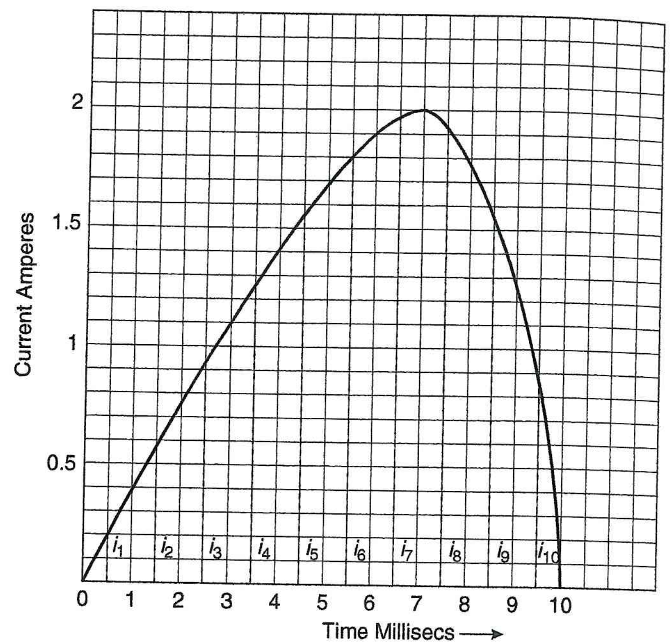


▲ **Figure 16**

9.5. The phasor diagram is shown (figure 17), but not to scale with the resultant is obtained mathematically. The second current is the reference along the horizontal.

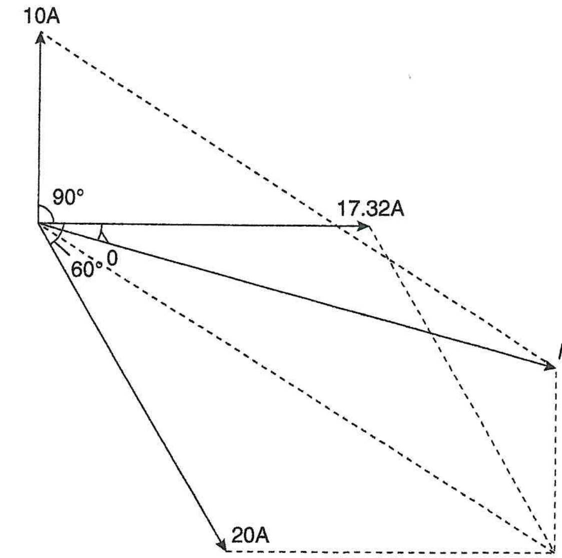
Sum of horizontal components

$$I_H = 17.32 \cos 0 + 20 \cos 60 + 10 \cos 90$$
$$= (17.32 \times 1) + (20 \times 0.5) + (10 \times 0)$$
$$= 17.32 + 10 + 0 = 27.32\text{A}$$

Sum of vertical components

$$I_V = 17.32 \sin 0 - 20 \sin 60 + 10 \sin 90$$
$$= (17 + 0) - (20 \times 0.866) + (10 \times 1)$$
$$= 0 - 17.32 + 10 = -7.32\text{A}$$

$$\text{Resultant } I = \sqrt{27.32^2 + 7.32^2}$$
$$= \sqrt{799.4} = 28.32\text{A}$$



▲ **Figure 17**

$\cos\theta = 0.965$ So $\theta = 15°12'$

As peak or maximum values were used for the individual phasors, the maximum value of the resultant current is 28.32A lagging 105°12′ behind the 10A current.

9.6. For a sine wave voltage applied to a resistor, a sinusoidal current results, and the maximum value of this current is at the instant of maximum voltage.

$$\text{Maximum current} = \frac{\text{maximum voltage}}{\text{resistance}} = \frac{340}{24}$$
$$= 14.14\text{A}$$

$$\text{R.m.s. value of current} = 0.707 \times 14.14 = 9.996\text{A}$$
$$= 10\text{A}$$

9.7. The phasor diagram is drawn as shown in figure 18. For the construction a scale of 10mm = 25V is used. The phase relation between the various phasors are determined thus:

$e_1 = 100 \sin \omega t$

$$e_2 = 50 \cos \omega t = 50 \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= 50 \sin(\omega t + 90°)$$

$$e_3 = 75 \sin\left(\omega t + \frac{\pi}{3}\right) = 75 \sin(wt + 60°)$$

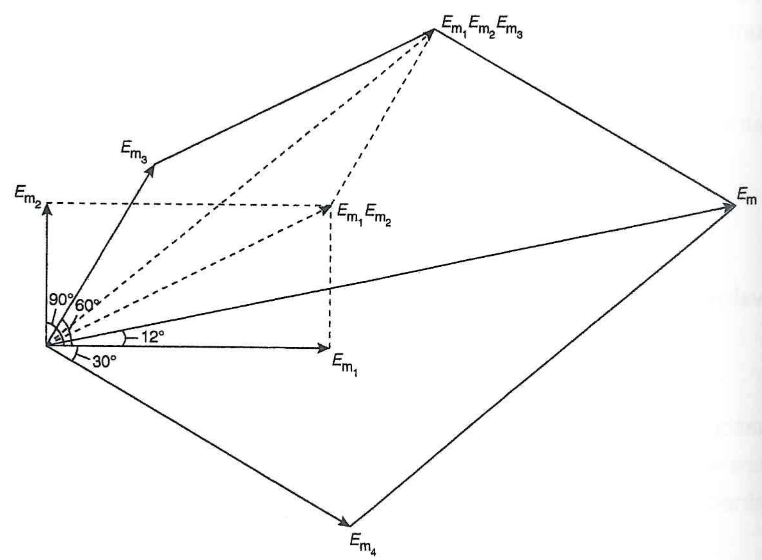
$$e_4 = 125 \cos\left(\omega t - \frac{2\pi}{3}\right)$$

$$= 125 \sin\left(\omega t - \frac{2\pi}{3} + \frac{\pi}{2}\right)$$

$$= 125 \sin\left(\omega t - \frac{\pi}{6}\right)$$

$$= 125 \sin(\omega t - 30°).$$

The diagram is drawn using the maximum value E_{1m} of the first voltage as the reference. For the diagram E_{1m} = 100V.



▲ Figure 18

E_{2m} = 50V E_{3m} = 75V E_{4m} = 125V

For the resultant E = 100.5mm = 252V

The angle θ = 12° (approx.)

But $12° = \dfrac{180}{15}$ or $\dfrac{\pi}{15}$ radians (leading)

The required expression can be written:

$$e_1 + e_2 + e_3 + e_4 \text{ or } e = 252 \sin\left(\omega t + \frac{\pi}{15}\right)$$

9.8. (a) Alternators in step V = 100 + 200 = 300.0V

(b) When the phase displacement is 60°

$$V = \sqrt{100^2 + 200^2 + 2 \times 100 \times 200 \cos 60}$$

$$= \sqrt{10\,000 + 40\,000 + (40\,000 \times 0.5)}$$

$$= 264.8\text{V}$$

(c) When the phase displacement is 90°

$$V = \sqrt{100^2 + 200^2}$$

$$= 223.7\text{V}$$

(d) When the phase displacement is 120°

$$V = \sqrt{50\,000 + 40\,000 \cos 120}$$

$$= \sqrt{50\,000 + 40\,000(-\cos 60)}$$

$$= 173.2\text{V}$$

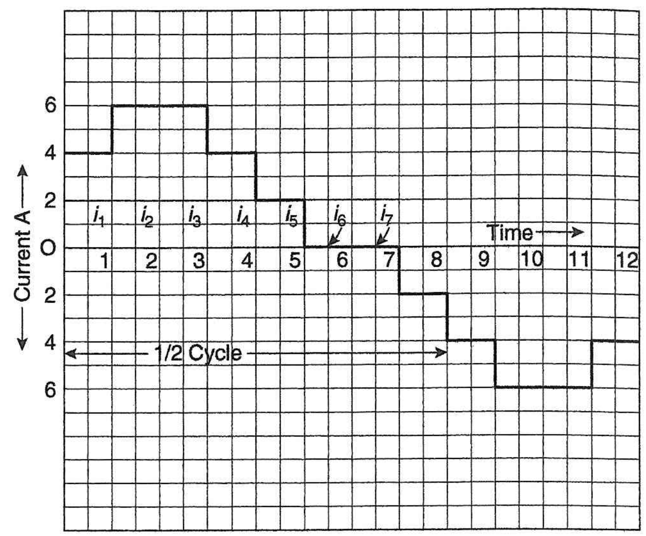
(e) When the phase displacement is 180°

$$\cos \theta = -1$$

$$V = \sqrt{50\,000 - 40\,000} = 100\text{V}$$

The above is obvious, i.e. since the voltages oppose the resultant is the arithmetical diff

9.9. The waveform, when plotted, is a stepped shape but each half wave is regular and similar to its other half, except that it is reversed. The r.m.s. value is obtained by considering a half wave only, as the reversal mentioned, will not affect this value.



▲ **Figure 19**

If the time interval 0–8 is considered as the base of the half wave, 8 mid-ordinates can be used, giving:

$i_1 = 4$ and $i_1^2 = 16$

$i_2 = 6$ and $i_2^2 = 36$ The sum of $i^2 = 112$

$i_3 = 6$ and $i_3^2 = 36$ The mean of $i^2 = \frac{112}{8} = \sqrt{14}$

$i_4 = 4$ and $i_4^2 = 16$ The r.m.s. value $= \sqrt{14}$

$i_5 = 2$ and $i_5^2 = 4 = 3.75\text{A}$

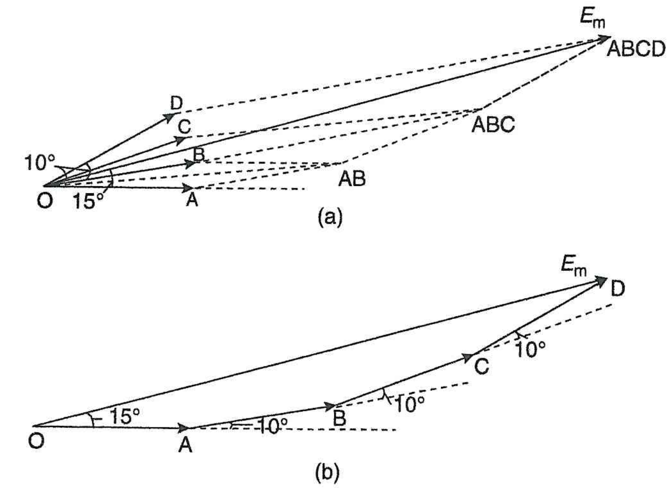
$i_6 = 0$ and $i_6^2 = 0$

$i_7 = 0$ and $i_7^2 = 0$

$i_8 = -2$ and $i_8^2 = 4$

The required value of D.C. will be 3.75A

9.10. The problem calls for a solution by phasor construction and is worked by drawing; the scale chosen being 10mm = 2V and the first e.m.f. 0A is used as the reference.



▲ **Figure 20**

The resultant OD or E_m is measured to be 15.6V and

$$\theta = 15° \text{ or } \frac{\pi}{12} \text{radians}$$

Thus $e = 15.6 \sin\left(\omega t + \frac{\pi}{12}\right)$

The above diagrams show **alternative solutions**, thus (a) uses the parallelogram method, (b) uses the polygon method.

