the problem.

Protection schemes are classified as either primary (main) protection or backup protection. Primary protection provides rapid and selective clearing of faults within the primary zone of protection. Backup protection provides the protection needed whenever the primary protection fails or is under maintenance.

Primary relaying is provided for each section or major piece of equipment, including generators, switchgears, transmission lines, transformers, and distribution feeders. The individual zones of protection overlap, so no possible section or area is left unprotected.

The backup relay is normally slower acting than the primary relay to allow the latter to perform its job. It is set to be energized at a higher level and its time setting is longer.

An important feature of any protection scheme is its selectivity. This pertains to its ability to search for the particular point or element of the system where the fault occurred and isolate the fault by tripping the nearest circuit breaker(s). This ensures minimum disruption of electric service to the customers. Selectivity is accomplished by coordinating the operating currents and time settings of the protective relays at adjacent and nearby sections or stations. With the complexity and size of modern power systems, this could prove to be a challenging task for the protection engineer.

2.3.5 Transmission Line Transients

The electric transients that occur on a power system are generally caused by a sudden change in the operating condition or configuration of the system. These transient overvoltages are caused either by lightning striking transmission lines or by switching operations. Lightning is always potentially harmful to equipment, but switching operations are also potentially damaging.

Overhead lines are usually protected from lightning by one or more wires, called *ground wires*, which are located above the phase conductors. The ground wires are connected to ground through the transmission towers. In most cases, lightning hits the ground wires instead of the phase conductors.

When a ground wire or a phase conductor is hit by lightning, a current is produced, half of which flows in one direction and the other half in the other. The crest value of current depends on the intensity of the lightning stroke, with typical values of 10,000 A and upward. When a phase conductor is hit by lightning, the damage to equipment at the line terminals is caused by the overvoltages resulting from the currents that travel along the line. The voltage and current waves propagate along the transmission line at a velocity near the speed of light. These voltages are typically above a million volts. When lightning hits a ground wire, high-voltage surges are also produced on the transmission line by electromagnetic induction.

Equipment at the line terminals is protected by surge arresters, also called lightning arresters. The arrester is connected from line to ground. When the voltage across the surge arrester becomes greater than a specified value, the arrester becomes conducting; thus, it serves to limit the voltage across its terminals to the specified value. The surge arrester becomes nonconducting again when the voltage drops below the specified value.

2.3.6 Stability Analysis

An electric power system is a dynamic nonlinear system. The dynamics are due to change in demand, generation, line switching, lightning surges, and faults. These dynamics are often classified by the speed of occurrence: the high-speed phenomena (less than 5 cycles = 5/60 s) include switching and lightning surges; the intermediate-speed occurrences (less than 100 cycles) are primarily electromechanical transients of the synchronous machine rotors. Slower phenomena, such as changes in load and generation, are virtually steady-state phenomena. The models needed to study these dynamics vary in detail, depending on the speed of occurrence.

The problem of stability is concerned with maintaining the synchronous operation of the generators and motors of the system. Stability studies are classified according to whether they involve steady-state or transient conditions. There is a definite limit to the amount of power that an AC generator can deliver and to the load that a synchronous motor can carry. Instability results from attempting to increase the mechanical input to a generator, or the mechanical load on a motor, beyond this definite amount of power, called the stability limit. A limiting value of power is reached even if the change is made gradually. Loss of synchronism may be due to suddenly applied loads, occurrence of faults, loss of field excitation of a generator, or equipment switching. Depending on whether instability is reached by a sudden and large change or a gradual change in system conditions, the limiting value of power is called the transient stability limit or steady-state stability limit, respectively.

2.4 THE COMPUTER CONNECTION

Many people believe that the only connection between power systems and computers is that computers must be plugged into an electrical outlet to operate. True, electricity does indeed power computers, either directly for desktop units or indirectly for large computers through uninterruptible power supplies that are DC batteries charged by the power system. There is far more, however, to the connection between computers and power systems.

The power industry is the third largest user of computers in the United States, next to the federal government and financial institutions. Computers are used heavily by electric utilities for customer billing, employee payrolls, and record keeping. They are also used to plan, design, control, and operate the power system.

Electric power systems are enormously large and complex. In the United States and Canada, three synchronously operated networks function independently of one another. The largest one ranges from Florida to Montana and Texas to the Hudson Bay. Power is generally produced in large central power-generating stations, sent over high-voltage transmission lines, and distributed over low-voltage lines to each customer.

There are, for example, over 600 gigawatts (600×10^9 W) of installed generating capacity in the United States. The typical cost of new generating capacity is \$4000 per kilowatt. Power plants are connected to load centers through 275,000 miles of overhead and underground transmission lines operating at 115 through 765 kV. Underground transmission, typically costing \$1.2 million per mile, is 5–15 times as costly as overhead transmission. Several factors make the power system a complex engineering problem:

- Power must be available the instant it is required because no economical method of storing electricity exists. This means that forecasting of the system loads must be sophisticated and system control fast.
- Customers expect 100% reliability. The high cost of supplying power prevents building a significant amount of redundancy into the power system, contrary to how other large complex engineering systems are built.
- Fossil-fueled power plants take some 10 years to build, and nuclear power plants take even longer. Hence, electric utility planners must make decisions on system expansions up to 20 years into the future.

The size of the industry and its complexities are important reasons why computers are used by the power industry. Another important reason is the enormous amount of money at stake. Revenues of the largest U.S. utilities in the mid-eighties were well over $100 billion, more than $40 billion was spent on fuel alone, and another $35 billion was spent on new equipment.

These staggering numbers make it obvious that reducing costs even a fraction of a percent would result in substantial savings. In an effort to control costs, the electric utilities use computers at all levels of the system. The system is too complex and stakes are too high to plan and operate a utility system without computers.

2.4.1 Typical Applications

Process control–type computers are used in power plants to assist plant operators with monitoring and control functions. These computers collect and process data from all over the plant. They alert the operator when various plant parameters such as temperatures, pressures, and flows are outside design operating values. They do performance calculations that tell operators how efficiently the plant and its individual components are operating.

In system control centers, operators (system dispatchers) use computers to assist them in selecting the least expensive mix of generators from the many possibilities. They are constantly monitoring and controlling the complex transmission network. They must be prepared for any emergency, such as storms or equipment failure. System changes for maintenance must be scheduled and controlled to avoid outages. Computers also help train both new and experienced system control dispatchers and power plant operators.

In load control centers, computers monitor lower-voltage transmission and major distribution circuits during periods of both normal and abnormal operation, such as during storms and system disturbances in which customers experience outages.

System planners use computers in the planning and design of future power system networks. Their requirements challenge the complex simulation techniques offered by mathematicians and computer scientists. Often the size and complexity of power system problems are beyond state-of-the-art capabilities.

Computer applications extend to electrical system components as well. Relays, for example, are used as sensors to control circuit breakers and remove shorted components from the system. At one time relays were electromechanical devices. They were, in turn, surpassed by solid-state devices. State-of-the-art relays now use microprocessor technology.

Microprocessors are also becoming commonplace in power plants to collect and process data for transfer to the plant computer. High reliability is expected from these microprocessors, even in the hostile physical and electromagnetic atmospheres of power plants and substations.

2.4.2 System Planning and Operation

Power system planning often extends up to 20 years from the present. Computers are invaluable tools in the efforts required to develop plans that are technically and economically feasible and environmentally and socially acceptable. Planning complications arise because of the enormous size of the systems studied, as well as the countless possible options and situations.

At this time, more than 40 application programs are available for planning the power system. Some examples of how these individual tools assist the planning effort follow.

The power flow (or load flow) program is used as the basis of steady-state analysis. Typical network study sizes range from 1000 to 4000 buses or nodes. A power flow program is typically written in Fortran and contains 10,000 to 40,000 Fortran statements using sparse matrix methods to reduce computer memory requirements. Large mainframe computers can perform a power flow computation in 10 to 30 s. Recently, personal computers have performed smaller power flow studies.

Another computer simulation tool is used extensively to analyze system response during the first 3 to 10 s after a major disturbance, such as loss of generating unit or transmission line. This transient stability program is substantially larger than the power flow program because it solves sets of both algebraic nonlinear equations and the tens of thousands of differential equations that describe system transient response. The stability program numerically integrates the differential equations at time steps of about 0.05 s. The number of equations and the small time step needed illustrate the challenge in computer applications of power system analysis.

Computer optimization is playing a larger role than ever before in planning tomorrow's power systems. Examples of the application of computer optimization include scheduling the operation of generating units while maintaining security constraints on individual transmission lines, coordinating the use of hydroelectric and fossil-fueled generating units, siting and scheduling of reactive power sources, planning generating system and transmission system expansions, and scheduling of fuel use.

Like the power system planner, the system dispatcher has more than 40 computer programs to assist in job performance. But in many ways the dispatchers are ahead of the planners in their use of computers.

The dispatchers are responsible for monitoring and controlling the power system. In addition to "keeping the lights on," dispatchers must operate the power system, that is, choose patterns of generation and energy transmission so that loss of any single system element such as a generator or transmission line will not lead to a blackout. When the power system is secure, the dispatcher can pay attention to operating the power system as efficiently as possible. A dedicated set of computer hardware and software assists the dispatcher.

The computer system used to monitor and control the power system has a large database management system at its heart. It is designed for redundant operation with automatic fail-over schemes. The hundreds of buses in the power system are scanned every 2 to 4 s to determine current operating conditions. Because it is impractical to monitor every single point, the newer system control centers use state estimation techniques to check for missing or bad data. From the available data, another computer program develops a model of the current operating condition.

Automatic generation control provides on-line scheduling of the power system generating units. Control signals are sent out to the plants every 6 to 10 s. Economic dispatch is a separate computer program that establishes set points for each generating unit that is on line and under automatic control. The unit commitment program determines which generating units should be operated on a daily basis; the program includes minimum run times and minimum down times for reducing maintenance requirements. Both economic dispatch and unit commitment are optimization programs.

An additional dispatcher responsibility is to purchase power from and sell power to other electric utilities. Buy and sell decisions are based on short-term load forecasts and knowledge of which generating units are operating and their operating costs.

Control centers make substantial use of computer graphics to display information to dispatchers. But so much information is available that a large application problem is determining exactly what information to display. Thus, the control center involves not only a computation and communication challenge but also the human problem of presenting information effectively.

2.4.3 The Computing Environment

Most power system simulation and analysis today is performed using either large mainframe computers or super-minicomputers. "Smart" terminals are often used as workstations. Computations are generally performed using an interactive data setup followed by batch solution execution, with interactive output analysis completing the computer run. Graphics are widely used but not yet to their full capability.

Personal computer applications are just beginning. These generally rely on commercial software packages, such as spreadsheets and word processing. Some engineering programs are becoming available, most of which have been developed by the utilities.

Database management systems are not yet in wide use by power system planners as a means of interfacing the various application computer programs. Almost all application programs are written in Fortran, although PL/1, Basic, C, and Pascal have been used.

The computing environment in the future will be significantly different. Desktop workstations in the form of personal computers will have the capability of present computer mainframes. Artificial intelligence methods, particularly expert systems, will become widely used. Program consolidation and increased use of company-wide databases using modern, relational database management systems will become common. These tools will allow studies to be performed with less human intervention. Office automation techniques will also become an integral part of the available tools.

2.5 THE ROLE OF THE POWER ENGINEER

This introduction shows that many challenges are waiting for the future electric power engineer. One of the objectives of this book is to help students choosing the area of power engineering as their career to prepare for further study and eventual employment in the industry. The power industry is different from most industries in that it is regulated by state and federal governments. So, not only does industry touch virtually everyone through the service it provides, but everyone has the ability to affect the industry through the regulatory bodies. Another objective of this book is to educate electrical engineers who do not choose power engineering as a profession in some of the basic principles of power system operation. In this way they can exercise their input into the industry from an informed point of view.

With revenues of over $100 billion, the electric utility industry is one of the nation's largest. Computer applications in the industry today are indeed extensive, but the coming years promise even greater challenges in such areas as computer modeling, process control, digital communications, mathematics, and software engineering. Thus, for electric utilities to make the best use of computers, they will require young engineers with diverse backgrounds. Your role in the industry can eventually become highly significant.

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Three

Basic AC Circuit Concepts

3.1 INTRODUCTION

The analysis of electric power systems involves the study of the performance of the system under both normal and abnormal conditions. The power system engineer regularly performs such analyses, which consider both single-phase and three-phase circuits. The power engineer, therefore, must be competent in steady-state AC circuit analytical techniques.

In this chapter, the notations used and a review of basic circuit analysis are presented first, followed by per-unit representation of the electrical quantities such as voltage, current, power, and impedance.

3.2 NOTATIONS

Single-subscript notation is discussed first. Then double-subscript notation is introduced to eliminate the need for the polarity markings for voltages and directions for currents.

3.2.1 Single-Subscript Notation

Consider the AC circuit shown in Fig. 3.1. The circle represents a voltage source with emf $\mathbf{E}_g$ and terminal voltage $\mathbf{V}_t$. The voltage across the load is designated as $\mathbf{V}_L$. The internal impedance of the source is Z_g, the impedance of the feeder is Z_{fdr}, and the impedance of the load is Z_L.

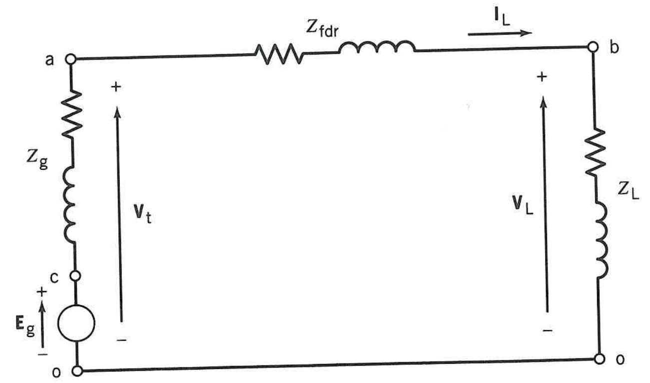


FIGURE 3.1 A simple AC circuit.