Chapter 10

10.1. Inductive circuit reactance $X_L = 2\pi fL$

$= 2 \times \pi \times 50 \times 0.01 = 3.14\Omega$

(a) Circuit impedance $= \sqrt{3^2 + 3.14^2} = 4.34\Omega$

(b) Power factor $= \dfrac{R}{Z} = \dfrac{3}{4.34} = 0.69$ (lagging)

(c) Power absorbed $= I^2R = \left(\dfrac{V}{Z}\right)^2 R = \dfrac{V^2}{Z} \times \dfrac{R}{Z} = \dfrac{V^2}{Z} \cos\phi$

$= \dfrac{60^2}{4.34} \times 0.69 = 572.3\text{W}$

10.2. Lamp current $= \dfrac{P}{V} = \dfrac{100}{100} = 1\text{A}$

Lamp resistance $= \dfrac{100}{1} = 100\Omega$

(a) Total resistance to give 1A with 220V applied $= \dfrac{220}{1} = 220\Omega$

$\therefore$ Series resistance $= 220 - 100 = 120\Omega$

Power absorbed by circuit $= I^2R = 1^2 \times 220 = 220\text{W}$

(b) When a coil is used for voltage dropping

Impedance of circuit $Z = \dfrac{220}{1} = 220\Omega$

Reactance of circuit $= \sqrt{220^2 - 100^2} = 196\Omega$

Also $X_L = 2\pi fL \;\therefore\; L = \dfrac{196}{2 \times \pi \times 50} = 0.624\text{H}$

Power absorbed by circuit, $P = I^2R = 1^2 \times 100 = 100\text{W}$

10.3. Power absorbed, $P = VI\cos\Phi$

$\therefore \cos\phi = \dfrac{P}{VI} = \dfrac{2500}{240 \times 15} = 0.694$ (lagging)

Current in the circuit $I = 15\text{A}$ (this data is given). Since $P = I^2R$

$\therefore R = \dfrac{P}{I^2} = \dfrac{2500}{15^2} = 11.1\Omega$

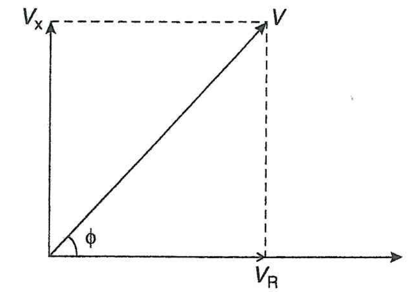
$Z = \dfrac{V}{I} = \dfrac{240}{15} = 16\Omega$

$X = \sqrt{16^2 - 11.1^2} = 11.6\Omega$

In figure 21 $V_R = IR = 15 \times 11.1 = 166.5\text{V}$

$V_X = IX = 15 \times 11.6 = 174\text{V}$

$I = 15\text{A} \qquad V = 240\text{V}$



▲ Figure 21

10.4. $R_A = 120\Omega \qquad R_B = 100\Omega$

$X_A = 2\pi fL \qquad X_B = 2\pi fL$

$= 2 \times \pi \times 50 \times 250 \times 10^{-3} = 2 \times \pi \times 50 \times 400 \times 10^{-3}$

$= 78.5\Omega \qquad = 125.6\Omega$

$Z_A = \sqrt{120^2 + 78.5^2} \qquad Z_B = \sqrt{100^2 + 125.6^2}$

$= 143\Omega \qquad = 160.5\Omega$

Total circuit $R = 120 + 100 = 220\Omega$

$X = 78.5 + 125.6 = 204.1\Omega$

$Z = \sqrt{220^2 + 204.1^2} = 300\Omega$

(a) $I = \dfrac{230}{300} = 0.766\text{A}$

(b) $\cos\phi = \dfrac{220}{300} = 0.733$ (lagging) and $\phi = 42°46'$

(c) Voltage across A $= 0.766 \times 143 = 109.6\text{V}$

Voltage across B $= 0.766 \times 160.5 = 122.9\text{V}$

(d) $\cos\phi_A = \dfrac{120}{143} = 0.838$ (lagging) or $\phi_A = 33°7'$

$\cos\phi_B = \dfrac{100}{160.5} = 0.623$ (lagging) or $\phi_B = 51°27'$

Thus phase difference $\Phi = 51°27' - 33°7' = 18°20'$

10.5. D.C. condition $R_A = \dfrac{20}{2} = 10\Omega \quad R_B = \dfrac{30}{2} = 15\Omega$

A.C. condition $Z_A = \dfrac{140}{2} = 70\Omega \quad Z_B = \dfrac{100}{2} = 50\Omega$

$$X_A = \sqrt{70^2 - 10^2} = 69.3\Omega$$

$$X_B = \sqrt{50^2 - 15^2} = 47.7\Omega$$

Since X is proportional to frequency

Therefore at 50Hz $X_A = 69.3 \times \dfrac{5}{4} = 86.6\Omega$

$$X_B = 47.7 \times \frac{5}{4} = 59.7\Omega$$

For the total series circuit $R = 10 + 15 = 25\Omega$

$X = 86.6 + 59.7 = 146.3\Omega$

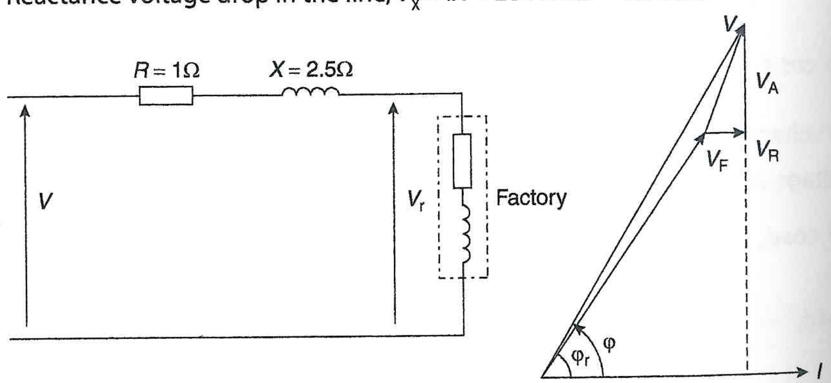
So $Z = \sqrt{25^2 + 146.3^2} = 148.1\Omega$

Current $I = \dfrac{230}{148.1} = 1.55\text{A}$

10.6. Current in the line $I = \dfrac{P}{V_F \cos\phi_F} = \dfrac{750 \times 1000}{3300 \times 0.8} = 284\text{A}$

Resistance voltage drop in the line, $V_R = IR = 284 \times 1 = 0.284\text{kV}$

Reactance voltage drop in the line, $V_X = IX = 284 \times 2.5 = 0.710\text{kV}$



▲ **Figure 22**

From the phasor diagram (figure 22)

$$V = \sqrt{(3.3 \times 0.8 + 0.284)^2 + (3.3 \times 0.6 + 0.71)^2}$$

$$= \sqrt{(2.64 + 0.284)^2 + (1.98 + 0.71)^2}$$

$$= \sqrt{2.924^2 + 2.69^2} = 3.98\text{kV}$$

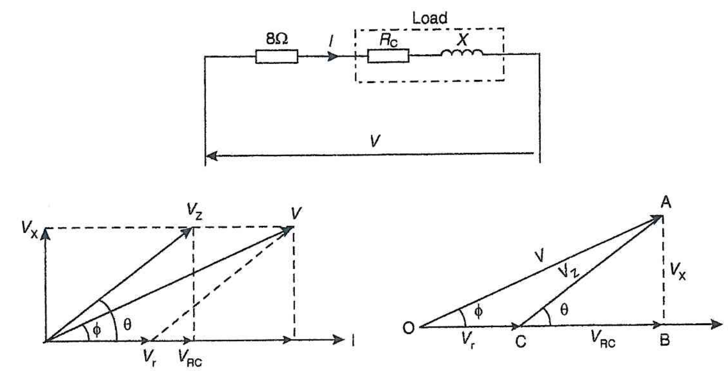
So the voltage at the generator = 3.98kV

Generator power factor $\cos\phi = \dfrac{2.924}{3.98} = 0.73$ (lagging)

Generator output $= \dfrac{3980 \times 284 \times 0.73}{1000}$ kilowatts $= 825\text{kW}$

10.7. Current in 8Ω resistor $= \dfrac{64}{8} = 8\text{A}$ = current in circuit

(b) Power absorbed in resistor $= I^2R = 8^2 \times 8 = 64 \times 8 = 512\text{W}$



▲ **Figure 23**

From the deduced diagrams (figure 23)

$OA^2 = OC^2 + CA^2 - 2 \times OC \times CA \times \cos(180 - \theta)$ (*Cosine formulae*)

or $100^2 = 64^2 + 48^2 + 2 \times 64 \times 48 \times \cos\theta$

$$\cos\theta = \frac{10\,000 - 4096 - 2304}{128 \times 48} = 0.586$$

(d) Power factor of load = 0.586 (lagging)

Voltage drop in resistance of inductive load $= V_{RC}$

$$= V_Z \cos\theta = 48 \times 0.586 = 28.128\text{V} = IR_C$$

$$\therefore R_C = \frac{28.128}{8} = 3.52\Omega$$

(a) Power absorbed by load = $I^2R_C = 8^2 \times 3.52 = 225.3$W

(c) Total Power = 512 + 225.3 = 737.3W

(d) Power factor of circuit or $\cos\phi = \frac{P}{VI} = \frac{737.3}{100\times 8} = 0.92$ (lagging)

10.8. Let X_1 ohms = the reactance at 40Hz

then $Z_1 = \sqrt{R_1^2 + X_1^2}$

and $Z_1 = \frac{200}{6.66} = 30.3\Omega$

Let X_2 ohms = the reactance at 50Hz

then $Z_2 = \sqrt{R^2 + X_2^2}$

and $Z_2 = \frac{200}{8} = 25\Omega$

Hence $30.3^2 = R_1^2 + X_1^2$

and $25^2 = R_2^2 + X_2^2$

but, since $R_1 = R_2$ then:

Subtracting $30.3^2 - 25^2 = X_1^2 - X_2^2$

or $(30.3 - 25)(30.3 + 25) = (X_1 - X_2)(X_1 + X_2)$

Hence $5.3 \times 55.3 = (X_1 - X_2)(X_1 + X_2)$

or $293.09 = (X_1 - X_2)(X_1 + X_2)$

Also since $X = \frac{1}{2\pi fC} \therefore X = \frac{k}{f}$ or $X \propto \frac{1}{f}$

Thus $X_1 = \frac{k}{f_1}$ and $X_2 = \frac{k}{F_2}$ or $\frac{X_1}{X_2} = \frac{(k/f_1)}{(k/f_2)} = \frac{f_2}{f_1}$

Hence $\frac{X_1}{X_2} = \frac{50}{40}$ and $X_1 = \frac{5}{4}X_2$ or $X_1 = 1.25X_2$

Substituting

$$293.09 = (1.25X_2 - X_2)(1.25X_2 + X_2)$$
$$= 0.25X_2 \times 2.25X_2 = 0.5625X_2^2$$

Whence $X_2^2 = \frac{293.09}{0.5625}$ and $X_2 = \sqrt{520} = 22.8$W

$$X_1 = \frac{5}{4} \times 22.8 = 28.5\Omega$$

Thus $R_2 = 25^2 - 22.8^2 = 105$

$$R = \sqrt{105} = 10.25\Omega$$

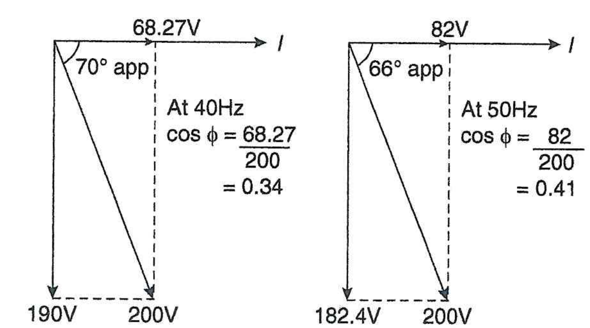
Also $X_C = \frac{10^6}{2\pi fC}$ or $22.8 = \frac{10^6}{2 \times \pi \times 50 \times C}$

Thus $C = \frac{10^6}{22.8 \times \pi \times 10^2} = 139$mF

For the diagram figure 24, as an example:

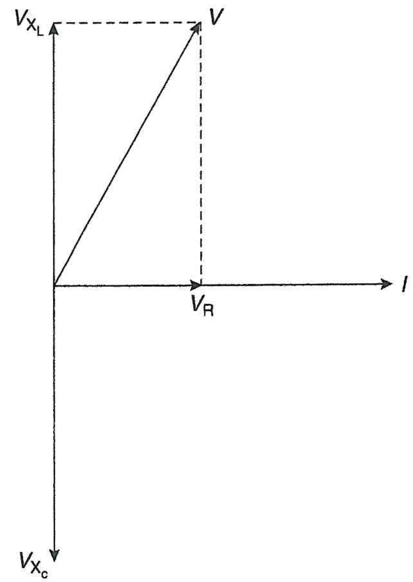
Resistance voltage drop = $IR = 6.66 \times 10.25 = 68.27$V

Reactance voltage drop = $IX_1 = 6.66 \times 28.5 = 190$V etc.



▲ Figure 24

10.9.



▲ **Figure 25**

At resonance $2\pi fL = \dfrac{1}{2\pi fC}$

and $f^2 = \dfrac{1}{(2\pi)^2 LC}$ or $f = \dfrac{1}{2\pi\sqrt{LC}}$

Here I can be obtained from $I = \dfrac{V}{X_C}$

or $I = \dfrac{V}{\dfrac{1}{2\pi fC}} = V2\pi fC$

Thus $I = 100 \times 2 \times \pi \times f \times 10 \times 10^{-6} = 2\pi \times f \times 10^{-3}$

$$= 2\pi \times 10^{-3} \times \frac{1}{2 \times \pi\sqrt{LC}}$$

$$= \frac{10^{-3}}{\sqrt{0.5 \times 10 \times 10^{-6}}} = 0.446\text{A}$$

For figure 25, if actual values were required then as an example:

$$V_R = IR = 0.446 \times 60 = 26.76\text{V}$$

$$X_L = X_C = 100\text{V}.$$

10.10. At 60Hz $R = 400\Omega$ $Z = 438\Omega$

$$X_L = \sqrt{438^2 - 400^2} = 178\Omega$$

At 50Hz $X_L = 178 \times \dfrac{50}{60} = 148.3\Omega$

Circuit impedance

$$Z = \sqrt{400^2 + (X_L - X_C)^2}$$

$$= \sqrt{400^2 + \left(148.3 - \frac{10^6}{2 \times \pi \times 50 \times 40}\right)^2}$$

$$= \sqrt{400^2 + 68.7^2} = 405\Omega$$

$$I = \frac{200}{405} = 0.495\text{A}$$

$$V_C = 0.495 \times \frac{10^6}{2 \times \pi \times 50 \times 40} = 39.5\text{V}$$

Impedance of coil $Z_L = \sqrt{400^2 + 148.3^2} = 426.5\Omega$

$V_{ZL} = IZ_L$

$= 0.495 \times 426.5\Omega = 211\text{V}$

Chapter 11

11.1. For the D.C. circuit, $P = VI$ or $300 = 60 \times I$

so D.C. current $= \dfrac{300}{60} = 5\text{A}$

and D.C. resistance $= \dfrac{V}{I} = \dfrac{60}{5} = 12\Omega$

For the A.C. circuit, $P = I^2R$

$\therefore 1200 = I^2 \times 12$

or $I^2 = 100$ and $I = 10$A

Since the current taken by the A.C. circuit = 10A

then impedance, $Z = \dfrac{V}{I} = \dfrac{130}{10} = 13\Omega$

Also $X = \sqrt{Z^2 - R^2}$ or $X = \sqrt{13^2 - 12^2}$

Thus $X = 5\Omega$ Coil reactance = 5 ohms

11.2. Impedance of branch A. $Z_A = \sqrt{12^2 + 3^2} = 12.4\Omega$

Current $I_A = \dfrac{V}{Z_A} = \dfrac{100}{12.4} = 8.08\text{A}$. Also $\cos\phi_A = \dfrac{12}{12.4} = 0.968$ (lagging)

and $\sin\phi_A = \dfrac{3}{12.4} = 0.242$

Impedance of branch B. $Z_B = \sqrt{8^2 + 20^2} = 21.6\Omega$

Current $I_B = \dfrac{100}{21.6} = 4.64\text{A}$ Also $\cos\phi_B = \dfrac{8}{21.6} = 0.372$ (lagging)

and $\sin\phi_B = \dfrac{20}{21.6} = 0.928$

Active components of current:

$$I_a = I_A \cos\phi_A + I_B \cos\phi_B$$
$$= (8.08 \times 0.968) + (4.64 \times 0.372) = 9.53\text{A}$$

Reactive component of current:

$$I_r = -I_A \sin\phi_A - I_B \sin\phi_B$$
$$= -(8.08 \times 0.242) - (4.64 \times 0.928) = -6.26\text{A}$$

11.3. Let the inductive circuit be circuit A, then:

$$X_A = 2\pi fL = 2 \times \pi \times 50 \times 0.02 = 6.28\Omega \quad R_A = 50\Omega$$

$$Z_A = \sqrt{50^2 + 6.28^2} = 50.5\text{W}$$

$$I_A = \frac{V}{Z_A} = \frac{200}{50.5} = 3.96\text{A}$$

$$\cos\phi_A = \frac{R_A}{Z_A} = \frac{50}{50.5} = 0.99 \text{ (lagging)}$$

$$\sin\phi_A = \frac{X_A}{Z_A} = \frac{6.28}{50.5} = 0124$$

Let the capacitive circuit be circuit B, then:

$$X_B = \frac{10^6}{2 \times \pi \times 50 \times 25} = 127\Omega$$

$$I_B = \frac{200}{127} = 1.575 \quad \cos\phi_B = 0 \quad \sin\phi_B = 1$$

Then $I_a = (3.96 \times 0.99) + (1.575 \times 0) = 3.92$A

$$I_r = -(3.96 \times 0.124) + (1.575 \times 1) = 1.084\text{A}$$

or $I = \sqrt{3.92^2 + 1.084^2} = 4.1$A

Total current $= 4.075\text{A}$ $\cos\phi = \dfrac{3.92}{4.075} = 0.962$ (leading)

Phase angle $\Phi = 15°50'$

11.4. Let the branches be A, B and C respectively. Then:

$$X_A = 2\pi fL = 2 \times \pi \times 50 \times 0.02 = 6.28\Omega$$

$$Z_A = \sqrt{8^2 + 6.28^2}$$

$$= 10.2\Omega$$

$$\text{and } \cos\phi_A = \frac{8}{10.2} = 0.785 \text{ (lagging) } \sin = \phi_A = \frac{6.28}{10.2} = 0.616$$

$$I_A = \frac{100}{10.2} = 9.8\text{A}$$

$$X_B = 2 \times \pi \times 50 \times 0.05 = 15.7\Omega$$

$$Z_B = \sqrt{10^2 + 15.7^2} = 18.6\Omega$$

$$\cos\phi_B = \frac{10}{18.6} = 0.537 \text{ (lagging) } \sin\phi_B = \frac{15.7}{18.6} = 0.845$$

$$I_B = \frac{100}{18.6} = 5.37\text{A}$$

$$X_C = \frac{1}{2\pi fC} = \frac{10^6}{2 \times \pi \times 50 \times 80} = 39.8\Omega$$

Then $Z_C = \sqrt{20^2 + 39.8^2} = 44.54\Omega$

$$\cos\phi_C = \frac{20}{44.54} = 0.449 \text{ (leading) } \sin\phi_C = \frac{39.8}{44.54}$$

$$I_C = \frac{100}{44.54} = 2.24\text{A}$$

Adding the active and reactive current components.

$$I_a = I_A \cos\phi_A + I_B \cos\phi_B + I_C \cos\phi_C$$
$$= (9.8 \times 0.785) + (5.37 \times 0.537) + (2.24 \times 0.449) = 11.59\text{A}$$
$$I_r = -I_A \sin\phi_A - I_B \sin\phi_B + I_C \sin\phi_C$$
$$= (9.8 \times 0.616) - (5.37 \times 0.845) + (2.24 \times 0.894) = -8.54\text{A}$$

Then $I = \sqrt{I_a^2 + I_r^2} = \sqrt{11.59^2 + 8.54^2} = 14.38\text{A}$

$$\cos\phi = \frac{11.59}{14.38} = 0.805 \text{ (lagging) } \phi = 36° \text{ (approx.)}$$

11.5. Apparent power = VI = 240 × 50.6 × 10^{-3} kilovolt amperes

= 12.1kVA

$$\text{Power factor} = \frac{\text{true power}}{\text{apparent power}} = \frac{10}{12.144}$$

$$= 0.823 \text{ (lagging)}$$

$$\text{Efficiency} = \frac{\text{output (power)}}{\text{input (power)}} = \frac{9\text{kW}}{10\text{kW}} = 0.9$$

or η = 90%

11.6. Output from motor = 1.5kW Efficiency = 80%

$$\text{Input to motor} = \frac{1500}{80} \times 100 = 1875\text{W}$$

Also power input to motor, $VI\cos\phi$ = 1875W

∴ 1875 = 230 × 11.6 cos ϕ

$$\therefore \cos\phi = \frac{1875}{230 \times 11.6} = 0.7 \text{(lagging) } \sin\phi = 0.714$$

Power component of input current, $I\cos\phi$ = 11.6 × 0.7 = 8.12A

Reactive component of input current, $I\sin\phi$ = 11.6 × 0.714 = 8.28A

At the new power factor, the power component of current

= 8.12A = $I_1 \cos\phi_1$. Also since $I_1 \cos\phi_1 = I\cos\phi$

$$\therefore I_1 \times 0.95 = 8.12 \text{ and } I_1 = \frac{8.12}{0.95} = 8.55\text{A}$$

Note. $\cos\phi_1 = 0.95$ $\sin\phi_1 = 0.327$

The reactive component of input current, at the new power factor = $I_1 \sin\phi_1$ = 8.55 × 0.327 = 2.8A

So reduction of reactive current = 8.28 – 2.8 = 5.48A

and capacitor current = 5.48A

$$\text{Capacitor reactance} = \frac{230}{5.48}\ \text{ohms} = \frac{10^6}{2\pi fC}$$

$$\text{or } C = \frac{10^6 \times 5.48}{2 \times \pi \times 50 \times 230} = 76\mu\text{F}$$

Rating of capacitor = $230 \times 5.48 \times 10^{-3} = 1.26$VAr

11.7. Load (a) Apparent power, $S_a = \dfrac{\text{active power}}{\text{power factor}} = \dfrac{10}{1} = 10\text{kVA}$

Reactive power, $Q_a = S_a \times \sin \Phi = 10 \times 0 = 0$kVAr

Load (b) Apparent power, $S_b = 80$kVA at a power factor of 0.8 (lagging)

Active power, $P_b = 80 \times 0.8 = 64$kW

Reactive power, $Q_b = 80 \times 0.6 = 48$kVAr (lagging)

Load (c) Apparent power, $S_c = 40$kVA at a power factor of 0.7 (leading)

Active power, $P_c = 28$kW

Reactive power, $Q_c = 40 \times 0.7143 = 28.57$kVAr

Total power taken from the supply, $P = 10 + 64 + 28 = 102$kW

Total reactive power, $Q = 0 - 48 + 28.57 = -19.43$kVAr

Total apparent power from supply, $S = \sqrt{102^2 + 19.43^2} = 104$kVA

Power factor of combined load, $\dfrac{P}{S} = \dfrac{102}{104} = 0.98$ (lagging)

$$\text{Mains current} = \frac{104\,000}{250} = 416\text{A}$$

11.8 (a) Phase voltage = 100V Impedance per phase of load = 10Ω

$\therefore$ Load current per phase $= \dfrac{100}{10}$ 10A

Line current = Phase current = 10A

Total power, $P = \sqrt{3}$ VI cos ϕ

But $V = \sqrt{3}\,V_{ph} = 1.732 \times 100 = 173.2$V and $I = 10$A

$$\therefore P = \frac{\sqrt{3} \times 173.2 \times 10 \times \cos 30}{1000} \quad P = 2.598\text{kW}$$

(b) Line voltage $= \sqrt{3} \times 100$

$\therefore$ Voltage per phase of load $= \sqrt{3} \times 100$volts

$$\text{Current per phase of load} = \frac{V_{ph}}{Z_{ph}} = \frac{\sqrt{3} \times 100}{10} = \sqrt{3} \times 10 \text{ amperes}$$

Line current = phase current $= \sqrt{3} \times \sqrt{3} \times 10 = 30$A

Total Power, $P = \sqrt{3}\ VI \cos \phi$

$$= \frac{\sqrt{3} \times \sqrt{3} \times 100 \times 30 \times 0.866}{1000} = 7.794\text{W}$$

(c) Line voltage = 100V

Voltage per phase of load = 100V

$$\text{Current per phase of load} = \frac{100}{10} = 10\text{A}$$

Line current $= \sqrt{3} I_{ph} = 1.732 \times 10 = 17.32$A

Total Power, $P = \sqrt{3}\ VI \cos \phi$

$$= \frac{\sqrt{3}\ 1000\ \sqrt{3}\ 10 \times 0.866}{1000} = 2.598\text{W}$$

(d) Line voltage = 100V

$$\text{Voltage per phase of load} = \frac{100}{\sqrt{3}}$$

$$\therefore \text{Current per phase of load} = \frac{100}{\sqrt{3} \times 10} \text{ amperes}$$

$$\text{Line current} = \text{Phase current} = \frac{10}{\sqrt{3}} = 5.77\text{A}$$

Total power $P = \sqrt{3}\ VI \cos \phi$

$$= \frac{\sqrt{3} \times 100 \times 10 \times 0.866}{1000 \times \sqrt{3}} \text{kilowatts} = 0.866\text{kW}$$

11.9. Motor output = 45kW = 45 000W

Efficiency of motor = 88%

Motor input $= 45\,000 \times \dfrac{100}{88} = 51\,140$W

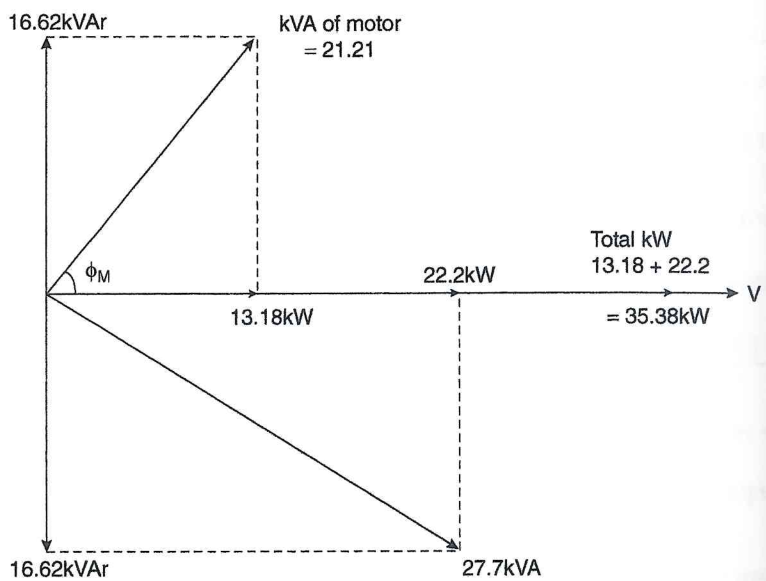
Since $P = \sqrt{3}\,VI\cos\phi$

(a) Line current, $I = \dfrac{51\,140}{\sqrt{3}\times 500 \times 0.9}$ $\quad I = 65.6$A

(b) Alternator output = input to motor = 51.14kW

(c) Input to alternator $= \dfrac{51\,140 \times 100}{80}$ watts or motor power of prime-mover = 64kW

11.10.