On the diagram, + and − polarity marks are assigned to each of the various voltages. The polarity marks specify that the voltage is positive when the terminal marked + is at a higher potential than the terminal marked −. In an AC circuit, this is the case during half of a cycle. During the next half-cycle, the voltage is negative because the terminal with the + marking is actually at a lower potential.

Alternatively, the voltages may be specified using arrows. In this case, the tip of the arrow points to the terminal corresponding to the + marking, whereas the tail corresponds to the − marking.

Similarly, an arrow is used to designate the positive direction of the flow of current. The current is considered negative if the actual direction of current flow is opposite to the arrow direction. For the given circuit, the phasor current can be calculated as follows:

$$\mathbf{I}_L = \frac{\mathbf{V}_t - \mathbf{V}_L}{Z_{fdr}} \tag{3.1}$$

The terminal voltage of the source is

$$\mathbf{V}_t = \mathbf{E}_g - \mathbf{I}_L Z_g \tag{3.2}$$

On the circuit diagram, the different nodes have been identified by letters. The voltages at these nodes may be referred to the reference node o, such that $\mathbf{V}_a$ is positive when node a is at a higher potential than reference node o. Thus,

$$\mathbf{V}_a = \mathbf{V}_t, \qquad \mathbf{V}_b = \mathbf{V}_L, \qquad \mathbf{V}_c = \mathbf{E}_g \tag{3.3}$$

Note that voltage and current phasors are set in boldface. Impedance is a complex quantity, but is not a phasor, and so is not set in boldface.

3.2.2 Double-Subscript Notation

The double-subscript notation eliminates the need for both polarity markings for voltages and direction arrows for currents. It is even more useful for representing voltages and currents in three-phase circuits, resulting in greater clarity and less confusion.

The voltage phasor with double subscripts represents the voltage across the two nodes identified by the two subscripts. The voltage is positive during the half-cycle in which the node named by the first subscript is at a higher potential than the node named by the second subscript.

The current phasor with the double subscript represents the current flowing between two nodes of a circuit. The current is considered positive when it flows in the direction from the node identified by the first subscript toward the node identified by the second subscript.

The voltage across the line impedance in Fig. 3.1 can be expressed in double-subscript notation as

$$\mathbf{V}_{ab} = \mathbf{I}_{ab} Z_{fdr} \tag{3.4}$$

Using double-subscript notation, Eqs. 3.1 and 3.2 can be rewritten as Eqs. 3.5 and 3.6.

$$\mathbf{I}_{ab} = \frac{\mathbf{V}_{ao} - \mathbf{V}_{bo}}{Z_{fdr}} \tag{3.5}$$

$$\mathbf{V}_{ao} = \mathbf{E}_{co} - \mathbf{I}_{ab} Z_g \tag{3.6}$$

Because node o is the reference node, it may be dropped from the subscripts without loss of clarity. Thus,

$$\mathbf{I}_{ab} = \frac{\mathbf{V}_a - \mathbf{V}_b}{Z_{fdr}} \tag{3.7}$$

$$\mathbf{V}_a = \mathbf{E}_c - \mathbf{I}_{ab} Z_g \tag{3.8}$$

3.3 SINGLE-PHASE AC CIRCUITS

In this section, basic concepts for the analysis of single-phase circuits are presented. The effective value of sinusoidal voltages and currents and definitions of power and complex power are discussed.

3.3.1 Effective or Root-Mean-Square Value

Consider the sinusoidal voltage $v(t) = V_m \cos(\omega t + \theta)$ whose peak value, or amplitude, is V_m. For this sinusoid, the period is $T = 2\pi/\omega$.

The *effective* or *root-mean-square (rms) value* is the value of the sinusoidal voltage that, when connected across a resistor, delivers the same amount of electric energy to the resistor in T seconds that a constant (DC) voltage would. The rms value is found by using the formula

$$V = \sqrt{\frac{1}{T}\int_0^T v^2(t)dt} \tag{3.9}$$

Substituting the expressions for the voltage and its period yields

$$V = \sqrt{\frac{V_m^2}{2\pi/\omega}\int_0^{2\pi/\omega} \cos^2(\omega t + \theta)dt} = \frac{V_m}{\sqrt{2}} \tag{3.10}$$

EXAMPLE 3.1

The voltage across a certain impedance load is given by $v(t) = 170\cos\omega t$, and the current flowing through the load is $i(t) = 14.14\cos(\omega t - 30°)$. Find the rms values of the voltage and current. Find the expressions for the voltage and current phasors.

Solution

a. The rms values of the voltage and current are

$$V = 170/\sqrt{2} = 120 \text{ V}$$
$$I = 14.14/\sqrt{2} = 10 \text{ A}$$

b. The voltage and current phasors are

$$\mathbf{V} = 120\underline{/0°} \text{ V}$$
$$\mathbf{I} = 10\underline{/-30°} \text{ A}$$

3.3.2 Power in a Single-Phase Circuit

Consider the impedance load consisting of a resistance R and an inductive reactance X_L connected in series as shown in Fig. 3.2.

Let the current flowing through the resistance and inductive reactance be expressed as

$$i(t) = \sqrt{2}I\cos\omega t \tag{3.11}$$

The applied voltage $v(t)$ is equal to the sum of the voltage drops across the resistance and reactance; that is,

$$v(t) = \sqrt{2}V_R\cos\omega t + \sqrt{2}V_X\cos(\omega t + 90°) \tag{3.12}$$

where $V_R = IR$ and $V_X = IX_L$. Combining the two cosine functions yields

$$v(t) = \sqrt{2}V\cos(\omega t + \theta) \tag{3.13}$$

where

$$V = \sqrt{V_R^2 + V_X^2} = I\sqrt{R^2 + X_L^2} = IZ \tag{3.14}$$

$$\theta = \tan^{-1}\left(\frac{V_X}{V_R}\right) = \tan^{-1}\left(\frac{X_L}{R}\right) = \arg(Z) \tag{3.15}$$

The instantaneous power $p(t)$ delivered to the load may be expressed as follows:

$$\begin{aligned} p(t) = v(t)i(t) &= [\sqrt{2}V\cos(\omega t + \theta)][\sqrt{2}I\cos\omega t] \\ &= 2VI\cos(\omega t + \theta)\cos\omega t \end{aligned} \tag{3.16}$$

The power $p(t) = v(t)i(t)$ absorbed by the load is positive when both $v(t)$ and $i(t)$ have the same sign (either both positive or both negative). This power becomes negative when $v(t)$ and $i(t)$ are of opposite sign.

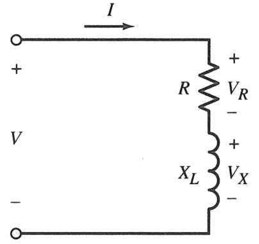


FIGURE 3.2 Series $R-X$ load.

In the case of a pure resistance load, the voltage and current are in phase and the power is always positive. For a load consisting of a pure inductance or pure capacitance, the power has alternately positive and negative, but equal, portions and its average value is zero.

Using the trigonometric identity $\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$ (see Appendix F), Eq. 3.16 reduces to

$$p(t) = VI\cos\theta + VI\cos(2\omega t + \theta) \tag{3.17}$$

The first term on the right-hand side of Eq. 3.17 has a constant value. The second term is a sinusoid of twice the frequency, and its average is zero. Thus, the average of the whole expression, that is, the *average power P* absorbed by the load, is

$$P = VI\cos\theta \tag{3.18}$$

The quantity P is also called *real power*, or active power, and is measured in watts (W), kW, or MW.

The product of the rms values of voltage and current, VI, is referred to as the *apparent power S*. It is measured in volt-amperes (VA), kVA, or MVA.

The cosine of the phase angle θ between the voltage and the current is called the *power factor*. It may be calculated from Eq. 3.18 as follows:

$$\text{Power factor} = \cos\theta = \frac{P}{VI} \tag{3.19}$$

An inductive circuit is said to have a *lagging power factor* because the current lags the voltage. On the other hand, a capacitive circuit is said to have a *leading power factor* because the current leads the voltage.

The instantaneous power may be expressed as the sum of two sinusoids of twice the frequency. Thus, Eq. 3.17 is written as

$$p(t) = VI\cos\theta(1 + \cos 2\omega t) + VI\sin\theta\cos(2\omega t + 90^\circ) \tag{3.20}$$

The first term on the right-hand side is a sinusoid displaced upward by an amount equal to the average power P.

The second term on the right-hand side of Eq. 3.20 is called the instantaneous reactive power. It has an average value of zero. Its peak value is designated as Q and is simply called *reactive power*. It is equal to

$$Q = VI\sin\theta \tag{3.21}$$

Reactive power is measured in volt-ampere reactive (VAR), kVAR, or MVAR.

From Eqs. 3.18 and 3.21, it is seen that the apparent power $S = VI$ may be computed from P and Q. Thus,

$$\sqrt{P^2 + Q^2} = \sqrt{V^2 I^2 \cos^2\theta + V^2 I^2 \sin^2\theta} = VI = S \tag{3.22}$$

The instantaneous power to the load of Fig. 3.2 may also be found as the product of Eqs. 3.11 and 3.12. Thus,

$$p(t) = 2I^2 R\cos^2\omega t + 2I^2 X_L\cos(\omega t + 90^\circ)\cos\omega t \tag{3.23}$$

Using the trigonometric identity cited previously, Eq. 3.23 reduces to

$$p(t) = I^2 R(1 + \cos 2\omega t) + I^2 X_L\cos(2\omega t + 90^\circ) \tag{3.24}$$

Comparing Eq. 3.24 with Eq. 3.20 yields the following:

$$P = VI\cos\theta = I^2 R \tag{3.25}$$

$$Q = VI\sin\theta = I^2 X_L \tag{3.26}$$

$$S = VI = \sqrt{P^2 + Q^2} = I^2 Z \tag{3.27}$$

Then the following are derived:

$$\begin{aligned} \text{Power factor} &= \cos\theta \\ &= \cos\left[\tan^{-1}\left(\frac{Q}{P}\right)\right] \\ &= \cos\left[\tan^{-1}\left(\frac{X_L}{R}\right)\right] \end{aligned} \tag{3.28}$$

3.3.3 Complex Power

Consider an arbitrary load. The voltage impressed across the load is $\mathbf{V} = V\,\angle\theta_V$, and the current flowing through it is $\mathbf{I} = I\,\angle\theta_I$. The *complex power* $\mathbf{S}$ is equal to the product of the voltage times the complex conjugate of the current; that is,

$$\mathbf{S} = \mathbf{V}\mathbf{I}^* = VI\,\angle\theta_V - \theta_I = VI\,\angle\theta \tag{3.29}$$

where $\theta = \theta_V - \theta_I$. Thus, it is seen that the magnitude of the complex power is the apparent power VI and the angle of the complex power is the phase angle difference between the voltage and the current, or the *power factor angle* θ.

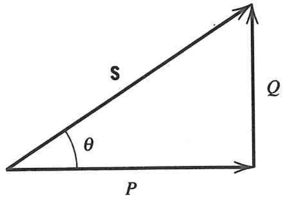


FIGURE 3.3 Power triangle.

The complex power may be expressed in rectangular form by using Euler's formula as follows:

$$\begin{aligned}\mathbf{S} &= VI(\cos\theta + j\sin\theta)\\ &= VI\cos\theta + jVI\sin\theta\\ &= P + jQ \end{aligned} \tag{3.30}$$

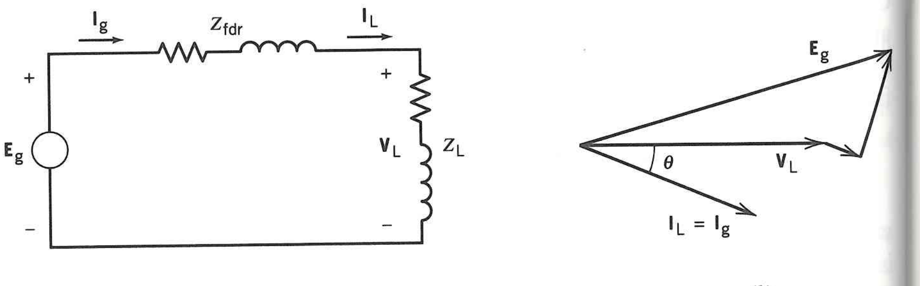
The reactive power Q is positive when $\theta_V > \theta_I$ or $\theta > 0$, which means that the current is lagging the voltage as in inductive loads. Conversely, Q is negative when $\theta_V < \theta_I$; that is, the current leads the voltage, as for the case of capacitive loads.

A power triangle for an inductive or lagging power factor load is shown in Fig. 3.3.

EXAMPLE 3.2

A generator supplies a load through a feeder whose impedance is $Z_{\text{fdr}} = 1 + j2\ \Omega$. The load impedance is $Z_{\text{L}} = 8 + j6\ \Omega$. The voltage across the load is 120 V. Find the real power and reactive power supplied by the generator. Take the load voltage $\mathbf{V}_{\text{L}}$ as the reference phasor.

Solution A schematic representation of the system and a diagram showing the various phasors are given in Fig. 3.4.



(a) (b)

FIGURE 3.4 (a) Power system; (b) phasor diagram of Example 3.2.

The impedances may be expressed in polar form as follows:

$$Z_{\text{fdr}} = 1 + j2 = 2.24\,\underline{/63.4^\circ}\ \Omega$$

$$Z_{\text{L}} = 8 + j6 = 10.0\,\underline{/36.9^\circ}\ \Omega$$

The voltage across the load is taken as reference phasor; thus,

$$\mathbf{V}_{\text{L}} = 120\,\underline{/0^\circ}\ \text{V}$$

The load current is computed as follows:

$$\mathbf{I}_{\text{L}} = \mathbf{V}_{\text{L}}/\mathbf{Z}_{\text{L}} = \frac{120\,\underline{/0^\circ}}{10\,\underline{/36.9^\circ}} = 12\,\underline{/-36.9^\circ}\ \text{A} = \mathbf{I}_{\text{g}}$$

That is, the generator current is the same as the load current.

The generator voltage is found by writing a Kirchhoff's voltage equation around the loop.

$$\begin{aligned}\mathbf{E}_{\text{g}} &= \mathbf{V}_{\text{L}} + \mathbf{I}_{\text{L}}Z_{\text{fdr}}\\ &= 120\,\underline{/0^\circ} + (12\,\underline{/-36.9^\circ})(2.24\,\underline{/63.4^\circ}) = 144.5\,\underline{/4.8^\circ}\ \text{V}\end{aligned}$$

The complex power is given by

$$\begin{aligned}\mathbf{S}_{\text{g}} &= \mathbf{E}_{\text{g}}\mathbf{I}_{\text{g}}^* = \mathbf{E}_{\text{g}}\mathbf{I}_{\text{L}}^* = (144.5\,\underline{/4.8^\circ})(12\,\underline{/-36.9^\circ})^*\\ &= 1734\,\underline{/41.7^\circ} = 1295 + j1154\ \text{VA}\end{aligned}$$

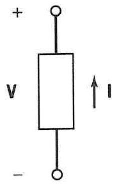
Therefore, the real power and reactive power are

$$P = 1295\ \text{W} \quad \text{and} \quad Q = 1154\ \text{VAR}$$

3.3.4 Direction of Power Flow

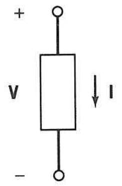
The convention used for positive power flow is described with the help of Fig. 3.5 and Eqs. 3.29 and 3.30. Figure 3.5*a* applies to a generator, and Fig. 3.5*b* is for a load.

For a generator, the electric current is assumed to flow out of the positive terminal in the direction of the voltage rise. Depending on the phase angle $\theta = \theta_V - \theta_I$, the values of P and Q may be positive or negative. If P is positive, the generator delivers positive real power. However, if P is negative,



S = VI* = P + jQ
If P is positive, then real power is delivered
If Q is positive, then reactive power is delivered
If P is negative, then real power is absorbed
If Q is negative, then reactive power is absorbed

(*a*) Generator condition



S = VI* = P + jQ
If P is positive, then real power is absorbed
If Q is positive, then reactive power is absorbed
If P is negative, then real power is delivered
If Q is negative, then reactive power is delivered

(*b*) Load condition

FIGURE 3.5 Convention for positive power flow.

the generator delivers negative real power; in other words, it absorbs positive real power. Similarly, if Q is positive, the generator delivers positive reactive power. However, if Q is negative, the generator delivers negative reactive power, or, alternatively, it absorbs positive reactive power.

For an electrical load, the current is assumed to enter the positive terminal in the direction of the voltage drop. Depending on the phase angle $\theta = \theta_V - \theta_I$, the values of P and Q may be positive or negative. If P is positive, the load absorbs positive real power. However, if P is negative, the load absorbs negative real power, or it delivers positive real power. Similarly, if Q is positive, the load absorbs positive reactive power. However, if Q is negative, the load absorbs negative reactive power, or it delivers positive reactive power.

DRILL PROBLEMS

D3.1 The instantaneous voltage $v(t)$ across an electrical device and the instantaneous current $i(t)$ entering the positive terminal of the circuit element are given by the following expressions:

$$v(t) = 110\cos(\omega t + 65^\circ)\ \text{V}$$
$$i(t) = 15\sin(\omega t - 20^\circ)\ \text{A}$$

Determine:

a. The maximum or peak value of the voltage and current
b. The rms value of the voltage and current
c. The phasor expressions for the voltage and current

D3.2 Determine the rms value of each of the following currents.

a. $12\sin 4t$
b. $3\cos 2t + 4\sin 2t$
c. $10 + 5\sin 3t$

D3.3 Given that $v(t) = \sqrt{2}V\cos(\omega t + \alpha)$, $i(t) = \sqrt{2}I\cos(\omega t + \beta)$, and $\omega = 2\pi/T$, show that the average power is given by

$$P = \frac{1}{T}\int_0^T v(t)i(t)dt = VI\cos(\alpha - \beta)$$

D3.4 For the voltage and current given in Drill Problem D3.1, find

a. The expression for instantaneous power
b. The average or real power, and state whether this power is absorbed or supplied by the impedance
c. The reactive power, and state whether absorbed or supplied
d. The power factor, and state whether lagging or leading

D3.5 An electrical load has an impedance of $10\,\underline{/30^\circ}\ \Omega$.

a. Compute the equivalent series resistance and reactance. State whether the reactance is inductive or capacitive.
b. The load is connected to a 60-Hz, 120-V source. Draw a phasor diagram showing the current, voltage across the resistance, voltage across the reactance, and voltage of the source as reference.

D3.6 A circuit consists of two impedances, $Z_1 = 50\,\underline{/45^\circ}\ \Omega$ and $Z_2 = 25\,\underline{/30^\circ}\ \Omega$, connected in parallel. They are supplied by a source whose voltage is $\mathbf{V} = 50\,\underline{/0^\circ}$ volts. Determine the following:

a. Current drawn by each impedance
b. Complex power absorbed by each impedance
c. Total current
d. Complex power supplied by the source

3.4 BALANCED THREE-PHASE AC CIRCUITS

In the United States, as in most parts of the world, electric bulk-power generation, transmission, and distribution are usually accomplished with three-phase systems. Although electric lights and small motors are single phase, they are assigned equally to the three phases of the distribution system so that the phases effectively form a balanced set.

A three-phase generator is shown in Fig. 3.6. The three voltage sources have equal magnitude, and their phase differences are each 120°. Each phase has a series resistance R_a and inductive reactance X_S. In the accompanying phasor diagram, $\mathbf{E}_{an}$ is seen leading $\mathbf{E}_{bn}$ by 120°, and $\mathbf{E}_{bn}$ is leading $\mathbf{E}_{cn}$ by 120°. Hence, the phase sequence is said to be positive sequence, or *an-bn-cn sequence*, or simply *abc sequence*.

3.4.1 Wye-Connected Load

A wye-connected load is shown in Fig. 3.7. Each phase impedance $Z_Y \angle \theta$ consists of a resistance R_Y and an inductive reactance X_Y. The *balanced set* of voltages applied to the terminals of the load is of abc phase sequence and is given by

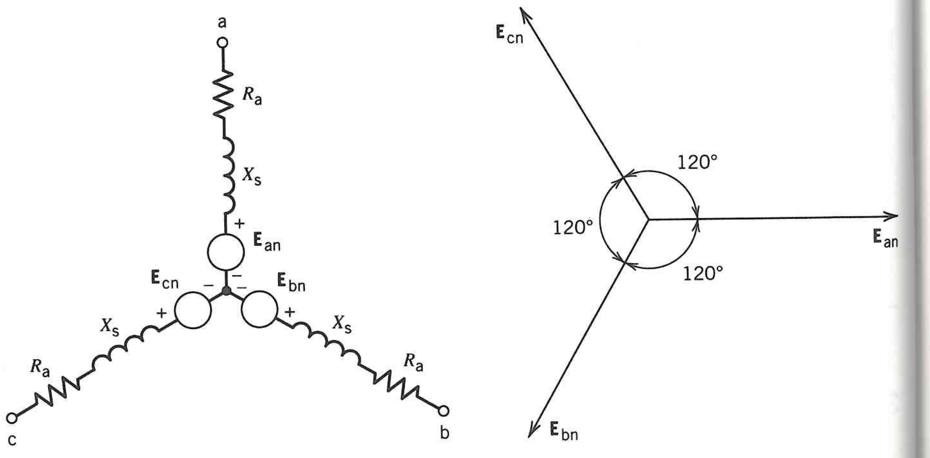
$$\mathbf{V}_{an} = V \angle 0^\circ, \qquad \mathbf{V}_{bn} = V \angle -120^\circ, \qquad \mathbf{V}_{cn} = V \angle -240^\circ \tag{3.31}$$

The line-to-line voltages are computed as follows:

$$\mathbf{V}_{ab} = \mathbf{V}_{an} - \mathbf{V}_{bn} = \sqrt{3}\,\mathbf{V}_{an} \angle 30^\circ = \sqrt{3}\,V \angle 30^\circ \tag{3.32}$$

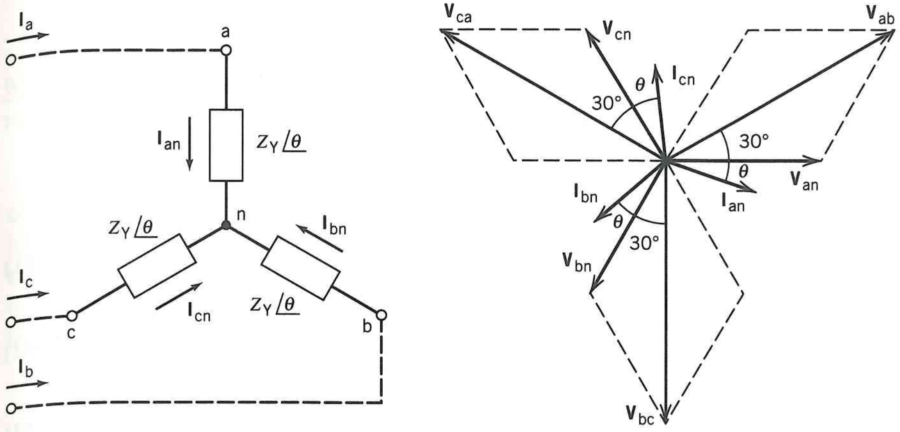
$$\mathbf{V}_{bc} = \mathbf{V}_{bn} - \mathbf{V}_{cn} = \sqrt{3}\,\mathbf{V}_{bn} \angle 30^\circ = \sqrt{3}\,V \angle -90^\circ \tag{3.33}$$

$$\mathbf{V}_{ca} = \mathbf{V}_{cn} - \mathbf{V}_{an} = \sqrt{3}\,\mathbf{V}_{cn} \angle 30^\circ = \sqrt{3}\,V \angle -210^\circ \tag{3.34}$$



(*a*) (*b*)

FIGURE 3.6 (*a*) Three-phase generator; (*b*) abc phase sequence.



(*a*) (*b*)

FIGURE 3.7 (*a*) Wye-connected load; (*b*) phasor diagram.

Thus, the line voltage has a magnitude equal to $\sqrt{3}$ times the magnitude of the phase voltage and it leads the corresponding phase voltage by 30°; that is,

$$\mathbf{V}_L = \sqrt{3}\mathbf{V}_P \angle 30^\circ \tag{3.35}$$

It may also be observed that the line currents are identical to the corresponding phase currents:

$$\mathbf{I}_L = \mathbf{I}_P \tag{3.36}$$

These currents are calculated as follows:

$$\mathbf{I}_a = \mathbf{I}_{an} = \frac{\mathbf{V}_{an}}{Z_Y \angle \theta} = \frac{V}{Z_Y} \angle -\theta = I \angle -\theta \tag{3.37}$$

$$\mathbf{I}_b = \mathbf{I}_{bn} = \frac{\mathbf{V}_{bn}}{Z_Y \angle \theta} = \frac{V}{Z_Y} \angle -\theta - 120^\circ = I \angle -\theta - 120^\circ \tag{3.38}$$

$$\mathbf{I}_c = \mathbf{I}_{cn} = \frac{\mathbf{V}_{cn}}{Z_Y \angle \theta} = \frac{V}{Z_Y} \angle -\theta - 240^\circ = I \angle -\theta - 240^\circ \tag{3.39}$$

It may be noted that the currents form a balanced set of phasors and the sum of the currents is zero:

$$\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 0 \tag{3.40}$$

3.4.2 Delta-Connected Load

A delta-connected load is shown in Fig. 3.8. Each phase impedance $Z_\Delta \angle\theta$ consists of a resistance R_Δ and an inductive reactance X_Δ. It is assumed that balanced abc phase sequence voltages are applied and $\mathbf{V}_{ab} = V \angle 0°$.

The phase currents will be

$$\mathbf{I}_{ab} = \frac{\mathbf{V}_{ab}}{Z_\Delta \angle\theta} = \frac{V}{Z_\Delta} \angle -\theta = I \angle -\theta \tag{3.41}$$

$$\mathbf{I}_{bc} = \frac{\mathbf{V}_{bc}}{Z_\Delta \angle\theta} = \frac{V}{Z_\Delta} \angle -\theta - 120° = I \angle -\theta - 120° \tag{3.42}$$

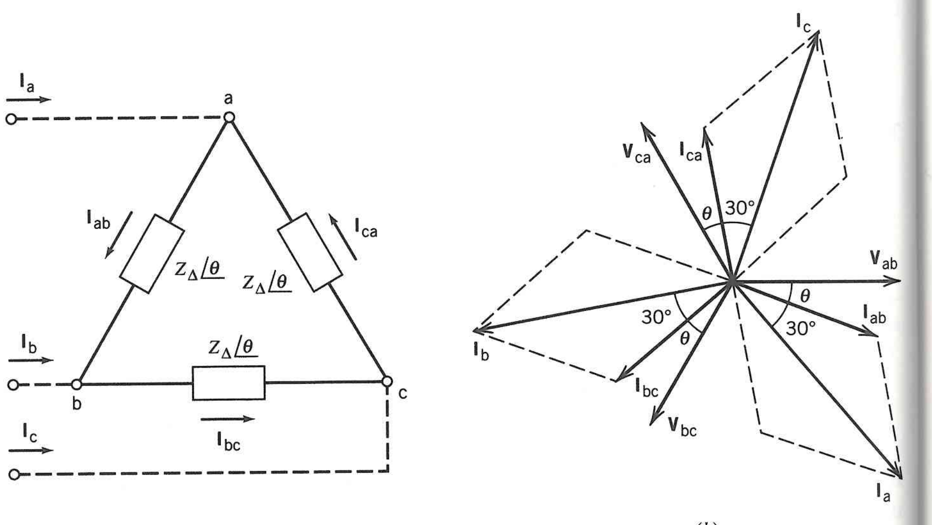
$$\mathbf{I}_{ca} = \frac{\mathbf{V}_{ca}}{Z_\Delta \angle\theta} = \frac{V}{Z_\Delta} \angle -\theta - 240° = I \angle -\theta - 240° \tag{3.43}$$

The line currents are calculated as follows:

$$\mathbf{I}_{a} = \mathbf{I}_{ab} - \mathbf{I}_{ca} = \sqrt{3}\mathbf{I}_{ab} \angle -30° = \sqrt{3} I \angle -\theta - 30° \tag{3.44}$$

$$\mathbf{I}_{b} = \mathbf{I}_{bc} - \mathbf{I}_{ab} = \sqrt{3}\mathbf{I}_{bc} \angle -30° = \sqrt{3} I \angle -\theta - 150° \tag{3.45}$$

$$\mathbf{I}_{c} = \mathbf{I}_{ca} - \mathbf{I}_{bc} = \sqrt{3}\mathbf{I}_{ca} \angle -30° = \sqrt{3} I \angle -\theta - 270° \tag{3.46}$$



(a) (b)

FIGURE 3.8 (a) Delta-connected load; (b) phasor diagram.