▲ **Figure 26**

Input power to the system, $P_s = \sqrt{3}\,VI\cos\phi$

$$= \frac{\sqrt{3}\times 400 \times 40 \times 0.8}{1000} = 22.2\text{kW}$$

Power output from the motor at 91% efficiency = 12kW

∴ Power input to the motor, $P_m = \dfrac{12\times 100}{\ldots} = 13.18$kW

Apparent power of system, $S_s = \dfrac{\sqrt{3}\times 400 \times 40}{1000} = 27.7$kVA

Reactive power of system, $Q_s = S_s \sin\Phi = 27.7 \times 0.6 = 16.62$kVAr

To improve the power factor to unity, the motor's reactive power must equal the reactive power of the system, so the motor's reactive power $Q_m = 16.62$kVAr.

Apparent power to motor, $S_m = \sqrt{P_m^2 + Q_m^2}$

$= \sqrt{13.18^2 + 16.62^2} = 21.21$kVA

Power factor of motor, $\cos\phi_m = \dfrac{P_m}{S_m} = \dfrac{13.18}{21.21} = 0.62$ (leading)

Total power taken from the mains = power supplied to the system + power supplied to the motor $= P_s + P_m = 22.2 + 13.18 = 35.38$kW

The phasor diagram shows the method of solution. Even though the problem is a 3-phase one, the diagram, as drawn, can be applied, as balanced conditions are assumed.

Chapter 12

12.1. E.m.f. generated $= \dfrac{Z\Phi N}{60} = \dfrac{P}{A}$volts

Here Z is 144 × 6 = 864 N = 600 rev/min

$P = 4$ and $A = 4$ since this is a *lap* winding

$\therefore 216 = \dfrac{864 \times \Phi \times 600}{60} \times \dfrac{4}{4}$

or $\Phi = \dfrac{216}{864 \times 10}$ webers

If the armature is *wave* wound $A = 2$

Substituting the value of Φ

then $E = 864 \times \dfrac{216}{864\times 10} \times \dfrac{600}{60} \times \dfrac{4}{2} = 432$V

12.2. Voltage applied to shunt field = 220V

Current through shunt field $= \dfrac{220}{176} = 1.25\text{A}$

Armature current = 250 + 1.25 = 251.25A

Voltage drop in armature and series field = 251.25(0.05 + 0.015)

= 251.25 × 0.04 = 10.05V

Total voltage drop = 10.05 + 2V (brush voltage drop)

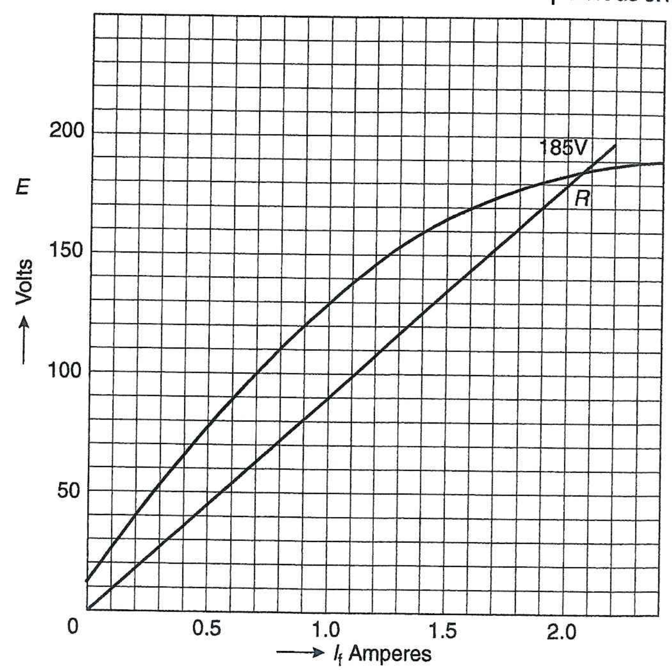
Induced e.m.f. $E = 220 + 10.05 + 2 = 232.05\text{V}$

12.3. It is noted that a change of speed is involved here and the solution cannot be affected before the O.C.C. at 900 rev/min is reached. Since $E \propto N$, the new values can be obtained by multiplying the original by a factor: $\dfrac{900}{1200} = \dfrac{3}{4}$

The table shows the adjustment for the 900 rev/min condition

Excitation current I_t (amperes)	0	0.4	0.8	1.2	1.6	2.0	2.4
E.m.f. at 1200 rev/min E (volts)	15	88	146	196	226	244	254
E.m.f. at 900 rev/min E (volts)	11.25	66	109.5	147	169.5	183	190.5

The field voltage-drop line is drawn by taking any current value and multiplying it by 90Ω. For example: 2A × 90Ω = 180V. Join this point R to the origin. The required answer 185V is obtained from the intersection point as shown.



▲ Figure 27

12.4. $I_f = \dfrac{220}{50} = 4.4\text{A}\ I_L = \dfrac{38\ 000}{220}$

$I_a = 172.72 + 4.4 = 177.12\text{A}$

Also $E = V + I_a R_a$

$= 220 + (177.12 \times 0.1) = 237.72\text{V}$

Again $E = \dfrac{Z\Phi N}{60} \times \dfrac{P}{A}$

$\therefore 237.72 = \dfrac{700 \times \Phi \times 800 \times 4}{60 \times 2}$

or $\Phi = \dfrac{237.71 \times 3}{56 \times 10^3}$

= 0.0128Wb or 12.8mWb

12.5. O.C. e.m.f = 440V = $k\Phi N$

So $k = \dfrac{440}{0.055 \times 620}$

Full-load current $= \dfrac{250\ 000}{480} = 520\text{A}$

Total voltage drop on full load = 520 (0.01 + 0.005 + 0.005)

= 520 × 0.02 = 10.4V

So e.m.f. generated on full load = 480 + 10.4 = 490.4V

$\therefore 490.4 = k\Phi_2 N_2 = \dfrac{440}{0.055 \times 620} \times \Phi_2 \times 600$

or $\Phi_2 = \dfrac{490.4 \times 0.055 \times 620}{440 \times 600} = 0.0633\text{Wb or } 63.3\text{mWb}$

12.6. On no load, the e.m.f. generated is caused by the shunt-field ampere-turns = 7900. These give a no-load voltage of 500V. But since the load voltage rises to 550V, the shunt-field current will rise and the shunt-field ampere-turns, on no load, increase to

$$7900 \times \frac{550}{500} = 8690\text{At}$$

It is found on a full-load test that 11 200At are needed and the extra (above

Series field must supply 11 200 – 8690 = 2510At

Now the full-load current $= \dfrac{500\,000}{550} = 910\text{A}$

$\therefore$ The series turns required $= \dfrac{2510}{910} = 2.76$ i.e. 3 turns

12.7. Here $Z = 90 \times 6 = 540$

$$\therefore E = \frac{540 \times 0.03 \times 1500}{60} \times \frac{4}{4} = 405\text{V}$$

If I_a = 25A. The armature voltage drop = 25 × 1.0 = 25 volts. Since the same field flux and speed are assumed, then the same e.m.f. is being generated or $V = E - I_aR_a$

$\therefore V = 405 - 25 = 380\text{V}.$

So shunt-field current $= \dfrac{380}{200} = 1.9\text{A}$

Machine output current = 25 – 1.9 = 23.1A

Let I_L = the load current

Then $I_L \times 40 = V$ (the terminal voltage)

Also $V = E - I_aR_a$

$= 380 - I_a \times 1.0$ also $I_a = I_f + I_L$

$\therefore V = 380 - 1.0\,(I_f + I_L)$

or $40 \times I_L = 380 - I_f - I_L$

and $41 \times I_L = 380 - I_f$

also $I_f = \dfrac{V}{200} = \dfrac{40 \times I_L}{200} = \dfrac{I_L}{5}$

So the above becomes:

$41 \times I_L = 380 - \dfrac{I_L}{5}$

$205 \times I_L = 1900 - I_L$

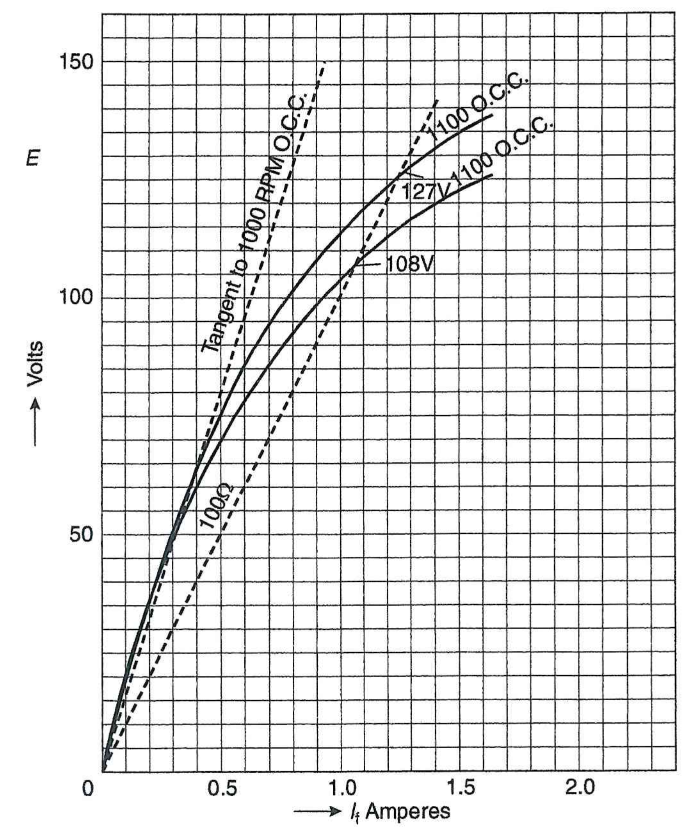
$206 \times I_L = 1900$ and $I_L = \dfrac{1900}{206} = 9.22\text{A}$

12.8. (a) The 1000 rev/min O.C.C. is plotted and cut by the 100Ω field voltage-drop line which is plotted by drawing a straight line through any deduced point and the origin. Thus consider a I_f value of 1A, then a field voltage-drop line point would

The point of intersection at 108V is the answer required.

(b) The tangent is drawn to the 1000 rev/min O.C.C. The critical resistance R_c is determined by taking any voltage value on this tangent and dividing by the current.

Thus $R_c = \dfrac{145\text{V}}{0.9\text{A}} = 161.1\Omega$. The critical resistance for this speed is 161Ω (approx.)