It may be observed that the line-to-line voltages are identical to the phase voltages for delta-connected loads:

$$\mathbf{V}_L = \mathbf{V}_P \tag{3.47}$$

The delta-connected load may be transformed into an equivalent wye-connected load so that the terminal behavior of the two configurations will be identical; that is, corresponding line-to-line voltages and line currents will be the same. A derivation of this Δ-to-Y transformation is given in Ref. 5. When the load is balanced, the impedance per phase of the wye-connected load will be one-third of the impedance per phase of the delta-connected load:

$$Z_Y = \tfrac{1}{3} Z_\Delta \tag{3.48}$$

3.4.3 Analysis of Balanced Three-Phase Systems

If a three-phase system is balanced, it may be analyzed by using a single-phase equivalent circuit. Since the sum of the phase currents is equal to zero, a neutral wire may be connected between the source neutral and the load neutral. This neutral wire would not affect voltages or currents in the circuit.

Figure 3.9 shows a wye-connected generator supplying a wye-connected load through a three-phase feeder. It can be shown that the voltages and currents belonging to a particular phase are identical to corresponding voltages and currents in the other phases except for 120° shifts in their respective phase angles. Therefore, a single circuit consisting of one phase and a neutral wire may be analyzed and the results applied to the other phases by including the corresponding 120° phase shift. This procedure will be illustrated in the following sample system.

When the three-phase source (or load) is delta connected, it is customary to transform it into an equivalent wye-connected source (or load) before applying the procedure. Subsequently, the results are reconverted into their delta equivalents.

EXAMPLE 3.3

A three-phase power system consists of a wye-connected ideal generator connected to a wye-connected load through a three-phase feeder. The load has an impedance $Z_L \angle\theta = 20 \angle 30°$ Ω /phase, and the feeder has an impedance $Z_{fdr} = 1.5 \angle 75°$ Ω /phase. The terminal voltage of the load is 4.16 kV. Determine (a) the terminal voltage of the generator and (b) the line current supplied by the generator.

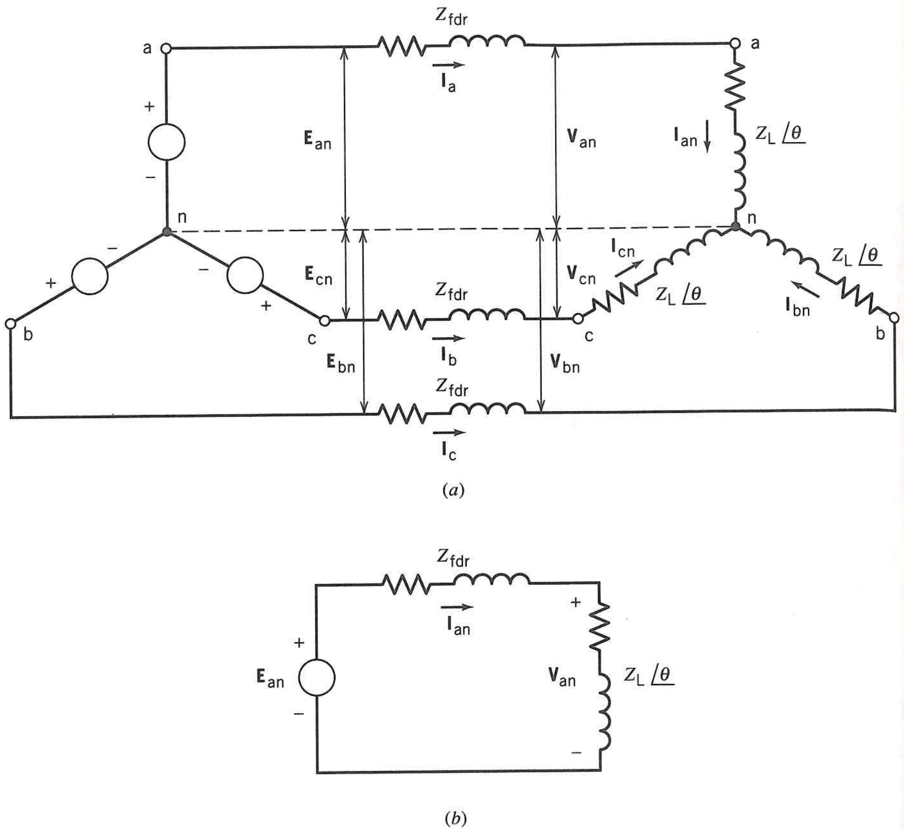
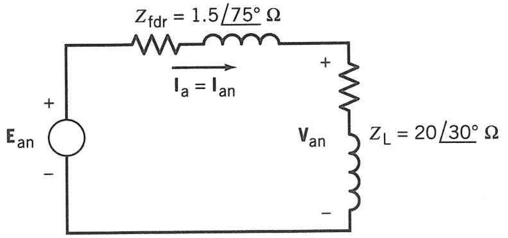


FIGURE 3.9 (*a*) Three-phase power system; (*b*) single-phase equivalent circuit.

Solution With both generator and load wye connected, the single-phase analysis is used in conjunction with the single-phase equivalent circuit shown in Fig. 3.10.

The phase a voltage at the load is chosen as reference phasor. It is given by

$$\mathbf{V}_{\text{an}} = \left(4160/\sqrt{3}\right)\angle 0^\circ = 2400\angle 0^\circ \text{ V} \qquad \text{(line-to-neutral)}$$



The phase a current, which is identical to the line a current, is found as follows:

$$\mathbf{I}_{\text{a}} = \mathbf{I}_{\text{an}} = \frac{\mathbf{V}_{\text{an}}}{Z_{\text{L}}\angle\theta} = \frac{2400\angle 0^\circ}{20\angle 30^\circ} = 120\angle -30^\circ \text{ A}$$

a. The phase a voltage of the generator is found as follows:

$$\begin{aligned}\mathbf{E}_{\text{an}} &= \mathbf{V}_{\text{an}} + \mathbf{I}_{\text{a}}Z_{\text{fdr}} = 2400\angle 0^\circ + (120\angle -30^\circ)(1.5\angle 75^\circ)\\ &= 2527 + j127 = 2530\angle 3^\circ \text{ V}\end{aligned}$$

Consequently, the phase voltages of the load and the generator are computed as follows:

$$\begin{aligned}\mathbf{V}_{\text{an}} &= 2400\angle 0^\circ \text{ V} & \mathbf{E}_{\text{an}} &= 2530\angle 3^\circ \text{ V}\\ \mathbf{V}_{\text{bn}} &= 2400\angle -120^\circ & \mathbf{E}_{\text{bn}} &= 2530\angle -117^\circ\\ \mathbf{V}_{\text{cn}} &= 2400\angle 120^\circ & \mathbf{E}_{\text{cn}} &= 2530\angle 123^\circ\end{aligned}$$

The line-to-line voltages are

$$\begin{aligned}\mathbf{V}_{\text{ab}} &= 4160\angle 30^\circ \text{ V} & \mathbf{E}_{\text{ab}} &= 4382\angle 33^\circ \text{ V}\\ \mathbf{V}_{\text{bc}} &= 4160\angle -90^\circ & \mathbf{E}_{\text{bc}} &= 4382\angle -87^\circ\\ \mathbf{V}_{\text{ca}} &= 4160\angle 150^\circ & \mathbf{E}_{\text{ca}} &= 4382\angle 153^\circ\end{aligned}$$

b. The load current and generator current are equal and are given by

$$\begin{aligned}\mathbf{I}_{\text{a}} &= \mathbf{I}_{\text{an}} = 120\angle -30^\circ \text{ A}\\ \mathbf{I}_{\text{b}} &= \mathbf{I}_{\text{bn}} = 120\angle -150^\circ\\ \mathbf{I}_{\text{c}} &= \mathbf{I}_{\text{cn}} = 120\angle 90^\circ\end{aligned}$$

3.4.4 Power in Balanced Three-Phase Systems

The total average power absorbed by a three-phase balanced load, or delivered by a three-phase generator, is equal to the sum of the powers in each phase. The voltages and currents in each phase are equal; that is,

$$V_{\text{P}} = V_{\text{an}} = V_{\text{bn}} = V_{\text{cn}} \qquad (3.49)$$

Therefore, the total three-phase power is

$$P_T = 3P_P = 3V_P I_P \cos\theta_P \tag{3.51}$$

where θ_P is the phase angle between the voltage and the current. Similarly, the total three-phase reactive power is

$$Q_T = 3Q_P = 3V_P I_P \sin\theta_P \tag{3.52}$$

Also the total three-phase apparent power is given by

$$S_T = 3S_P = \sqrt{P_T^2 + Q_T^2} = 3\sqrt{P_P^2 + Q_P^2} = 3V_P I_P \tag{3.53}$$

For a three-phase wye-connected generator, or wye-connected load, $I_L = I_P$, and $V_L = \sqrt{3}V_P$. Thus the real, reactive, and apparent powers may be expressed as follows:

$$P_T = 3\left(\frac{V_L}{\sqrt{3}}\right) I_L \cos\theta_P = \sqrt{3}V_L I_L \cos\theta_P \tag{3.54}$$

$$Q_T = 3\left(\frac{V_L}{\sqrt{3}}\right) I_L \sin\theta_P = \sqrt{3}V_L I_L \sin\theta_P \tag{3.55}$$

$$S_T = 3\left(\frac{V_L}{\sqrt{3}}\right) I_L = \sqrt{3}V_L I_L \tag{3.56}$$

For a three-phase delta-connected generator, or delta-connected load, $I_L = \sqrt{3}I_P$ and $V_L = V_P$. In terms of the line quantities, the power expressions may be written as

$$P_T = 3V_L\left(\frac{I_L}{\sqrt{3}}\right) \cos\theta_P = \sqrt{3}V_L I_L \cos\theta_P \tag{3.57}$$

$$Q_T = 3V_L\left(\frac{I_L}{\sqrt{3}}\right) \sin\theta_P = \sqrt{3}V_L I_L \sin\theta_P \tag{3.58}$$

$$S_T = 3V_L\left(\frac{I_L}{\sqrt{3}}\right) = \sqrt{3}V_L I_L \tag{3.59}$$

Ratings of three-phase equipment, such as generators, motors, transformers, and transmission lines, are normally given as total or three-phase real power in MW, or as total apparent power in MVA, and as line-to-line voltage in kV.

EXAMPLE 3.4

A three-phase motor draws 20 kVA at 0.707 lagging power factor from a 220-V source. It is desired to improve the power factor to 0.90 lagging by connecting a capacitor bank across the terminals of the motor.

a. Calculate the line current before and after the addition of the capacitor bank.

b. Determine the required kVA rating of the capacitor bank.

Solution The real and reactive powers of the load are

$$P_M = (20)(0.707) = 14.14 \text{ kW}$$
$$Q_M = 20\sin(\cos^{-1}0.707) = 14.14 \text{ kVAR}$$

a. The line current of the motor is

$$I_M = \frac{S_T}{\sqrt{3}V_L} = \frac{20{,}000}{\sqrt{3}\,220} = 52.5 \text{ A}$$

For a power factor $\text{PF}_{\text{corr}} = 0.90$, the new value of line current is

$$I_{\text{corr}} = \frac{P_M}{\sqrt{3}V_L \text{PF}_{\text{corr}}} = \frac{14{,}140}{\sqrt{3}(220)(0.9)} = 41.2 \text{ A}$$

b. The corrected value of reactive power is

$$Q_{\text{corr}} = P_M \tan(\cos^{-1}0.90) = 14.14\tan 25.8^\circ = 6.85 \text{ kVAR}$$

The kVA rating of the capacitor bank required to bring the power factor from 0.707 to 0.90 lagging is found as

$$Q_{\text{cap}} = Q_{\text{corr}} - Q_M = 6.85 - 14.14 = -7.29 \text{ kVAR}$$

A power triangle depicting power factor correction by using capacitors is shown in Fig. 3.11.

3.4.5 Instantaneous Power in Balanced Three-Phase Systems

For a balanced three-phase load, the total instantaneous power is equal to the sum of the individual powers of the three phases. Thus,

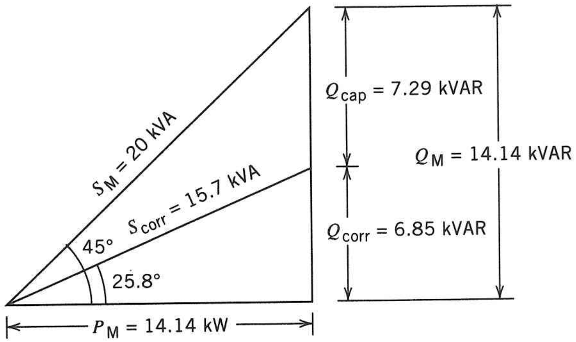


FIGURE 3.11 Power triangle of Example 3.4.

$$p_T = p_a + p_b + p_c \tag{3.60}$$

In terms of the instantaneous phase voltages and currents, the total power is

$$p_T = v_{an}i_a + v_{bn}i_b + v_{cn}i_c \tag{3.61}$$

Assuming a positive phase sequence with v_{an} taken as the reference, the total power may be expressed as follows:

$$\begin{aligned} p_T &= V_mI_m \cos\omega t \cos(\omega t - \theta_P) \\ &\quad + V_mI_m \cos(\omega t - 120°)\cos(\omega t - \theta_P - 120°) \\ &\quad + V_mI_m \cos(\omega t - 240°)\cos(\omega t - \theta_P - 240°) \end{aligned} \tag{3.62}$$

where V_m and I_m are the peak values of the phase voltages and currents, respectively, and θ_P is the phase angle by which the current lags the voltage in each phase. By using the trigonometric identity on the product of two cosine functions that was cited previously in Section 3.3.2, Eq. 3.62 can be written as

$$\begin{aligned} p_T &= \tfrac{3}{2}V_mI_m \cos\theta_P + \tfrac{1}{2}V_mI_m[\cos(2\omega t - \theta_P) \\ &\quad + \cos(2\omega t - \theta_P + 120°) + \cos(2\omega t - \theta_P - 120°)] \end{aligned} \tag{3.63}$$

Since the second term on the right-hand side of Eq. 3.63 is identically equal to zero, it reduces to

$$p_T = \tfrac{3}{2}V_mI_m \cos\theta_P \tag{3.64}$$

This demonstrates an important property of a balanced three-phase system: the total instantaneous power is time invariant.

3.4.6 Three-Phase Power Measurements

In three-phase power systems, two wattmeters can be used to measure total power. The connection and phasor diagrams are shown in Fig. 3.12 for an assumed abc phase sequence and lagging power factor.

The wattmeter readings are given by

$$W_1 = V_{ab}I_a \cos \underline{\angle(V_{ab}, I_a)} = V_LI_L\cos(30° + \theta) \tag{3.65}$$

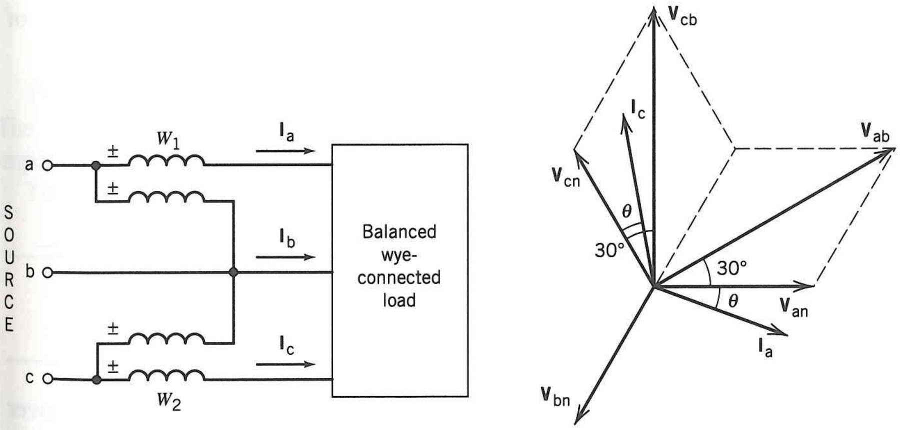
$$W_2 = V_{cb}I_c \cos \underline{\angle(V_{cb}, I_c)} = V_LI_L\cos(30° - \theta) \tag{3.66}$$

The sum of the two wattmeter readings gives the total three-phase power:

$$\begin{aligned} P_T = W_1 + W_2 &= V_LI_L[\cos(30° + \theta) + \cos(30° - \theta)] \\ &= \sqrt{3}V_LI_L\cos\theta \end{aligned} \tag{3.67}$$

The difference of the two readings is

$$\begin{aligned} W_2 - W_1 &= V_LI_L[\cos(30° - \theta) - \cos(30° + \theta)] \\ &= V_LI_L\sin\theta \end{aligned} \tag{3.68}$$



(*a*) (*b*)

FIGURE 3.12 Two-wattmeter method: (*a*) connection diagram; (*b*) phasor diagram.

which is $1/\sqrt{3}$ times the total three-phase reactive power. Thus, Q_T can be found from

$$Q_T = \sqrt{3}(W_2 - W_1) \tag{3.69}$$

The power factor angle can also be found from

$$\theta = \tan^{-1}\left(\frac{Q_T}{P_T}\right) = \tan^{-1}\left[\frac{\sqrt{3}(W_2 - W_1)}{W_2 + W_1}\right] \tag{3.70}$$

DRILL PROBLEMS

D3.7 A balanced, three-phase load is connected to a 2200-V feeder. The load draws a line current of 60 A at a power factor of 0.90 lagging. Calculate the real, reactive, and apparent power absorbed by the load.

D3.8 A delta-connected load consists of three identical impedances of $8 + j6\ \Omega$ each and is supplied from a three-phase, 200-V source. Calculate

- **a.** The phase current and the line current
- **b.** The power factor
- **c.** The real, reactive, and apparent power taken by the load

D3.9 Repeat Problem D3.8 when the load impedances are wye connected.

D3.10 Two balanced wye-connected loads of $8 + j5\ \Omega$/phase and $6 - j2\ \Omega$/phase are supplied by a three-phase source at a line-to-line voltage of 440 V. Find

- **a.** The line current drawn by each load
- **b.** The total line current supplied by the source
- **c.** The real, reactive, and complex power absorbed by each load
- **d.** The real, reactive, and complex power delivered by the source

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3.5 PER-UNIT ANALYSIS

The per-unit method of power system analysis offers distinct advantages over the use of actual amperes, volts, and ohms. It eliminates the need for conversion of the voltages, currents, and impedances across every transformer in the circuit; thus, there is less chance of computational errors. Second, the need to transform from three-phase to single-phase equivalents, and vice versa, is avoided with the use of per-unit quantities; hence, there is less confusion in handling and manipulating the various parameters in three-phase systems.

In the per-unit system, any electrical quantity may be expressed in per unit as the ratio of the actual quantity and the chosen base value for that quantity. This quotient is presented either in decimal form or as a percentage. However, care must be taken when handling percent quantities. Although the product of two quantities in per unit is expressed in per unit itself, the product of two quantities in percent must be divided by 100 to obtain the correct result in percent.

Four base electrical quantities must be considered: power, voltage, current, and impedance bases. Just like the actual quantities, these bases satisfy the electrical laws.

In single-phase systems, the relationships among the base quantities are

$$S_{\text{base}} = V_{\text{base}} I_{\text{base}} \tag{3.71}$$

$$V_{\text{base}} = I_{\text{base}} Z_{\text{base}} \tag{3.72}$$

With only two equations relating the four base quantities, it is necessary to specify two base values. The power and voltage bases are usually chosen equal to the rated values, or nominal values, and the other two are computed from the foregoing electrical relationships as follows:

$$I_{\text{base}} = \frac{S_{\text{base}}}{V_{\text{base}}} \tag{3.73}$$

$$Z_{\text{base}} = \frac{V_{\text{base}}}{I_{\text{base}}} = \frac{(V_{\text{base}})^2}{S_{\text{base}}} = \frac{(\text{kV}_{\text{base}})^2}{\text{MVA}_{\text{base}}} \tag{3.74}$$

The specified power base is applicable to all parts of the power system. The voltage base varies across a transformer, and so do the current base and impedance base.

The per-unit electrical quantities are calculated as follows:

$$\mathbf{S}_u = \frac{P + jQ}{S_{\text{base}}} = P_u + jQ_u \tag{3.75}$$

$$\mathbf{V}_u = \frac{\mathbf{V}}{V_{\text{base}}} \tag{3.76}$$

$$\mathbf{I}_u = \frac{\mathbf{I}}{I_{\text{base}}} \tag{3.77}$$

$$Z_u = \frac{Z}{Z_{\text{base}}} \tag{3.78}$$

The complex power into a lossless transformer is equal to the complex power out. On the other hand, actual voltages, currents, and impedances change across the transformer. However, because base voltage, base current, and base impedance also change across the transformer, the per-unit values are the same on both sides of the transformer. Furthermore, since the voltage base is usually chosen to be either the nominal voltage or the rated voltage, the per-unit value of voltage is almost always near unity.

EXAMPLE 3.5

Solve Example 3.2 using per-unit representation. Choose $S_{\text{base}} = 1500$ VA and $V_{\text{base}} = 120$ V.

Solution The base current is calculated as

$$I_{\text{base}} = S_{\text{base}}/V_{\text{base}} = 1500/120 = 12.5 \text{ A}$$

The base impedance is

$$Z_{\text{base}} = (V_{\text{base}})^2/S_{\text{base}} = (120)^2/1500 = 9.6\ \Omega$$

The per-unit values are computed as follows:

$$\mathbf{V}_{\text{L}} = \frac{120\angle 0^\circ}{120} = 1.0\angle 0^\circ \text{ pu V}$$

$$Z_{\text{L}} = \frac{10\angle 36.9^\circ}{9.6} = 1.04\angle 36.9^\circ \text{ pu } \Omega$$

$$\mathbf{I}_{\text{L}} = \frac{\mathbf{V}_{\text{L}}}{Z_{\text{L}}} = \frac{1.0\angle 0^\circ}{1.04\angle 36.9^\circ} = 0.96\angle -36.9^\circ \text{ pu A}$$

$$Z_{\text{fdr}} = \frac{2.24\angle 63.4^\circ}{9.6} = 0.233\angle 63.4^\circ \text{ pu } \Omega$$

The generator voltage in per unit is computed as follows:

$$\begin{aligned}\mathbf{E}_{\text{g}} &= \mathbf{V}_{\text{L}} + \mathbf{I}_{\text{L}}Z_{\text{fdr}} \\ &= 1.0\angle 0^\circ + (0.96\angle -36.9^\circ)(0.233\angle 63.4^\circ) \\ &= 1.204\angle 4.8^\circ \text{ pu} \\ &= (1.204\angle 4.8^\circ)(120) = 144.5\angle 4.8^\circ \text{ V}\end{aligned}$$

The complex power is then calculated as follows:

$$\begin{aligned}\mathbf{S}_{\text{g}} &= \mathbf{E}_{\text{g}}\mathbf{I}_{\text{g}}^* = \mathbf{E}_{\text{g}}\mathbf{I}_{\text{L}}^* \\ &= (1.204\angle 4.8^\circ)(0.96\angle -36.9^\circ)^* = 1.156\angle 41.7^\circ \\ &= 0.863 + j0.769 \text{ pu}\end{aligned}$$

Therefore,

$$\begin{aligned}P_{\text{g}} &= (0.863)(1500) = 1295 \text{ W} \\ Q_{\text{g}} &= (0.769)(1500) = 1154 \text{ VAR}\end{aligned}$$

For three-phase systems, the base power is total three-phase power and the base voltage is line-to-line voltage. With a wye connection assumed, the base line current is equal to the base phase current. The base impedance is per phase. The three-phase base quantities are related to the single-phase bases as follows:

$$\text{Base power:} \quad S_{\text{T,base}} = 3S_{\text{P,base}} \tag{3.79}$$

$$\text{Base voltage:} \quad V_{\text{L,base}} = \sqrt{3}V_{\text{P,base}} \tag{3.80}$$

$$\text{Base current:} \quad I_{\text{L,base}} = \frac{S_{\text{T,base}}}{\sqrt{3}V_{\text{L,base}}} = \frac{S_{\text{P,base}}}{V_{\text{P,base}}} = I_{\text{P,base}} \tag{3.81}$$

$$\text{Base impedance:} \quad Z_{\text{base}} = \frac{(V_{\text{L,base}})^2}{S_{\text{T,base}}} = \frac{(V_{\text{P,base}})^2}{S_{\text{P,base}}} \tag{3.82}$$

The impedance characteristic of an electrical equipment or device is usually expressed as a percentage based on its ratings. When such a device is connected in a power system in which the selected base values are different from the machine ratings, the per-unit quantities have to be expressed in terms of the system bases. The per-unit value of impedance may be converted to the new bases as follows:

$$Z_{\text{u,new}} = Z_{\text{u,old}}\left(\frac{S_{\text{base,new}}}{S_{\text{base,old}}}\right)\left(\frac{V_{\text{base,old}}}{V_{\text{base,new}}}\right)^2 \tag{3.83}$$

EXAMPLE 3.6

A three-phase, 60-Hz, 30-MVA, 13.8-kV, wye-connected synchronous generator has an armature resistance $R_{\text{a}} = 2\ \Omega$ per phase and a synchronous reactance $X_{\text{s}} = 10\ \Omega$ per phase.

a. Express the machine impedance in per unit based on the machine ratings.

b. Using the results of part (a), find the per-unit impedance based on a new $S_{\text{base}} = 50$ MVA and $V_{\text{base}} = 34.5$ kV.

Solution

a.

$$\begin{aligned} Z_{\text{base}} &= (13.8)^2/30 = 6.35\ \Omega \\ Z_s &= R_a + jX_s = 2 + j10 = 10.2\underline{/78.7^\circ}\ \Omega \\ Z_{\text{su}} &= Z_s/Z_{\text{base}} = (2 + j10)/6.35 = 0.315 + j1.575 \\ &= 1.606\underline{/78.7^\circ}\ \text{pu}\ \Omega \end{aligned}$$

b.

$$\begin{aligned} Z_{\text{su,new}} &= 1.606\underline{/78.7^\circ}(50/30)(13.8/34.5)^2 \\ &= 0.428\underline{/78.7^\circ} = 0.084 + j0.420\ \text{pu}\ \Omega \end{aligned}$$

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PROBLEMS

3.1 Given the complex numbers $A_1 = 5\underline{/30^\circ}$ and $A_2 = -3 + j4$, calculate the following, giving the answers in both rectangular and polar forms.

a. $A_1 + A_2$

b. A_1A_2

c. $A_1/(A_2)^*$

d. $(A_2)^2$

e. $A_1[1 + (A_2)^2]$

3.2 For a given electrical circuit, the expressions for voltage and current as functions of time are given as follows:

$$\begin{aligned} v(t) &= 283 \sin \omega t\ \text{V} \\ i(t) &= 35 \cos(\omega t + 25^\circ)\ \text{A} \end{aligned}$$

a. Find the rms values of the voltage and current.

b. Find the phasor expressions for the voltage and current in both polar and rectangular form.

c. State whether the circuit is inductive or capacitive.