▲ **Figure 28**

(c) The 1100 rev/min O.C.C. is obtained by multiplying the original 1000 rev/min values by a factor: $\dfrac{11}{10} = 1.1$ to give the new table:

Excitation current I_f (amperes)	0.2	0.4	0.6	0.8	1.1	1.2	1.4	1.6
E.m.f. at 1000 rev/min E (volts)	32	58	78	93	104	113	120	125
E.m.f. at 1100 rev/min E (volts)	35.2	63.8	58.8	102.3	114.4	124.3	132	137.4

This 1100 rev/min characteristic when plotted is cut by the 100Ω field voltage-

12.9. Output current $= \dfrac{50 \times 1000}{230} = 217.39\text{A}$

Shunt-field current $= \dfrac{230}{55} = 4.18\text{A}$

Armature current = 217.39 + 4.18 = 221.57A

Armature voltage drop = 221.57 × 0.034 = 7.53A

Induced e.m.f. = 230 + 7.53 + 2 = 239.53V

Electrical power required to be generated

= 239.53 × 221.57 watts = 53.18kW

Total input power = electrical power input + mechanical loss

= 53.18 + 1.6 = 54.78kW

Thus input power = 54.78kW

12.10. Since the answers required are at a different speed condition, the new O.C.C. at 400 rev/min is obtained by multiplying the original values by $\dfrac{400}{200} = 2$. Thus:

Excitation current I_f (amperes)	0	1	2	3	4	5	6	7	8	9
E.m.f. at 200 rev/min *E* (volts)	10	38	61	78	93	106	115	123	130	135
E.m.f. at 400 rev/min *E* (volts)	20	76	122	156	186	212	230	246	260	270

(a) E.m.f. to which machine self-excites = 243V

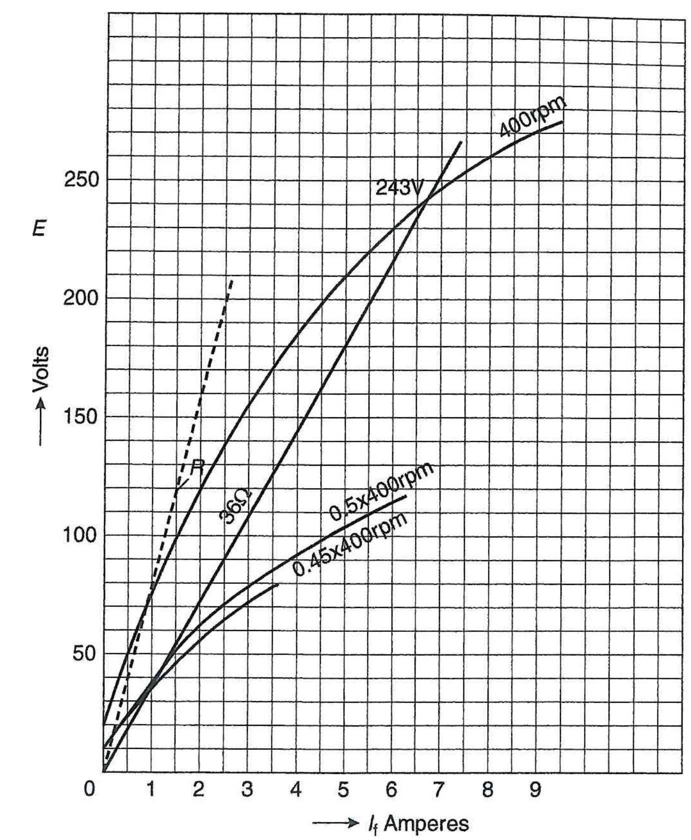
This is the point of intersection between the 400 rev/min O.C.C. and the 36Ω field voltage-drop line. The latter is drawn by taking any current, say 5A and finding the voltage drop 5 × 36 = 180V, and joining this point to the origin.

(b) Draw the tangent to 400 rev/min O.C.C. This may be difficult. Disregard the bottom part of the characteristic (due to residual magnetism) and assume that the characteristic would pass from the second point given (76V at 1A), through the origin. A straight line drawn as shown, through this point and the origin, will be sufficiently tangential to give a suitable answer.

Consider a point such as X. Then critical resistance $R_C = \dfrac{120}{1.5} = 80\Omega$

(c) For the e.m.f. to reduce to 220V, the field voltage-drop line should cut the O.C.C. at this point. The field current would then be 5.3A. So field-current resistance would be $\dfrac{220}{5.3} = 41.5\Omega$. Thus additional resistance required = 41.5 − 36 = 5.5Ω.

(d) This answer is obtained by assuming the machine slows down and is best obtained by trial and error. Multiply the O.C.C. values by various fractions of



▲ **Figure 29**

the speed to obtain a magnetisation curve which makes the 36Ω voltage-drop line a tangent. The 200 rev/min O.C.C. is a suitable starting point and is plotted. It is seen to be cut at the bottom by the 35Ω line, which therefore is not quite tangential. The required O.C.C. must be at a lower speed and 400 × 0.45 may be tried to give values of 9, 34.2, 54.9, 70.2, etc. If plotted this will be approximately correct and the answer 180 rev/min (approx.).

Chapter 13

13.1. The output is 7.5kW = 7500W.

Input is $\dfrac{7500 \times 100}{85} = 8824\text{W}$

Motor current $= \dfrac{8824}{110} = 80.2\text{A}$

$E_b = V - I_a(R_a + R_{se})$

$= 110 - (80.2 \times 0.12) = 100.4\text{V}$

13.2. Power input = 90 000W

Current input to motor $= \dfrac{90\,000}{500} = 180\text{A}$

Shunt-field current $I_f = \dfrac{500}{100} = 5\text{A}$

Armature current = 180 – 5 = 175A

Back e.m.f. $= E_b = V - I_aR_a$

$= 500 - (175 \times 0.1) = 482.5\text{V}$

13.3. No load. Back e.m.f. $E_{b0} = V - I_{a0}R_a$

$= 460 - (10 \times 0.025) = 459.75\text{V}$

Full load. Back e.m.f. $E_{b1} = V - I_{a1}R_a$

$= 460 - (300 \times 0.025) = 452.5\text{V}$

13.4. (a) Current in 1 parallel path $= \dfrac{40}{4} = 10\text{A}$

Current in 1 conductor = current per parallel path

So force on 1 conductor is given by $BI\ell$

$= 1.2 \times 10 \times 0.4 = 4.8\text{N}$

Torque of 1 conductor $\dfrac{4.8 \times 0.3}{2} = 0.72\text{Nm}$

(b) Total torque due to all conductors

$= 240 \times 0.72 = 172.8\text{Nm}$

(c) Power output $= \dfrac{2 \times \pi \times 800 \times 172.8}{60}$ joules/sec

$= \dfrac{6.28 \times 80 \times 172.8}{6}$

$= 14\,469\text{W} = 14.5\text{W}$

13.5. On full load $E_{b1} = 220 - (25 \times 0.2) = 220 - 5 = 215\text{V}$

Also, since torque is unchanged, and $T_1 = T_2$

and since $T \propto \Phi I_a$ or $T = k\Phi I_a$, we can write:

$k\Phi_1 I_{a1} = k\Phi_2 I_{a2}$

But $\Phi_2 = 0.9\Phi_1$

so $\Phi_1 I_{a1} = 0.9\Phi_1 I_{a2}$

or $I_{a2} = \dfrac{I_{a1}}{0.9} = \dfrac{25}{0.9} = 27.77\text{A}$

Thus $E_{b2} = 220 - (27.77 \times 0.2) = 214.446\text{V}$

Also since $\dfrac{E_{b2}}{E_{b1}} = \dfrac{k\Phi_2 N_2}{k\Phi_1 N_1}$

$\therefore N_2 = \dfrac{E_{b2} \times \Phi_1 \times N_1}{E_{b1} \times \Phi_2} = \dfrac{214.446 \times \Phi_1 \times 725}{215 \times 0.9\Phi_1}$

$= 804$ rev/min

13.6. *As a generator.* 50kW at 250V gives a load current of

$$I_L = \frac{50\,000}{250} = 200\text{A}$$

Field current $I_f = \dfrac{250}{50} = 5\text{A}$

$\therefore$ Armature current $I_a = I_L + I_f = 200 + 5 = 205\text{A}$

General voltage $E = V + I_aR_a$ + brush voltage drop

or $E = 250 + (205 \times 0.02) + 2 = 256.1\text{V}$

As a motor. Input current $I_L = \dfrac{50\,000}{250} = 200\text{A}$

Field current I_f (as before) $= \dfrac{250}{50} = 5\text{A}$

$\therefore$ Armature current $I_a = I_L - I_f = 200 - 5 = 195\text{A}$

Back e.m.f. $E_b = V - I_aR_a$ – brush voltage drop

$= 250 - (195 \times 0.02) - 2$

or $E_b = 244.1\text{V}$

Again since E and E_b are proportional to flux and speed then,

$E = k\Phi N$ and $E_b = k\Phi N$

Thus $\dfrac{E}{E_b} = \dfrac{5 \times 400}{5 \times N}$ or $\dfrac{256.1}{244.1} = \dfrac{400}{N}$

or $N = \dfrac{400 \times 244}{256.1} = 382$ rev/min

Note. Flux is assumed to be proportional to the field ampere-turns, and hence the exciting current, with the current value substituted for the flux values Φ.

13.7. *On no load.* $I_f = \dfrac{105}{90} = 1.17\text{A}$ $I_{10} = 3.5\text{A}$

$I_{a0} = 3.5 - 1.17 = 2.33\text{A}$

$E_{b0} = 105 - (2.33 \times 0.25) = 104.42\text{V}$

On full load. Output = 3 × 1000 watts

Input = $\dfrac{3 \times 1000 \times 100}{82} = 3660\text{W}$

Input line current $I_{L1} = \dfrac{3660}{105} = 34.86\text{A}$

and we can write $I_{a1} = 34.86 - 1.17 = 33.7\text{A}$

$E_{b1} = 105 - (33.7 \times 0.25) = 96.57\text{V}$

Now $E \propto \Phi N$ or $E = k\Phi N$

and we can write $\dfrac{E_{b0}}{E_{b1}} = \dfrac{k\Phi_0 N_0}{k\Phi_1 N_1}$

or $N_0 = \dfrac{E_{b0} \times \Phi_1 \times N_1}{E_{b1} \times \Phi_0} = \dfrac{104.42 \times 1.17 \times 1000}{96.57 \times 1.17} = 1080$ rev/min

Again, since T is constant and $T \propto \Phi I_a$, we can write

$T_2 = k\Phi_2 I_{a2}$ and $T_1 = k\Phi_1 I_{a1}$

or $\dfrac{T_2}{T_1} = \dfrac{k\Phi_2 I_{a2}}{k\Phi_1 I_{a1}}$ But $T_2 = T_1$

and $\Phi_2 = \Phi_1$

$\therefore I_{a2} = I_{a1} = 33.7\text{A}$

If R is the added resistance to reduce speed then

$E_{b2} = 105 - 33.7\,(R + 0.25)$

Also since $E_b \propto \Phi$ and N, since flux is constant

then $\dfrac{E_{b2}}{E_{b1}} = \dfrac{800}{1000}$ or $E_{b2} = 96.57 \times \dfrac{800}{1000}$

Back e.m.f. (at reduced speed) = 77.26V

Thus $77.26 = 105 - 33.7\,(R + 0.25)$

$= 105 - 33.7R - 8.43$

or $33.7R = 105 - 85.69$

$R = \dfrac{19.31}{33.7} = 0.57\Omega$

Note. As for Q13.6, flux being proportional to field current, I_f is substituted for Φ.

13.8. Cold condition

$I_{f0} = \dfrac{230}{200} = 1.15\text{A}$

$I_{a0} = 50 - 1.15 = 48.85\text{A}$

$E_{b0} = 230 - (48.85 \times 0.2) = 220.23\text{V}$

Hot condition

Temp rise = 60 − 15 = 45°C

$\therefore R_{a1} = 0.2\left[1 + \left(\dfrac{0.4}{100} \times 45\right)\right] = 0.236\Omega$

Similarly $R_{f1} = 200\left[1 + \left(\dfrac{0.4}{100} \times 45\right)\right]$

$= 200 + (200 \times 45 \times 0.004) = 236\Omega$

So $I_{f1} = 50 - 0.975 = 49.025\text{A}$

$E_{b1} = 230 - (49.025 \times 0.236) = 218.43\text{V}$

Again since $E_b \propto \Phi$, and N and since $\Phi \propto I_f$

we write $E_{b0} = kI_{f0}N_0$

and $E_{b1} = kI_{f1}N_1$

or $\dfrac{N_1}{N_2} = \dfrac{E_{b1}}{E_{b0}} \times \dfrac{I_{f0}}{I_{f1}}$ or $N_1 = \dfrac{E_{b1} \times I_{f0} \times N_0}{E_{b0} \times I_{f1}}$

and $N_1 = \dfrac{218.43 \times 1.15 \times 1000}{220.23 \times 0.975} = 1160$ rev/min

13.9. (a) $V = E_b + I_a R_a$ or $E_b = V - I_a R_a$

$= 230 - (200 \times 0.35)$

and $E_b = 160\text{V}$

Also since $E_b = \dfrac{Z\Phi N}{60} \times \dfrac{P}{A}$ then $N = \dfrac{E_b}{Z\Phi} \times \dfrac{60A}{P}$

or $N = \dfrac{160 \times 60 \times 2}{294 \times 0.025 \times 4} = 653\text{rev/min}$

(b) Again torque is given by:

$$T = 0.159 \times Z\Phi I_a \frac{P}{A}\text{Nm}$$

$$= 0.159 \times 294 \times 0.025 \times 200 \times \frac{4}{2} = 467.5\text{Nm}$$

13.10. $I_{f0} = \dfrac{230}{104.5} = 2.2\text{A}\ I_{a0} = 5 - 2.2 = 2.8\text{A}$

Also since $I_{f1} = 2.2\text{A}$ then $I_{a1} = 50 - 2.2 = 47.8\text{A}$

Again $E_{b1} = 230 - (47.8 \times 0.4) - 2 = 208.88\text{V}$

And $E_{b0} = 230 - (2.8 \times 0.4) - 2 = 226.88\text{V}$

(a) Since $\dfrac{E_{b1}}{E_{b0}} = \dfrac{k\Phi_1 N_1}{k\Phi_0 N_0}$

Then $N_0 = \dfrac{E_{b0} N_1}{E_{b1}}$ assuming constant flux.

or $N_0 = \dfrac{226.88 \times 600}{208.88} = 648\ \text{rev/min}$

(b) At 600 rev/min $E_{b1} = 208.88\text{V}$

Assuming a constant flux, then for 500 rev/min

$$E_{b2} = 208.88 \times \frac{5}{6}$$

$\therefore E_{b2} = 174.07\text{V}$

$\therefore$ The voltage across the armature has to be reduced by:

$230 - 174.07 = 55.93\text{V}$

or since $V = E_b + I_a(R_a + R) + 2$

then $V - E_b = I_a(R_a + R) + 2$

or $55.93 = I_a R_a + I_a R + 2$

So $I_a R = 55.93 - 2 - (47.8 \times 0.4) = 34.81$

$R = \dfrac{34.81}{47.8} = 0.73\Omega$

(c) Under the new condition, Φ must be altered, hence I_f

$\therefore \dfrac{E_{b3}}{E_{b1}} = \dfrac{k\Phi_3 N_3}{k\Phi_1 N_1}$ or $E_{b3} = 230 - (30 \times 0.4) - 2 = 216\text{V}$

$\dfrac{216}{208.88} = \dfrac{I_{f3} \times 750}{2.2 \times 600}$

So $I_{f3} = \dfrac{216 \times 2.2 \times 600}{208.88 \times 750} = 1.82\text{A}$

Thus current – and hence the flux, is reduced to:

$\dfrac{1.82}{2.2} = 0.827 = 82.7\%$

SELECTION OF TYPICAL SECOND CLASS EXAMINATION QUESTIONS

1. A thin rectangular plate 3.5cm × 2.5cm is totally covered on both sides with Nickel 0.12mm thick in 8.25h. The current required if supplied to a voltameter causes 0.0805kg of silver to be deposited in 1h. If the Nickel E.C.E. is 304 × 10^{-9}kg/C, calculate the Nickel density in kg/m^3, if the E.C.E. of silver is 1118 × 10^{-9}kg/C.

2. A 110V D.C. lighting system comprises six 150W and forty 60W lamps. Calculate the inductance of a coil of negligible resistance which, if placed in series would operate on 230V, 50Hz mains.

3. The resistance of the armature, field coils and starter of a 220V shunt motor are 0.2, 165 and 9.8Ω respectively, the field being connected across the first stud of the starter and an armature terminal. Calculate (a) the field current at the instant of starting, (b) the field current when running and (c) the total current taken by the

4. If the instantaneous value of a current is represented by $i = 70.7 \sin 520t$, calculate the current's (a) maximum value, (b) r.m.s. value, (c) frequency and (d) instantaneous value 0.0015s after passing through zero.

5. The maximum value of a 50Hz sinusoidal current wave is 170A. Find graphically the instantaneous currents after 0.001s, 0.003s, 0.006s and 0.008s after zero and increasing positively.

6. The O.C. voltage of a cell, measured by a voltmeter of 100Ω resistance, was 1.5V, and the P.D. when supplying current to a 10Ω resistance was 1.25V, measured by the same voltmeter. Determine the cell's e.m.f. and internal resistance.

7. An alternating current series circuit consists of a coil A that has an inductance of 0.3H and negligible resistance and a resistor B of 100Ω. The supply voltage is 200V with a frequency of 50Hz. Determine (a) the circuit's impedance, (b) the current flowing and (c) the power factor.

8. What is meant by the term 'back e.m.f.' as applied to an electric motor? A 40kW, 220V shunt motor has a full-load efficiency of 90%, an armature resistance of 0.075Ω and a shunt-field resistance of 55Ω. When 'at starting', the starter handle is moved onto the first stud. It is desired to limit the current through the armature to 1.5 times the value which it has when the motor is on full load. What must be the total value of the starting resistance? If, on overload, the speed falls to 90% of its normal full-load value, what would be the armature current? Neglect the effect of armature reaction.

9. State Lenz's law. An iron ring is wound with a coil of 84 turns and carries 0.015mWb of residual magnetism. When the coil is excited, the magnetic flux increases to 0.3mWb in 0.12s. Calculate the average value of the e.m.f. which will be self-induced in the exciting coil while the flux is increasing, and state the *direction* in which it will act relative to the supply voltage, giving reasons.

10. Find the impedance and power factor of an A.C. circuit consisting of 2 electronic components, A and B. A has a resistance of 2Ω and inductive reactance of 14Ω, and B has a resistance of 10Ω and a capacitive reactance of 6Ω.

11. Define the temperature coefficient of resistance of a conductor. State a conductor which has a negative temperature coefficient. When first switched on, the field winding of a 200V shunt motor takes 2A. After running for 2 hours the field current decreases to 1.7A, the supply voltage having remained constant and the shunt regulator setting not having changed. If the ambient air temperature is 15°C,

calculate the average temperature rise in the windings. Temperature coefficient of resistance of copper is 0.004 28 at 0°C.

12. A 440V single-phase motor is rated at 7.5kW and operates at a power factor of 0.8 (lagging) with an efficiency of 88%. Find the current taken from the supply.

13. A coil consumes 300W when the voltage is 60V D.C. On an A.C. circuit the consumption is 1200W when the voltage is 130V. What is the coil's reactance?

14. A motor has 4 poles, its armature is 0.36m in diameter and has 720 conductors whose effective lengths are 0.3m. The field flux density under the poles is 0.7T. Each conductor carries 30A. If the armature turns at 680 rev/min, find the torque in Newton metres and the power developed if two-thirds of the conductors are effective.

15. Define the average value and r.m.s. value of an alternating quantity. Calculate the average r.m.s. value for the stepped half wave given.

Time (ms)	0–10	10–20	20–30	30–40	40–50	50–60	60–70
Steady current (a)	2	4	6	8	6	4	2

16. Two 200V lamps are connected in series across a 400V supply. One lamp is 75W, the other is 40W. What resistance must be connected so that each lamp gives its correct illumination? What will be the power loss in the resistance?

17. Explain the meaning of the following: (a) self-inductance and (b) back e.m.f.

18. Explain the term 'power factor'. An alternator supplies 560kW at a power factor of 0.7 (lagging). What extra power would be available if the power factor is increased to 0.8 (lagging) for the same kVA output?

19. The armature winding of a 6-pole, lap-wound generator is made up from wire 250m long and 7mm² cross-sectional area. If the specific resistance of copper is 1.7 × 10^{-8}Ωm, find the armature's resistance.

20. 100V A.C. is applied to a circuit of 3Ω resistance and 4Ω reactance. Find (a) the circuit current, (b) the circuit's active e.m.f. (resistance voltage drop) and (c) the self-induced e.m.f. (reactive voltage drop).

21. A coil of 125Ω impedance has a resistance of 100Ω when connected across a 50Hz supply. Find its inductance. If the impedance falls to 120.6Ω when the frequency is varied, find the new frequency value.

22. 4.5 litres of fresh water at 17°C is heated to boiling point in 15min. If the heater is 80% efficient and the supply voltage is 220V. Find the current taken from the mains and the heater's resistance. Take the density of water as 1kg/litre and the specific heat capacity as 4.2kJ/kg°C.

23. An alternating voltage of r.m.s. value 100V is applied to a circuit with negligible resistance and an inductive reactance of 25Ω. Determine the r.m.s. variation of current flowing. Show graphically the variation of current and voltage during one cycle of applied voltage. What is the value of the current when the voltage is at its *maximum* value?

24. A generator has 8 brush arms, each with 6 brushes, 30mm long and each with a bearing surface of 30mm by 20mm. The current density is 0.054A/mm² in the brushes. Find the sectional area of the cables, if the leads to the switchboard are each 9.2m and the current density must not exceed 1.0A/mm². Find the power lost in the brushes and cables. The resistivity for carbon and copper is 2550 × 10^{-8} and 1.7 × 10^{-8}Ωm respectively.

25. An iron conductor and an aluminium conductor are connected in *parallel* to a supply. The iron conductor is 10% longer than, and half the diameter of, the aluminium conductor. Given that the ratio of the resistivities of iron to aluminium is 40 to 13, find the ratio of the currents in the 2 conductors.

26. Two currents I_1 = 14.14A and I_2 = 8.5A with a phase difference of 30° are fed into a common conductor. Find the resultant current and the heating effect in joules when it passes through a resistor of value 4Ω for a period of 2min.

27. A 6-pole D.C. generator has 498 conductors, the e.m.f. per conductor being 1.5V and the current in each 100A. Find the e.m.f. and current output of the armature, if it is (a) lap wound and (b) wave wound.

28. A circuit takes a current of 10A from 220V, 50Hz mains at a power factor of 0.866 (lagging). Find the value of the current when the voltage is (a) passing through its maximum value and (b) 0.005s later.

29. An inductance coil has a resistance of 19.5Ω and when connected to a 220V, 50Hz supply, the current passing is 10A. Find the coil's inductance.

30. State Faraday's and Lenz's laws of electromagnetic induction. A 4-pole, 250V motor has its armature removed in order to test the continuity of the field windings which

are connected in series and consist of 2000 turns each. What is the average e.m.f. induced when the current is switched off, if the flux falls from 0.026Wb to 0.001Wb in 0.2s?

31. A coil when connected across 206V A.C. mains passed a current of 10A and dissipates 500W. If it is connected in series with an impedance of 5Ω and a capacitive reactance of 4Ω, find the impedance and power factor of the complete circuit.

32. A piece of copper wire is bent to form a circle and another piece of the same wire is placed across to form a diameter, all the junctions being electrically connected. If the resistance of the straight wire is 2Ω, find the total current flowing when a P.D. of 220V is applied across the junctions.

33. A 15kW motor of efficiency 90% is supplied at 240V by a 2-wire system. The supply cables are 500m long and are of diameter 5mm. Find the current taken by the motor, the voltage at the supply point and the efficiency of the distribution system. Take the specific resistance of copper as $1.7 \times 10^{-8}\Omega$m.

34. If the impedance of a circuit is 20Ω, the resistance is 16Ω and the inductance 0.047 75H, find the frequency of the supply.

35. Differentiate clearly between the kilowatt and the kilowatt-hour. A heater with an efficiency of 85% develops 10MJ in 30min at 200V. Find the energy consumption in kilowatt-hours and the current taken. Find also the length of wire in the element if its resistance is 0.26 ohm per metre.

36. An electric heater of resistance 6.5Ω is connected in series with a coil of inductance value 0.1H. If the mains frequency is 50Hz, find the voltage to be applied to the arrangement in order to maintain 110V across the heater. If the frequency was increased by 5%, keeping the applied voltage constant, find the voltage across the heater.

37. A resistor of ohmic value 3Ω is connected in series with a coil of inductance 0.1H and resistance 1Ω. If 100V at a frequency of 50Hz is applied to the circuit, find the current flowing.

38. A 500V, D.C. shunt motor has a full-load armature current of 20A. A 3% of the input power is dissipated as heat in the armature. What would be the current on starting if 500V is applied across the armature? Find the value of starting resistance needed to *limit* the starting current to twice the full-load current.

39. Find the total effective reactance of a 50Hz circuit made up from a coil of inductance 100mH, in series with a capacitor of 20μF. If the coil has a resistance of 10Ω, find the circuit's impedance.

40. The armature resistance of a 200V, shunt motor is 0.4Ω and the no-load armature current is 2A. When fully loaded and with an armature current of 50A, the speed is 1200 rev/min. Find the no-load speed and state the assumption made in the calculation.