3.3 The electrical network shown in Fig. 3.13 has a voltage source $\mathbf{V} = 100\underline{/0^\circ}$, and the values of the impedances are as follows:

$$Z_1 = 8 - j6\ \Omega, \qquad Z_2 = 3 - j4\ \Omega, \qquad Z_3 = 5 + j5\ \Omega$$

Determine (a) the real power absorbed by each impedance and (b) the reactive power taken by each impedance.

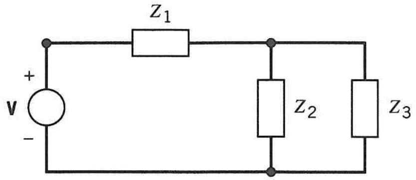

FIGURE 3.13 Electrical network of Problem 3.3.

3.4 A single-phase source supplies a load consisting of a resistor $R = 20\ \Omega$ and a capacitive reactance $X_C = 10\ \Omega$, which are connected in parallel. The instantaneous voltage of the source is given by

$$v(t) = 120 \cos(\omega t + 45^\circ)\ \text{V}$$

Find the following:

a. Phasor voltage **V** of the source

b. Phasor current **I** supplied by the source

c. Instantaneous current $i(t)$ supplied by the source

3.5 Repeat Problem 3.4 if the resistor and capacitor are connected in series.

3.6 For the parallel $R - C$ load of Problem 3.4, determine

a. Instantaneous power absorbed by the resistor

b. Instantaneous power absorbed by the capacitor

c. Real power absorbed by the resistor

d. Reactive power absorbed by the capacitor

e. Power factor of the combined load

3.7 Repeat Problem 3.6 if the resistor and capacitor are connected in series.

3.8 Consider a single-phase load with an applied voltage $v(t)$ and load current $i(t)$ specified as follows:

$$\begin{aligned} v(t) &= 220 \cos(\omega t + 20^\circ)\ \text{V} \\ i(t) &= 40 \cos(\omega t - 30^\circ)\ \text{A} \end{aligned}$$

a. Find the real and reactive power absorbed by the load.

b. Draw the power triangle.

c. Find the power factor, and state whether it is lagging or leading.

d. Calculate the reactance in ohms of capacitors to be connected in parallel with the load in order to improve the power factor to 0.9 lagging.

3.9 A series circuit has an impedance of $25\angle 53.1^\circ$ and is connected to a single-phase 220-V source.

a. Find the resistance and reactance of the load.

b. Find the real and reactive power absorbed by the load.

c. Find the power factor of the circuit, and state whether lagging or leading.

3.10 A single-phase source has a terminal voltage $\mathbf{V} = 120\angle -15^\circ$. It supplies a current of $15\angle 45^\circ$ to an electrical load.

a. Find the complex power supplied by the source.

b. Determine the real power. State whether the source is delivering or absorbing.

c. Determine the reactive power, and state whether the source is delivering or absorbing.

3.11 Two ideal voltage sources are connected to each other through a feeder with impedance $Z = 1.5 + j6\ \Omega$ as shown in Fig. 3.14. Let $\mathbf{E}_1 = 120\angle 0^\circ$ V and $\mathbf{E}_2 = 110\angle 45^\circ$ V.

a. Determine the real power of each machine, and state whether the machine is supplying or absorbing real power.

b. Determine the reactive power of each machine, and state whether each machine is delivering or receiving reactive power.

c. Determine the real and reactive power of the impedance, and state whether supplied or consumed.

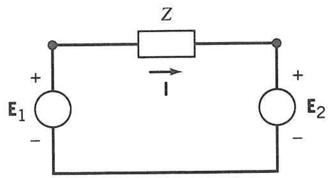

FIGURE 3.14 Electric circuit of Problem 3.11.

3.12 Repeat Problem 3.11 if the feeder impedance between the voltage sources of Fig. 3.14 is $Z = 1.5 - j6\ \Omega$.

3.13 A single-phase electrical load draws 10 MW at 0.6 power factor lagging.

a. Find the real and reactive power absorbed by the load.

b. Draw the power triangle.

c. Determine the kVAR of a capacitor to be connected across the load to raise the power factor to 0.95.

3.14 A capacitor is connected across the series impedance of Problem 3.9. This capacitor supplies 1000 VARs.

a. Find the real and reactive power supplied by the source.

b. Find the resultant power factor.

3.15 An industrial plant consists of several induction motors. The plant absorbs 300 kW at 0.6 PF lagging from the substation bus.

a. Compute the required kVAR rating of the capacitor connected across the load to raise the power factor to 0.9 lagging.

b. A 200-hp, 90% efficiency, synchronous motor is operated from the same bus at rated conditions and 0.8 power factor leading. Calculate the resulting power factor.

3.16 A 230-V source supplies two loads in parallel. One draws 5 kVA at a lagging power factor of 0.80, and the other draws 3 kW at a lagging power factor of 0.90. Find the source current.

3.17 A 440-V, 30-hp, three-phase motor operates at full load, 88% efficiency, and 65% power factor lagging.

a. Find the current drawn by the motor.

b. Find the real and reactive power absorbed by the motor.

3.18 A three-phase, 50-kVA, 600-V, 60-Hz generator operates at rated terminal voltage and supplies a line current of 48 A per phase at a 0.8 lagging power factor to a balanced three-phase load. Determine the real, reactive, and apparent power.

3.19 A 345-kV, three-phase transmission line delivers 500 MVA, 0.866 power factor lagging, to a three-phase load connected to its receiving-end terminals. Assume that the load is Δ connected and the voltage at the receiving end is 345 kV.

a. Find the complex load impedance per phase.

b. Calculate the line and phase currents.

c. Find the real and reactive power per phase.

d. Find the total real and reactive power.

3.20 Repeat Problem 3.19 assuming that the load is wye connected.

3.21 A three-phase load draws 120 kW at a power factor of 0.85 lagging from a 440-V bus. In parallel with this load is a three-phase capacitor bank that is rated 50 kVAR. Find (a) the total line current and (b) the resultant power factor.

3.22 A three-phase motor draws 40 kVA at 0.65 power factor lagging from a 230-V source. A capacitor bank is connected across the motor terminals to make the combined power factor 0.95 lagging.

a. Determine the required kVAR rating of the capacitor bank.

b. Determine the line current before and after the capacitors are added.

3.23 Two balanced wye-connected loads are connected in parallel with each other. The first draws 15 kVA at 0.8 PF lagging, and the second requires 20 kW at 0.9 PF leading. The two loads are supplied by a balanced three-phase, wye-connected, 2400-V source.

a. Determine the phasor current drawn by each load.

b. Find the real and reactive power absorbed by each load.

c. Compute the phasor current supplied by the source.

d. Calculate the total real and reactive power drawn by the combined load.

e. What is the overall power factor?

3.24 The motor of Problem 3.17 is connected to a substation bus through a three-phase feeder with an impedance $0.5 + j1.5$ Ω per phase. Find the line-to-line voltage at the bus if the voltage at the motor terminals is 440 V.

3.25 A three-phase substation bus supplies two wye-connected loads that are connected in parallel through a three-phase feeder with an impedance of $0.5 + j2.0$ Ω per phase. Load 1 draws 50 kW at 0.866 lagging power factor, and load 2 draws 36 kVA at 0.9 leading power factor. The line-to-line voltage at the loads is 460 V. Find the following:

a. Impedance of each load per phase

b. Total line current flowing through the feeder

c. Line-to-line voltage at the substation bus

d. Total real and reactive power supplied by the bus

3.26 A delta-connected load consists of three identical impedances $Z_\Delta = 45\angle 60^\circ$ Ω per phase. It is connected to a three-phase, 208-V source by a three-phase feeder with conductor impedance $Z_{\text{fdr}} = (1.2 + j1.6)$ Ω per phase.

a. Calculate the line-to-line voltage at the load terminals.

b. A delta-connected capacitor bank with a reactance of 60 Ω per phase is connected in parallel with the load at its terminals. Find the resulting line-to-line voltage at the load terminals.

3.27 The total power being absorbed by a balanced three-phase load is measured using the two-wattmeter method. The phase sequence is abc. The current coils of the wattmeters are connected in lines a and b. Let the readings of the two meters be P_a and P_b, respectively. The line voltage is 2400 V, and the load is 30 kVA.

a. Show a wiring diagram.

b. Calculate P_a and P_b if the power factor is 1.0.

c. Calculate P_a and P_b if the power factor is 0.2 lagging.

d. Calculate P_a and P_b if the power factor is 0.5 leading.

e. Sketch the phasor diagram showing all voltages and currents for each case.

3.28 Using 100 MVA and 115 kV as base values, express 110 kV, 75 MVA, 375 A, and 26.5 Ω in per-unit and percent values.

3.29 The per-unit impedance of a single-phase electric load is 0.5. The base power is 200 kVA, and the base voltage is 12 kV.

a. Find the per-unit impedance of the load if 400 kVA and 24 kV are selected as base values.

b. Find the ohmic value of the impedance.

3.30 A three-phase, 350-MVA, 13.8-kV AC generator has a synchronous reactance of 1.20 per unit. The generator is connected to a circuit for which the specified bases are 100 MVA, 13.2 kV.

a. Find the per-unit value of the generator synchronous reactance on the specified bases.

b. Find the ohmic value of the synchronous reactance.

3.31 A single-phase source is connected to an electrical load. The load draws a 0.6 pu current at 1.10 pu voltage while taking a real power of 0.4 pu at a lagging power factor. Choose a base voltage of 8 kV and a base current of 125 A. Calculate the following:

a. Real power in kW

b. Reactive power in kVAR

c. Power factor

d. The ohmic values of the resistance and the reactance of the load

Four

Magnetic Circuits and Transformers

4.1 INTRODUCTION

A transformer is a device used to convert AC electric energy at one voltage and current level to AC electric energy at another voltage and current level. This conversion takes place in an electromagnetic system consisting of two or more windings supported by a ferromagnetic structure.

Other forms of electromechanical energy conversion take place in rotating machines, both AC and DC machines, and other devices such as transducers, solenoids, and relays. In these machines and devices, electromagnetic fields that are confined in magnetic structures also play an important role in the conversion process. In the first few sections of this chapter, some basic concepts of electromagnetic theory are reviewed, typical magnetic circuits are analyzed, and other parameters, including flux linkages and inductances, are defined. These parameters are used in the discussion of the theory of energy conversion in rotating machines and transformers.

In the succeeding sections of this chapter, the principle of operation of a transformer is discussed, equivalent circuits for the transformer at steady state are developed, and the operating performance of the transformer is analyzed. Also, electrical tests for determining the parameters of the transformer are described. Finally, three-phase interconnections of transformers for three-phase voltage transformations are discussed.

4.2 MAGNETIC CIRCUITS

Ferromagnetic materials constitute a large portion of any electrical machine, including transformers. Hence, the study and design of electrical machinery includes the analysis of the magnetic circuits involved in these machines.

To be able to apply electric circuit concepts to these magnetic circuits, the following assumptions are made:

a. The frequencies involved—60 Hz or less—and the sizes (dimensions) of the magnetic structures are such that the displacement term in Maxwell's equation based on *Ampère's law* can be neglected. Thus,

$$\oint \mathbf{H} \cdot \mathbf{dl} = I \tag{4.1}$$

b. A three-dimensional magnetic field is reduced to a one-dimensional circuit equivalent, that is, magnetic circuit. Equation 4.1 states that the line integral of the *magnetic field* **H** over a closed path is equal to the net current enclosed by the path. **H** is expressed in amperes per meter when the current is given in amperes.

A *magnetic circuit* consists of a magnetic structure built mainly of high-permeability magnetic material. Thus, magnetic flux is confined to the paths presented by the high-permeability material, just as electric current is confined to the paths presented by the high-conductivity conductors of the electric circuit.

A simple magnetic circuit is shown in Fig. 4.1. The source of magnetic flux, the *magnetomotive force (mmf)*, is the electric current flowing in the N-turn winding. Applying Eq. 4.1 to a closed path yields

$$\oint \mathbf{H}_c \cdot \mathbf{dl} = NI \tag{4.2}$$

The magnetic field $\mathbf{H}_c$ is approximately constant, and its direction is the same as that of the magnetic flux ϕ , as shown in the figure. Hence, the integral in Eq. 4.2 reduces to $H_c l_c$. Therefore, the magnetomotive force (mmf), designated as

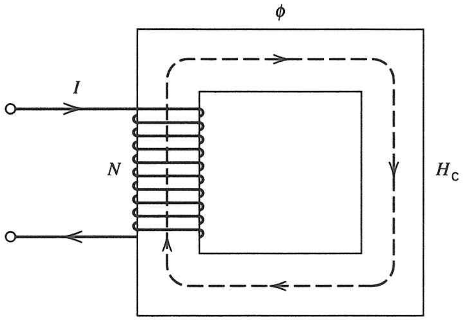

FIGURE 4.1 A magnetic circuit.

F, is given by

$$F = NI = H_c l_c \tag{4.3}$$

where l_c is the average length of the magnetic path or core.

The core is usually made of ferromagnetic material. The *magnetic flux density* B (expressed in tesla or weber/m^2) in the core is related to the magnetic field H according to the saturation curve, or B-H curve, of Fig. 4.2. The slope of this curve is designated as μ, the permeability of the material in henries per meter (H/m). Therefore, the relationship between $\mathbf{B}$ and $\mathbf{H}$ may be expressed as

$$\mathbf{B} = \mu \mathbf{H} \tag{4.4}$$

It may be seen from Fig. 4.2 that the slope of the B-H curve depends on the operating value of magnetic flux density. However, if the range of operating values is confined below the knee of the curve, the relationship between B and H can be approximated as linear with reasonable accuracy. Therefore, in the following discussions, it is assumed that the permeability of the magnetic materials used for core is a constant.