SOLUTIONS TO TYPICAL SECOND CLASS EXAMINATION QUESTIONS

1. When used in the voltameter: $m = zIt$

$$\text{or } I = \frac{0.0805}{1118\times10^{-9}\times3.6\times10^{3}} = 20\text{A}$$

When used for plating:

Area of coating = 350 × 250 × 2 = 175 000mm^2

Volume of coating = 175 × 10^3 × 12 × 10^{-2}

= 21 000mm^3 = 21 × 10^{-6}m^3

Also mass of nickel deposited

= 304 × 10^{-9} × 20 × 8.25 × 3600

= 0.181kg

$$\text{So density} = \frac{0.181}{21\times10^{-6}}\ \text{kg/m}^3$$

= 8600kg/m^3

2. Total wattage = (6 × 150) + (40 × 60)

= 3300W

$$\text{System current} = \frac{3300}{110} = 30\text{A}$$

$$\text{Impedance required on 230V A.C.} = \frac{230}{30} = 7.66\Omega$$

$$\text{Resistance of lamps} = \frac{110}{30} = 3.66\Omega$$

$$\text{Reactance of coil} = \sqrt{7.66^2 - 3.66^2} = 6.73\Omega$$

$$\text{Inductance} = \frac{6.73}{2\times\pi\times50}$$

= 0.0214H

3. (a) Field current at instant of starting $= \frac{220}{165} = 1.33\text{A}$

(b) When running, the starter resistance is inserted into the field circuit by virtue of the position of the contact arm. Field-circuit resistance = 165 + 9.8 = 174.8Ω

$$\text{Field current when running} = \frac{220}{174.8} = 1.26\text{A}$$

(c) Armature-circuit resistance, at instant of starting = 0.2 + 9.8 = 10Ω

$$\text{Armature current} = \frac{220}{10} = 22\text{A}$$

Total current taken by motor = 22 + 1.33 = 23.33A

4. (a) Maximum value of current = 70.7A

(b) The current is sinusoidal

∴ r.m.s. value = 0.707 × maximum value

= 0.707 × 70.7 = 49.94A

(c) $i = 70.7 \sin(520 \times 0.0015)$ is in the form $i = I_m \sin \omega t$

where ω = 520 radians/second. Also ω equals $2\pi f$

or $2\pi f = 520$

Thus $f = \dfrac{520}{2 \times \pi} = 82.8\text{Hz}$

(d) Again $i = 70.7 \sin 2\pi ft$, and if degrees are used for the angle, then

$i = 70.7 \sin(2 \times 180 \times 82.8 \times 0.0015) = 49.65\text{A}$

5. Plot a sine wave on graph paper with a maximum value of 170A and a base of 0.01s. A frequency of 50Hz gives the time of a half wave as 0.01s.

The wave can be plotted from a phasor of length equal to 170A or by use of tables to obtain ordinates. Thus for 30° the instantaneous value or ordinate would be 170 × sin 30° = 170 × 0.5 = 85A. So for 45° i = 170 × 0.707 = 120.2A.

For 60° i = 147.02A. For 90° i = 170A, etc.

Answers from the deduced waveform:

When time t = 0.001s, i = 57.5A.

When time t = 0.003s, i = 132A.

When time t = 0.006s, i = 162A.

When time t = 0.008s, i = 102A.

6. Let E = the e.m.f. of the cell and R_i = the internal resistance.

With the voltmeter across the cell terminals only,

Current taken by voltmeter $= \dfrac{1.5}{100} = 0.015\text{A}$

Then $E = 1.5 + (0.015) \times R_i$ … (i)

With the voltmeter and resistor across the cell terminals,

Current taken by resistor $= \dfrac{1.25}{10} = 0.125\text{A}$

Current taken by voltmeter $= \dfrac{1.25}{100} = 0.0125\text{A}$

Current supplied by cell = 0.125 + 0.0125 = 0.1375A

Thus $E = 1.25 + 0.1375R_i$ … (ii)

Solving (i) and (ii) then $E = 1.25 + 0.1375R_i$

and $E = 1.5 + 0.015R_i$

Subtracting $0 = -0.25 + 0.1225R_i$

or $R_i = \dfrac{0.25}{0.1225} = 2.04\,\Omega$

and $E = 1.5 + (0.015 \times 2.04) = 1.5 + 0.031$

Thus cell e.m.f. = 1.531V

7. $R_A = 0\Omega \quad X_A = 2\pi fL = 2 \times \pi \times 50 \times 0.3$

$= 94.2\Omega$

$R_B = 100\Omega \qquad Z = \sqrt{100^2 + 94.2^2}$

$= 137.4\Omega$

Current $I = \dfrac{200}{137.4} = 1.45\text{A}$

Power factor, $\cos\phi = \dfrac{R}{Z} = \dfrac{100}{137.4} = 0.73$ (lagging)

8. Output = 40 × 1000 watts

$$\text{Input} = 40 \times 10^3 \times \frac{100}{90}\text{watts} = 44\,444\text{W}$$

$$\text{Input current} = \frac{44\,444}{220} = 202.02\text{A}$$

$$\text{Shunt-field current} = \frac{220}{55} = 4\text{A}$$

Armature current = 202.02 – 4 = 198.2A

Armature starting current = 198.2 × 1.5 = 297.3A

$$\text{Resistance of armature circuit} = \frac{220}{297.3} = 0.74\Omega$$

Resistance to be added = 0.74 – 0.075 = 0.665Ω

On normal load E_b = 220 – (198.2 × 0.075) volts

= 205.13V

On 90% speed E_{b1} = 0.9 × 205.13 = 184.62V

∴ Armature voltage drop = 220 – 184.62 = 35.38V

$$\text{Armature current} = \frac{35.38}{0.075} = 471.7\text{A}$$

9. $$E_{av} = \frac{N(\Phi_2 - \Phi_1)}{t}\text{ volts}$$

$$= \frac{84(3\times10^{-4} - 1.5\times10^{-5})}{0.12}$$

= 0.1995V or 199.5mV

The induced voltage will *oppose* the applied supply voltage, thus reducing the rate of current growth.

10. Total circuit resistance = 2 + 10 = 12Ω

Total circuit reactance = 14 – 6 = 8Ω (inductive)

From impedance relationship:

$$Z = \sqrt{12^2 + 8^2} = 14.4\Omega$$

The circuit impedance is 14.4Ω

$$\cos\phi = \frac{R}{Z} = \frac{12}{14.4} = 0.832$$

Thus power factor = 0.832 (lagging)

11. Carbon has a negative temperature coefficient.

$$\text{Resistance of field (cold)} = R_1 = \frac{200}{2} = 100\Omega$$

$$\text{Resistance of field (hot)} = R_2 = \frac{200}{1.7} = 117.64\Omega$$

Then 100 = R_0 (1 + 0.004 28 × 15)

and 117.64 = R_0 (1 + 0.004 28 × T)

$$\text{Dividing } \frac{117.64}{100} = \frac{1+0.0024\,28T}{1+0.0642}$$

or 1.1764 × 1.0642 = 1 + 0.004 28T

and 1.2519 – 1 = 0.004 28T

$$\therefore T = \frac{0.2519}{0.00428} = 58.85°\text{C}$$

Temperature rise of winding = 58.85 – 15°C

= 43.85°C

12. Motor output = 7.5 × 1000 = 7500W

$$\text{Motor input} = 75000 \times \frac{100}{88} = 8523\text{W}$$

$$\text{But } VI \cos\phi = P \quad \therefore I = \frac{P}{V\cos\phi} = \frac{8523}{440 \times 0.8} \text{ amperes}$$

$$\text{or } I = \frac{8523}{352} = 24.21\text{A}$$

Thus motor current = 24.2A

13. When on D.C. $P = \frac{V^2}{R} \quad \therefore R = \frac{V^2}{P} = \frac{60 \times 60}{300} = 12\Omega$

When on A.C. $P = 1200$W also $P = I^2R$

$$\text{So } I^2 = \frac{1200}{12} = 100 \text{ or } I = 10\text{A}$$

$$\text{The impedance Z of the circuit} = \frac{V}{I} = \frac{130}{10} = 13\Omega$$

$$\text{So reactance } X = \sqrt{13^2 - 12^2} = 5\Omega$$

Thus reactance of coil = 5Ω

14. Force on 1 conductor is given by $F = BI\ell$ newtons

or $F = 0.7 \times 30 \times 0.3 = 6.3$N

$$\text{No of conductors in the field at any instant} = \frac{2}{3} \times 720 = 480$$

Total force = 480 × 6.3 = 3024N

Torque = force × radius = 3024 × 0.18 Nm

= 544.32Nm

So torque exerted = 544Nm

$$\text{Power developed} = \frac{2\pi NT}{60}$$

$$= \frac{2 \times \pi \times 680 \times 544}{60}$$

= 38.7kW

15. If plotted, this waveform will be found to be made up of 7 rectangular blocks, the mid-ordinates of which are 2, 4, 6, etc., as given.

$$\therefore \text{Average value} = \frac{2+4+6+8+6+4+2}{7} = \frac{32}{7}$$

= 4.57A

$$\text{Also r.m.s. value} = \sqrt{\frac{2^2+4^2+6^2+8^2+6^2+4^2+2^2}{7}}$$

$$= \sqrt{\frac{4+16+36+64+36+16+4}{7}}$$

$$= \sqrt{\frac{176}{7}} = \sqrt{25.14} = 5\text{A}$$

16. 75W lamp. $I = \frac{75}{200} = 0.375$A

40W lamp. $I = \frac{40}{100} = 0.2$A

With lamps in series 40W lamp will only pass 0.2A

∴ (0.375 – 0.2) amperes must be passed through a shunt resistor connected across the 40W lamp. This resistor is also to be suitable for 200V and its resistance value:

$$= \frac{200}{0.175} = 1143\Omega$$

Power loss in this resistor = 200 × 0.175 = 35W

17. Standard explanations, written in the student's own words, should be drawn from the relevant chapter to provide a satisfactory explanation of the meaning of both (a) self-inductance and (b) back e.m.f.

18. Since $P = VI\cos\phi$, then $VI = \dfrac{P}{\cos\phi}$

or kilovolt amperes (S) $= \dfrac{\text{kilowatts } (P)}{\text{power factor } (\cos\phi)}$

Thus $S = \dfrac{560}{0.7} = 800\text{kVA}$

If this kVA or S value is to be maintained

New $P = 800 \times 0.7 = 640\text{kW}$

Thus extra power available $= 640 - 560 = 80\text{kW}$

19. Here $\rho = 1.7 \times 10^{-8}\,\Omega\text{m}$. $\ell = 250\text{m}$. $A = 7 \times 10^{-6}\,\text{m}^2$.

Then $R = \dfrac{\rho\ell}{A} = \dfrac{1.7\times10^{-8}\times250}{7\times10^{-6}} = 0.607\Omega$

This would be the resistance of the length of wire, but this is a lap-wound generator, with 6 parallel paths in the armature. Thus resistance of 1 parallel path $= \dfrac{0.607}{6} = 0.101\,16\Omega$.

But there are 6 paths in parallel so the equivalent resistance is one-sixth of the above

$= \dfrac{0.101\,16}{6} = 0.016\,86\Omega$

20. Impedance of circuit $= \sqrt{3^2 + 4^2}$

$= 5\Omega$

Thus $Z = 5\Omega$

(a) Current $I = \dfrac{V}{Z} = \dfrac{100}{5} = 20\text{A}$

(b) Resistive voltage drop $V_R = IR = 20 \times 3 = 60\text{V}$

(c) Reactive voltage drop $V_X = IX = 20 \times 4 = 80\text{V}$

21. Reactance X_L at 50Hz $= \sqrt{Z^2 - R^2} = 100\sqrt{1.25^2 - 1^2} = 75\Omega$

Also $X_L = 2\pi fL \quad \therefore L = \dfrac{75}{2\times\pi\times50} = 0.24\text{H}$

At new frequency $X_L = \sqrt{120.6^2 - 100^2} = 67.4\Omega$

So $\dfrac{\text{new frequency}}{50} = \dfrac{67.4}{75}$

or new frequency $= 67.4 \times \dfrac{50}{75}$

$= 44.93\text{Hz}$. The new frequency value will be 45Hz

22. Mass of water = volume × density = 4.5 × 1 = 4.5kg

Heat received by water = 4.5 × 4.2 (100 − 17)

= 18.9 × 83 kilojoules = 1569kJ

Electrical energy supplied to heater $= 1569 \times \dfrac{100}{80}$

$= 1961\text{kJ}$

Power rating of heater $= \dfrac{\text{energy}}{\text{time}} = \dfrac{1961}{15\times60}$

$= 2.18\text{kW}$

Current taken from mains $= \dfrac{2180}{220} = 9.9\text{A}$

Mains current = 10A (approx.)