The permeability of a magnetic material is usually given relative to the permeability μ_0 of free space. Thus,

$$\mu = \mu_r \mu_0 \tag{4.5}$$

where μ_r is the relative permeability. In SI units, the permeability of free space $\mu_0 = 4\pi \times 10^{-7}$ H/m.

The *magnetic flux* ϕ (expressed in webers) through a given surface is found as follows:

$$\phi = \int_S \mathbf{B} \cdot \mathbf{dS} \tag{4.6}$$

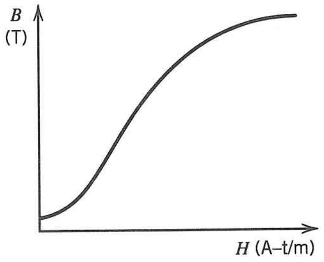

Table 4.1 Analogy Between Magnetic and Electric Circuits

Electric Circuit	Magnetic Circuit
i = current (A)	ϕ = flux (Wb)
V = emf (V)	F = mmf (A-t)
R = resistance (Ω)	$\mathcal{R}$ = reluctance (A-t/Wb)
σ = conductivity (S/m)	μ = permeability (H/m)

The direction of the differential area is along the perpendicular to the cross-sectional area A_c of the core itself. Since the flux density B_c has the same direction as $\mathbf{dS}$ and is approximately uniform over A_c, Eq. 4.6 reduces to

$$\phi = B_c A_c = \mu H_c A_c = \mu \left(\frac{NI}{l_c} \right) A_c \tag{4.7}$$

Rearranging Eq. 4.7 gives

$$\mathcal{R}\phi = NI = F \tag{4.8}$$

where $\mathcal{R} = l_c/(\mu A_c)$ = *reluctance* of the magnetic circuit in A-t/Wb.

Equation 4.8 is analogous to Ohm's law for resistive circuits. The analogies between other magnetic and electric circuit quantities are presented in Table 4.1.

EXAMPLE 4.1

The magnetic circuit shown in Fig. 4.1 has the following dimensions: A_c = 16 cm^2, l_c = 40 cm, and N = 350 turns. The relative permeability of the core is μ_r = 50,000. For a magnetic flux density of 1.5 T in the core, determine

a. The flux ϕ

b. The total flux linkage $\lambda = N\phi$

c. The required current through the coil

Solution

a. An electric circuit analog is drawn for the magnetic circuit. This is shown in Fig. 4.3.

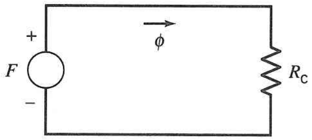

FIGURE 4.3 Electric circuit analog for Example 4.1

The magnetic flux is given by

$$\phi = BA = (1.5)(16 \times 10^{-4}) = 2.4 \text{ mWb}$$

b. The total flux linkage is

$$\lambda = N\phi = (350)(2.4 \times 10^{-3}) = 0.84 \text{ Wb-t}$$

c. The reluctance of the magnetic circuit is computed as

$$\begin{aligned} R_c &= l_c/(\mu_r \mu_0 A_c) \\ &= \frac{40 \times 10^{-2}}{(50{,}000)(4\pi \times 10^{-7})(16 \times 10^{-4})} = 3979 \text{ A-t/Wb} \end{aligned}$$

By using Eq. 4.8, the current is computed as follows:

$$I = \frac{R_c \phi}{N} = \frac{(3979)(2.4 \times 10^{-3})}{350} = 27.3 \text{ mA}$$

A magnetic core of this type may be used in a single-phase transformer.

EXAMPLE 4.2

An air gap of length $g = 0.5$ mm is cut in the right leg of the magnetic circuit of Fig. 4.1. If a current of 1.2 A flows through the coil, calculate

a. The flux ϕ

b. The total flux linkage $\lambda = N\phi$

c. The magnetic flux density B

Solution

a. The electric circuit analog is shown in Fig. 4.4.

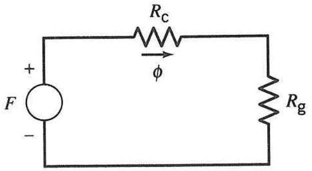

FIGURE 4.4 Electric circuit analog for Example 4.2.

The mmf is given by

$$F = NI = (350)(1.2) = 420 \text{ A-t}$$

The reluctance of the core is computed as follows:

$$R_c = \frac{l_c - g}{\mu_c A_c} \cong \frac{l_c}{\mu_c A_c} = 3979 \text{ A-t/Wb}$$

Assuming that fringing in the air gap is negligible, that is, $A_g = A_c$, the reluctance of the air gap is computed as follows:

$$\begin{aligned} R_g &= \frac{l_g}{\mu_0 A_c} \\ &= \frac{0.5 \times 10^{-3}}{(4\pi \times 10^{-7})(16 \times 10^{-4})} = 248{,}680 \text{ A-t/Wb} \end{aligned}$$

For this magnetic circuit, the total reluctance of the flux path is

$$R_t = R_c + R_g = 3979 + 248{,}680 = 252{,}659 \text{ A-t/Wb}$$

The magnetic flux is calculated as

$$\phi = F/R_t = 420/252{,}659 = 1.66 \times 10^{-3} \text{ Wb}$$

b. The flux linkage is computed as follows:

$$\lambda = N\phi = (350)(1.66 \times 10^{-3}) = 0.58 \text{ Wb-t}$$

c. The magnetic flux density is

$$B = \phi/A = (1.66 \times 10^{-3})/(16 \times 10^{-4}) \cong 1.04 \text{ T}$$

A similar magnetic circuit is used in electrical machines, electric meters, and protective relays.

DRILL PROBLEMS

D4.1 The magnetic circuit shown in Fig. 4.1 has an air gap cut in the right leg of the core. The air gap is 0.1 mm long. The coil is connected to a voltage source, and the current drawn is adjusted so that the magnetic flux density in the air gap is 1.5 T. Assume that flux fringing in the air gap is negligible. Use the dimensions and the relative permeability of the magnetic core specified in Example 4.1.

a. Find the value of the current.

b. Calculate the magnetic flux.

c. Determine the flux linkage of the coil.

D4.2 A magnetic core is built in the form of a circular ring having a mean radius of 10 cm. A coil containing 150 turns is wound uniformly throughout the length of the core. The coil is connected to a voltage source, and it draws a current of 15 A.

a. Determine the mmf of the coil.

b. Calculate the magnetic field intensity in the core.

D4.3 The circular ring of Drill Problem D4.2 has a mean cross-sectional area of 25 cm^2. The relative permeability of the material of the ring is 1500. Calculate

a. The magnetic flux in the core

b. The magnetic flux density in the core

c. The flux linkage of the coil

d. The reluctance of the core

D4.4 A magnetic core has a circular cross-sectional area of 2.0 in^2, a mean path length of 10 in, and an air-gap length of 0.125 in. A 350-turn coil is wound around the magnetic core, and a current of 5 A is supplied to the coil. Assume that the relative permeability of the core is infinite and fringing of flux in the air gap is negligible.

a. Calculate the reluctance of the magnetic circuit.

b. Find the magnetic flux density in the air gap.

D4.5 Repeat Drill Problem D4.4 assuming that the core has a relative permeability $\mu_r = 5000$.

4.3 FARADAY'S LAW

In 1820 Oersted observed that a compass needle is deflected by a current-carrying conductor. In 1831 Faraday discovered the principles of induced electromotive force (emf) on which the design and operation of generators, motors, and transformers are based. Faraday's experiments and findings can be described as follows:

A time-varying magnetic field induces an electromotive force that produces a current in a closed circuit. This current flows in a direction such that it produces a magnetic field that tends to oppose the changing magnetic flux of the original time-varying field.

Mathematically, *Faraday's law* can be stated as follows:

$$\text{emf} = \frac{d\lambda}{dt} \tag{4.9}$$

where λ is the total flux linkage of the closed path. A nonzero value of $d\lambda/dt$ may result from any of the following conditions:

a. A time-varying flux linking a stationary path

b. Relative motion between a steady flux and a closed path

c. A combination of the first two

If the closed path consists of a winding with N turns and we assume that each turn links the same flux ϕ, then the induced emf is given by

$$e = \text{emf} = \frac{d\lambda}{dt} = N\frac{d\phi}{dt} \tag{4.10}$$

where $\lambda = N\phi$ = total flux linkage.

EXAMPLE 4.3

The flux density **B** is normal to the plane of the rectangular loop and directed outward as shown in Fig. 4.5, and it is equal to

$$\mathbf{B} = B_0 \cos \omega t \ \mathbf{u}$$

where

B_0 = maximum value of flux density in tesla
ω = constant radian frequency
t = time in seconds
$\mathbf{u}$ = unit vector normal to the loop

a. Find an expression for the induced emf e_{xy}, between terminals x and y.

b. If a resistor R is connected between terminals x and y, determine the magnitude and direction of current in the resistor at $t = 0$ and at $t = \pi/(2\omega)$ s.

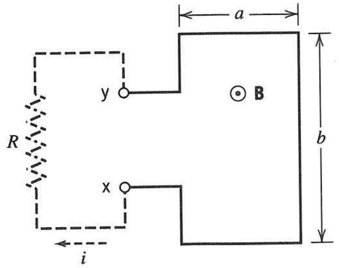

FIGURE 4.5 Rectangular loop.

Solution

a. If flux increases outward, point x will have a higher potential than point y. The magnetic flux is given by

$$\phi = \int \mathbf{B} \cdot d\mathbf{S} = Bab = B_0 ab \cos \omega t$$

where ab = area of the loop.

The induced voltage is found as

$$\begin{aligned} e_{xy} &= N\frac{d\phi}{dt} \\ &= N\frac{d}{dt}(B_0 ab \cos \omega t) \\ &= -B_0 ab\omega \sin \omega t \\ &= B_0 ab\omega \cos (\omega t + \pi/2) \end{aligned}$$

since the loop contains $N = 1$ turn.

b. The current $i(t)$ that will flow through a resistor R connected across terminals x and y is given by

$$i(t) = e_{xy}(t)/R$$

At $t = 0$:

$$i(0) = \frac{B_0 ab\omega}{R} \cos\left(\frac{\pi}{2}\right) = 0$$

At $t = \pi/(2\omega)$:

$$i\left(\frac{\pi}{2\omega}\right) = \frac{B_0 ab\omega}{R} \cos\left[\omega\left(\frac{\pi}{2\omega}\right) + \frac{\pi}{2}\right] = -\frac{B_0 ab\omega}{R}$$

Faraday's law is used to derive the expression for the induced emf in magnetic circuits. Such magnetic circuits have cores that are made of magnetic materials. Hence, a few remarks are given here on the losses associated with magnetic materials.

There are two sources of losses associated with magnetic materials. The first loss is called eddy current loss. Because of the time variation of flux in the core, eddy current loops are induced in the core. Since the core is made of ferromagnetic materials that contain resistances, current flow results in power loss in the form of heat. To reduce the effects of eddy currents, magnetic structures are usually built of laminations, or thin sheets, that are insulated from each other by a thin coat of insulating varnish.

The second loss is called hysteresis loss. This loss is the energy required to move the magnetic dipoles in the material. This energy is also dissipated as heat. To reduce hysteresis loss, core material is usually made of good-quality electrical steel having a narrow hysteresis characteristic loop. These losses due to hysteresis and eddy currents are collectively known as *core losses* or *iron losses*.

4.4 INDUCTANCE AND MAGNETIC ENERGY

Consider the magnetic circuit shown in Fig. 4.6. If the coil is connected to a voltage source of voltage v, a current i will flow. The current produces magnetic flux ϕ, as shown in the figure. The total *flux linkage* λ of the coil containing N turns is given by

$$\lambda = N\phi \qquad (4.11)$$

If leakage flux is neglected, the magnetic flux is equal to the mmf of the coil divided by the reluctance of the flux path:

$$\phi = \frac{Ni}{R} = \frac{Ni}{l/(\mu A)} \qquad (4.12)$$

where

l = mean length of the flux path
A = cross-sectional area of the core
μ = permeability of the core

The *self-inductance* L of the coil is defined as follows:

$$L = \frac{\text{total flux linkage of the coil}}{\ } = \frac{\lambda}{\ } \qquad (4.13)$$

Substituting Eq. 4.12 into Eq. 4.11 and dividing by the current i yields the coil inductance:

$$L = \frac{\lambda}{i} = \frac{N^2\mu A}{l} \tag{4.14}$$

When the voltage applied to the winding shown in Fig. 4.6 is a time-varying voltage, the current that flows and the magnetic flux produced are also time varying. Therefore, by Faraday's law, an emf is induced across the coil. This induced voltage is given by

$$e = \frac{d\lambda}{dt} = \frac{d(Li)}{dt} = L\frac{di}{dt} \tag{4.15}$$

The energy delivered to the inductor over the time interval from t_0 to t_1 is calculated as the integral of the power. If it is assumed that at time $t_0 = 0$, $i_0 = 0$, and at time t_1, $i = i_1$, then the energy is found as follows:

$$W = \int_{t_0}^{t_1} p\,dt = \int_{t_0}^{t_1} i e\,dt = \int_{t_0}^{t_1} i\left(\frac{d\lambda}{dt}\right)dt \tag{4.16}$$

$$W = \int_{\lambda_0}^{\lambda_1} i\,d\lambda = \frac{1}{L}\int_{\lambda_0}^{\lambda_1} \lambda\,d\lambda = \left(\frac{1}{2L}\right)\lambda_1^2 \tag{4.17}$$

$$W = L\int_{i_0}^{i_1} i\,di = \frac{1}{2}Li_1^2 \tag{4.18}$$

where $\lambda_0 = i_0 = 0$ at $t_0 = 0$ and $\lambda_1 = Li_1$ at time t_1.

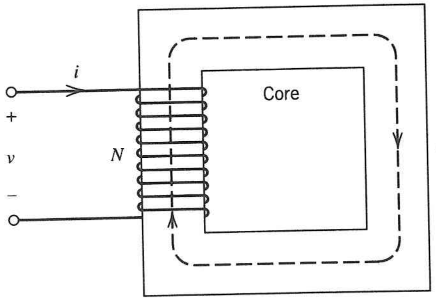

FIGURE 4.6 A simple inductor.

Next, suppose a second coil is added on the right leg of the magnetic circuit as shown in Fig. 4.7. If the first coil is excited ($i_1 \neq 0$) and the second coil is left unenergized ($i_2 = 0$), there is flux linking coil 2. The source of this flux is the current i_1 in coil 1. Then, similar to Eq. 4.13, the *mutual inductance* L_{21} is defined as

$$L_{21} = \frac{\text{total flux linking coil 2}}{\text{current flowing through coil 1}} = \frac{\lambda_{21}}{i_1} \tag{4.19}$$

Assuming leakage flux is negligible, the total flux linkage in coil 2 may be expressed as follows:

$$\lambda_{21} = N_2\phi_{21} = N_2\phi \tag{4.20}$$

where

$$\phi = \frac{N_1 i_1}{R} = \frac{N_1 i_1}{l/(\mu A)}$$

Substituting the expression for ϕ into Eq. 4.20 and dividing the result by i_1, the mutual inductance L_{21} is obtained.

$$L_{21} = N_2 N_1\left(\frac{\mu A}{l}\right) \tag{4.21}$$

On the other hand, if the second coil is excited ($i_2 \neq 0$) and the first coil is unenergized ($i_1 = 0$), the flux produced by coil 2 will link coil 1. Therefore, the mutual inductance L_{12} is defined as

$$L_{12} = \frac{\text{total flux linking coil 1}}{\text{current flowing through coil 2}} = \frac{\lambda_{12}}{i_2} \tag{4.22}$$

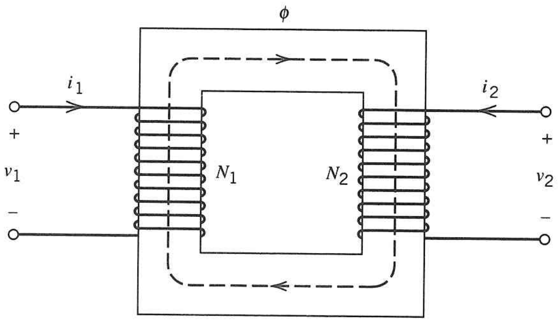

FIGURE 4.7 Mutual inductance.

Again assuming that leakage flux is negligible, the total flux linkage is

$$\lambda_{12} = N_1\phi_{12} = N_1\phi \tag{4.23}$$

where

$$\phi = \frac{N_2 i_2}{R} = \frac{N_2 i_2}{l/(\mu A)}$$

Substituting the expression for ϕ into Eq. 4.23 and dividing the result by i_2, the mutual inductance L_{12} is obtained.

$$L_{12} = N_1 N_2\left(\frac{\mu A}{l}\right) \tag{4.24}$$

It may be observed from Eqs. 4.21 and 4.24 that $L_{12} = L_{21}$.

EXAMPLE 4.4

The magnetic circuit shown in Fig. 4.7 has the following dimensions: $A_c = 12\ \text{cm}^2$ and $l_c = 50$ cm. The magnetic core has a relative permeability $\mu_r = 20{,}000$. The first coil has $N_1 = 500$ turns and the second has $N_2 = 1000$ turns.

a. The first coil is supplied with a current $i_1 = 10$ A, while the second is left unenergized (open circuited). Calculate the self-inductance L_{11} of coil 1 and the mutual inductance L_{21} between the two coils.

b. The first coil is de-energized ($i_1 = 0$), while the second is connected to a source from which it draws a current $i_2 = 8$ A. Calculate the self-inductance L_{22} of coil 2 and the mutual inductance L_{12} between the two coils.

Solution

a. The reluctance of the magnetic circuit is

$$\begin{aligned} R_c &= \frac{l_c}{\mu_r\mu_0 A_c} \\ &= \frac{50 \times 10^{-2}}{(20{,}000)(4\pi \times 10^{-7})(12 \times 10^{-4})} = 16.58 \times 10^3 \end{aligned}$$

The magnetic flux ϕ_1 due to the current in coil 1 is found as follows:

$$\phi_1 = \frac{N_1 i_1}{R_c} = \frac{(500)(10)}{16.58 \times 10^3} = 0.30\ \text{Wb}$$

The flux linkages of the two coils are given by

$$\begin{aligned} \lambda_{11} &= N_1\phi_1 = (500)(0.30) = 150\ \text{Wb-t} \\ \lambda_{21} &= N_2\phi_1 = (1000)(0.30) = 300\ \text{Wb-t} \end{aligned}$$

Therefore, the self-inductance of coil 1 is

$$L_{11} = \lambda_{11}/i_1 = 150/10 = 15\ \text{H}$$

The mutual inductance is given by

$$L_{21} = \lambda_{21}/i_1 = 300/10 = 30\ \text{H}$$

b. The magnetic flux ϕ_2 due to the current in coil 2 is found as follows:

$$\phi_2 = \frac{N_2 i_2}{R_c} = \frac{(1000)(8)}{(16.58 \times 10^3)} = 0.48\ \text{Wb}$$

The flux linkages of the two coils are given by

$$\begin{aligned} \lambda_{12} &= N_1\phi_2 = (500)(0.48) = 240\ \text{Wb-t} \\ \lambda_{22} &= N_2\phi_2 = (1000)(0.48) = 480\ \text{Wb-t} \end{aligned}$$

Therefore, the self-inductance of coil 2 is

$$L_{22} = \lambda_{22}/i_2 = 480/8 = 60\ \text{H}$$

The mutual inductance is given by

$$L_{12} = \lambda_{12}/i_2 = 240/8 = 30\ \text{H}$$

DRILL PROBLEMS

D4.6 Find the inductance of the coil in the magnetic circuit of Problem D4.3.

D4.7 Calculate the inductance of the coil of the magnetic circuit of Problem D4.4.

D4.8 In the magnetic circuit of Example 4.4, an air gap of length 0.5 mm is cut in the lower leg of the core. Find the self-inductances and mutual inductance of the two coils.

4.5 TRANSFORMERS

The transformer is an indispensable component of power systems. It is one of the main reasons for the widespread use of AC power systems. It makes possible electric power generation at the most economical voltage, transmission and distribution at the most economical voltage levels, and power utilization at the most suitable voltage. The transformer is also used in measurements of high voltages (potential transformers) and large currents (current transformers). Other uses of transformers include impedance matching, insulating one circuit from another, or insulating DC circuits from AC circuits.

A *single-phase transformer* basically consists of two or more windings coupled by a magnetic core. When one of the windings (primary) is connected to an AC voltage source, a time-varying flux is produced in the core. This flux is confined within the magnetic core, and it links the second winding. Therefore, a voltage is induced in the second winding (secondary). When an electrical load such as a resistor is connected to the secondary winding, a secondary current flows.

A single-phase transformer is illustrated in Fig. 4.8. The primary winding has N_1 turns and the secondary has N_2 turns. The voltages and currents are expressed in phasor form. In the following analysis, the capacitances of the transformer windings are not included; they do become important at higher frequencies.

4.5.1 Ideal Transformer

An *ideal transformer* is characterized by the following:

1. There is zero leakage flux. This implies that the fluxes produced by the primary and secondary currents are confined within the core. This assumption was also made in magnetic circuit analysis (see Section 4.2).

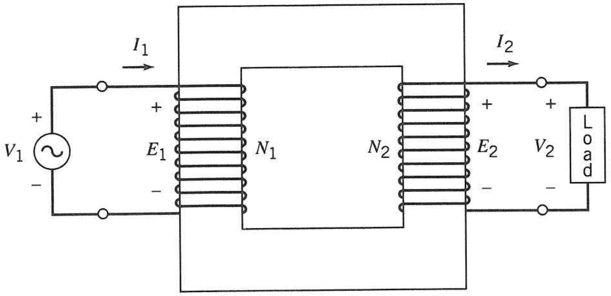

FIGURE 4.8 A transformer circuit.

2. The windings have no resistances. Therefore, the applied voltage v_1 equals the induced primary voltage e_1; that is, $v_1 = e_1$. Similarly, $v_2 = e_2$.
3. The core has infinite permeability. This implies that the reluctance of the core is zero. Hence, negligible current is required to set up the magnetic flux.
4. The magnetic core is lossless. Hysteresis and eddy current losses are, therefore, negligible.

Let the mutual flux linking both windings be sinusoidal, that is,

$$\phi_m = \Phi_p \sin \omega t \tag{4.25}$$

Then, according to Faraday's law of electromagnetic induction, the induced emfs may be expressed as

$$e_1 = \frac{d\lambda_1}{dt} = N_1 \frac{d\phi_m}{dt} = \omega \Phi_p N_1 \cos \omega t \tag{4.26}$$

$$e_2 = \frac{d\lambda_2}{dt} = N_2 \frac{d\phi_m}{dt} = \omega \Phi_p N_2 \cos \omega t \tag{4.27}$$

The rms values of the induced voltages are

$$E_1 = \frac{1}{\sqrt{2}} \omega \Phi_p N_1 = 4.44 f \Phi_p N_1 \tag{4.28}$$

$$E_2 = \frac{1}{\sqrt{2}} \omega \Phi_p N_2 = 4.44 f \Phi_p N_2 \tag{4.29}$$

where $f = \omega/(2\pi)$ cycles per second or hertz.

The polarities of the induced voltages are given by Lenz's law; that is, the emfs produce currents that tend to oppose the flux change. The ratio of the induced voltages is

$$\frac{E_1}{E_2} = \frac{N_1}{N_2} = a \tag{4.30}$$

where a is called the *turns ratio*. Since the transformer is ideal, the induced voltages are equal to their corresponding terminal voltages; that is, $E_1 = V_1$ and $E_2 = V_2$. Hence,

$$\frac{V_1}{V_2} = \frac{E_1}{E_2} = a \tag{4.31}$$

The assumption that the magnetic circuit of the ideal transformer is lossless implies that the mmfs produced by the windings balance or cancel each other; that is, primary mmf equals secondary mmf. In terms of the winding currents, this may be expressed as

$$N_1 I_1 = N_2 I_2 \tag{4.32}$$

Equation 4.32 shows that the winding currents are in phase and that their magnitudes are related by

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a} \tag{4.33}$$

The primary voltage and current may be expressed in terms of their secondary counterparts as follows:

$$V_1 = aV_2 \tag{4.34}$$

$$I_1 = \left(\frac{1}{a}\right) I_2 \tag{4.35}$$

Multiplying Eqs. 4.34 and 4.35 together yields

$$V_1 I_1 = V_2 I_2 \tag{4.36}$$

Equation 4.36 states the *power invariance law* across the ideal transformer. In other words, the power input to the transformer is equal to its power output.

Dividing Eq. 4.34 by Eq. 4.35 gives the secondary impedance referred to the primary side.

$$\frac{V_1}{I_1} = \frac{a^2 V_2}{I_2} \tag{4.37}$$

$$Z_1 = a^2 Z_2 \tag{4.38}$$

The equivalent circuit of an ideal transformer is illustrated in Fig. 4.9, with all quantities referred to the same side.

EXAMPLE 4.5

A 60-Hz ideal transformer is rated 220/110 V. An inductive load $Z_2 = 10 + j10\ \Omega$ is connected across the low-voltage side at rated secondary voltage. Calculate the following:

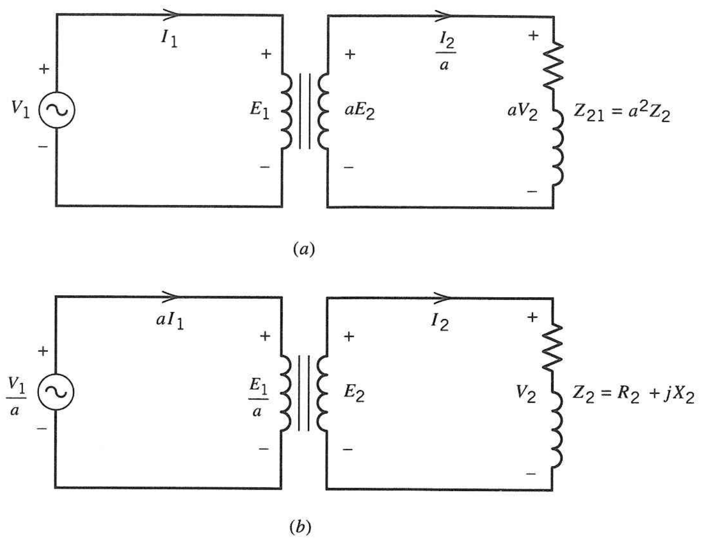

FIGURE 4.9 Equivalent circuit of an ideal transformer: (*a*) all quantities referred to primary side; (*b*) all quantities referred to secondary side.

b. Load impedance referred to the primary

c. Power supplied by the source

Solution

a. The turns ratio is

$$a = V_1/V_2 = 220/110 = 2$$

The primary and secondary currents are found as follows:

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{Z_2} = \frac{110\angle 0^\circ}{(10 + j10)} = 7.78\angle -45^\circ \text{ A}$$

$$\mathbf{I}_1 = \left(\frac{1}{a}\right)\mathbf{I}_2 = (1/2)(7.78\angle -45^\circ) = 3.89\angle -45^\circ \text{ A}$$

b. The load impedance referred to the primary side is

$$Z_1 = a^2 Z_2 = (2)^2(10 + j10) = 40 + j40\ \Omega$$

c. The power supplied by the source is computed as follows:

$$P_1 = P_2 = V_2 I_2 \cos\theta = (110)(7.78)\cos 45^\circ = 605 \text{ W}$$

$$P_1 = P_2 = I_2^2 R_2 = (7.87)^2(10) = 605 \text{ W}$$