Resistance of heater $= \dfrac{220}{9.9} = 22.2\Omega$

23. Here $R = 0\Omega$ and $Z = X = 25\Omega$

$\therefore$ r.m.s. value of current $I = \dfrac{V}{Z} = \dfrac{100}{25} = 25\text{A}$

The graphical solution consists of a sinusoidal voltage wave with a sinusoidal current wave lagging it by 90°, since the circuit is wholly inductive. When voltage is maximum current is zero. When voltage has fallen to zero, the current has risen to its maximum value and as voltage rises to its negative maximum the current falls to zero.

When V is a maximum the current value is zero.

24. Bearing surface of 1 brush = 30 × 20 mm² = 600mm²

With a current density of 0.054A/mm², the current carried by one brush = 600 × 0.054 = 32.4A

With 8 brush arms, there are 4 positive and 4 negative brush arms. Also since there are 6 brushes per arm, the number of brushes in parallel carrying current into or out of the armature = 4 × 6 = 24 brushes, or total current carried by brushes = 32.4 × 24 = 777.6A.

Current display density in the cable is limited to 1.6A/mm².

$\therefore$ The cable is able to carry 777.6A $= \dfrac{777.6}{1.6} = 486\text{mm}^2$

Resistance of cable $= \dfrac{\rho\ell}{A} = \dfrac{1.7\times10^{-8}\times9.2\times2}{486\times10^{-6}} = 0.643\times10^{-3}$ ohms

Voltage drop in cable = 777.6 × 0.643 × 10^{-3} = 0.5V

Power loss in cable = 777.6 × 0.5 = 388.8W

Resistance of the brushes is given by $R = \dfrac{\rho\ell}{A}$ where ℓ is the length of a +ve plus a −ve brush and A is the area of half the total number of brushes.

Thus $R = \dfrac{2550}{10^8}\times\dfrac{2\times30\times10^{-3}}{600\times10^{-6}\times24} = 0.106\times10^{-3}$ ohms

Power loss in the brushes is given by I^2R

$= 777.6^2 \times 0.106 \times 10^{-3} = 64.1$ watts

Thus power loss in brushes = 64.1W

25. The equations for the iron and aluminium conductors can be written as $R_i = \dfrac{\rho_i\ell_i}{A_i}$ and $R_a = \dfrac{\rho_a\ell_a}{A_a}$

$\therefore \dfrac{R_i}{R_a} = \dfrac{\rho_i L_i}{A_i} \Big/ \dfrac{\rho_a\ell_a}{A_a} = \dfrac{\rho_i\ell_i A_a}{\rho_a\ell_a A_i}$

But $\ell_i = 1.1\,\ell_a$

and $d_i = \frac{1}{2} d_a \qquad \therefore A_i = \dfrac{A_a}{4}$ since area ∝ diameter².

So $\dfrac{R_i}{R_a} = \dfrac{40\times1.1\times\ell_a\times A_a\times4}{13\times\ell_a\times A_a}$

$= \dfrac{13.54}{1}$

Since the resistance ratio of iron to aluminium is 13.54 to 1, and as the wires are in *parallel*, the currents in the wires are in the ratio iron:

aluminium = 1: 13.54.

26. Resultant current $I = \sqrt{I_1^2 + I_2^2 + (2\times I_1\times I_2\times\cos 30)}$

$= \sqrt{14.14^2 + 8.5^2 + (2\times14.14\times8.5\times0.866)}$

= 21.92A

Resultant current = 21.92A

Power dissipated = $I^2R = 21.9^2 \times 4$ watts

Energy at heat = I^2Rt joules

$$= 21.92^2 \times 4 \times 2 \times 60$$

$$= 230\,602\text{J} = 230.602\text{kJ}$$

27. (a) *Lap wound.* A = *P* = 6

$$\text{Conductors in series} = \frac{\text{total conductors}}{\text{parallel paths}} = \frac{498}{6} = 83$$

E.m.f. of 1 parallel path = e.m.f. of the machine

$$= 1.5 \times 83 = 124.5\text{V}$$

Current per parallel path = current in 1 conductor = 100A

Current of 6 paths in parallel = 6 × 100 = 600A

(b) *Wave wound. A* = 2

$$\text{Conductors in series} = \frac{\text{total conductors}}{\text{parallel paths}} = \frac{498}{2} = 249$$

E.m.f. of 1 parallel path = e.m.f. of machine

$$= 1.5 \times 249 = 373.5\text{V}$$

Current per parallel path = current in 1 conductor = 100A

Current of 2 paths in parallel = 2 × 100 = 200A

28. The power factor of this circuit is 0.866 (lagging) or cos ϕ = 0.866 (lagging) and ϕ = 30°, where ϕ is the angle of lag between the voltage and the current – the latter lagging the former.

The values of 10A and 220V as given can be assumed to be r.m.s. values. So the maximum value of current is given by $I = 0.707\, I_m$ (here I_m is the maximum value). Sine-wave working is assumed.

$$\text{or } I_m = \frac{10}{0.707} = 14.14\text{A}$$

Also $V = 0.707\, V_m$ (maximum value)

$$\text{or } V_m = \frac{220}{0.707} = 311.08\text{V}$$

The voltage and current are written as:

$v = V_m \sin \omega t$

and $i = I_m \sin(\omega t - \phi)$ ϕ is in radians

At a frequency of 50Hz, time for 1 cycle $= \frac{1}{50}$ seconds

(a) When the voltage is at a maximum, the time is for $\frac{1}{4}$ cycle or

$$t = \frac{1}{4 \times 50} = 0.005\text{s}$$

Current at this instant is given by substituting in

$i = 14.14 \sin(2\pi 50 \times 0.005 - \phi)$

or $i = 14.14 \sin(2 \times 180 \times 50 \times 0.005 - 30)$. π and ϕ in degrees

$= 14.14 \sin(90 - 30)$

$= 14.14 \sin 60° = 12.25\text{A}$

(b) At an instant 0.005s later *t* would be 0.01s

$\therefore i = 14.14 \sin(2 \times 180 \times 50 \times 0.01 - 30)$

or $i = 14.14 \sin(180 - 30) = 14.14 \sin 150°$

$i = 14.14 \sin 30° = 7.07\text{A}$

29. From the information given, the impedance *Z* of the circuit is $= \frac{V}{I}$ or

$$Z = \frac{220}{10} = 22\Omega$$

The resistance *R* is 19.5Ω. Therefore the reactance *X* is obtained from $X = \sqrt{Z^2 - R^2} = \sqrt{22^2 - 19.5^2} = 10.15\Omega$

Also $X = 2\pi fL \quad \therefore L = \dfrac{10.15}{2\times\pi\times 50} = 0.032\text{H}$

30. From Faraday's law.

$$E_{av} = \frac{N(\Phi_1 - \Phi_2)}{t} \text{ volts}$$

$$= \frac{200\times 4(2.6\times 10^{-2} - 0.1\times 10^{-2})}{0.2}$$

Induced e.m.f. = 1000V or 1kV

31. Z of coil $= \dfrac{206}{10} = 20.6\Omega$

Also since $P = I^2R$, then R of coil $= \dfrac{500}{10^2} = 5\Omega$

Thus reactance X of coil $= \sqrt{Z^2 - R^2}$

or $X = \sqrt{20.6^2 - 5^2}$

$= 19.98\Omega$

Z of additional circuit $= 5\Omega$

X of additional circuit $= 4\Omega$ (capacitive)

$\therefore R$ of additional circuit $= \sqrt{5^2 - 4^2} = 3\Omega$

Total resistance of circuit $= 5 + 3 = 8\Omega$

Total reactance of circuit $= 19.98 - 4 = 15.98\Omega$

Note. The inductive and capacitive reactances have been subtracted.

Total circuit impedance $= \sqrt{8^2 + 15.98^2} = 17.88\Omega$

$\dfrac{8}{\quad}$ 0.44 (lagging)

Thus power factor = 0.44 (lagging), since circuit is net inductive

32. Since the circuit is built up from wire of the same material and cross-sectional area, then the resistances of various parts of the circuit are proportional to length.

The resistance of the diameter = 2 ohms

$\therefore$ The resistance of the circumference $= \pi d = 2\pi$ ohms.

The resistance of 1/2 circumference $= \pi$ ohms

The circuit is made up of a diameter and two $\frac{1}{2}$ circumferences in parallel. $\therefore$ if R is the circuit resistance $\dfrac{1}{R} = \dfrac{1}{2} + \dfrac{1}{\pi} + \dfrac{1}{\pi} = \dfrac{\pi + 2 + 2}{2\pi} = \dfrac{\frac{22}{7}+4}{2\times\frac{22}{7}}$

or $\dfrac{1}{R} = \dfrac{\frac{22+28}{7}}{\frac{44}{7}} = \dfrac{50}{44} = \dfrac{25}{22}$

or $R = \dfrac{22}{25} = 0.88\Omega$

With 220V applied across R, the current would be $= \dfrac{220}{0.88} = 250\text{A}$

33. Output of motor = 15kW

Input to motor $= \dfrac{15}{0.9}$ kilowatts $= 16\,666\text{W}$

Input current or current in cables $= \dfrac{16\,666}{240} = 69.44\text{A}$

Resistance of cable is given by $\dfrac{\rho\ell}{A}$

or $R = \dfrac{1.7\times 10^{-8}\times 500\times 2}{\frac{\pi}{4}\times(5\times 10^{-3})^2} = 0.866\Omega$

Voltage drop in cable = 69.44 × 0.866 = 60.14V

Input voltage at supply cables = 240 + 60.14 = 300.14V

$$\text{Efficiency of distribution} = \frac{\text{Power output from cables}}{\text{Power input to cables}}$$

$$= \frac{240 \times 69.44}{300.14 \times 69.44}$$

$$= 0.799 = 79.9\%$$

34. Here $Z = 20\Omega$ and $R = 16\Omega$ $\quad \therefore X = \sqrt{20^2 - 16^2} = 12\Omega$

Also $X = 2\pi f L = 2 \times \pi \times f \times 0.047\,75 = 12$

$$\text{Thus } f = \frac{12}{2 \times \pi \times 0.047\,75}$$

$$= 40\text{Hz}$$

The frequency of the supply is 40Hz.

35. Output of heater = 10MJ

also 1kWh = 3600 × 1000 = 36 × 10^5 joules

$$\text{Now energy output of heater} = \frac{10 \times 10^6}{36 \times 10^5}$$

$$= 2.78\text{kWh}$$

$$\text{Energy input} = \frac{\text{output}}{\text{efficiency}} = \frac{2.78}{0.85} = 3.27\text{kWh}$$

$$\text{Power input} = \frac{\text{energy}}{\text{time}} = \frac{3.27}{0.5} = 6.54\text{kW}$$

Also, since $P = I^2R = I \times R \times I = V \times \frac{V}{R} = \frac{V^2}{R}$

$$\therefore R = \frac{V^2}{P}$$

$$\text{Thus } R = \frac{200^2}{6540} = 6.12\Omega$$

$$\text{Length of element at } 0.26\ \Omega/\text{m} = \frac{\text{resistance}}{\text{ohms per metre}} = \frac{6.12}{0.26}$$

$$= 23.54$$

$$\text{Current taken} = \frac{P}{V} = \frac{6540}{200} = 32.7\text{A}$$

36. If 110V is maintained across the heater, the current will be $\frac{110}{6.5} = 16.92\text{A}$

At 50Hz reactance of coil = $2\pi f L$

$$= 2 \times \pi \times 50 \times 0.1 = 31.4\Omega$$

Impedance Z of complete circuit $= \sqrt{6.5^2 + 31.4^2} = 32.07\Omega$

Applied voltage for 16.92A = 16.92 × 32.07 = 542.6V

If frequency rises 5%, reactance rises 5%.

New reactance = 3.14 × 1.05 = 32.97Ω

New impedance $= \sqrt{6.5^2 + 32.97^2} = 33.6\Omega$

$$\text{Circuit current} = \frac{542.6}{33.6} = 16.15\text{A}$$

Voltage across heater = 16.15 × 6.5 = 104.97 = 105V (approx.)

37. Total resistance of circuit = 3 + 1 = 4Ω

Reactance of circuit is given by $X = 2\pi fL$

or $X = 2 \times \pi \times 50 \times 0.1 = 31.4\Omega$

Circuit impedance $Z = \sqrt{4^2 + 31.4^2} = 31.65\Omega$

Current flowing $= \dfrac{100}{31.65} = 3.16\text{A}$

38. Neglecting the field current, as it will be small

Input power = VI

$= 500 \times 20 = 10\,000\text{W}$

3% of the input power $= \dfrac{3}{100} \times 10\,000 = 300\text{W}$

This is dissipated as heat in the armature, i.e. it is a copper or I^2R loss.

$\therefore\ I^2R_a = 300$ or $20^2R_a = 300$

and $R_a = \dfrac{300}{400} = 0.75\Omega$

The starting current with only armature resistance to limit the armature current:

$I_{as} = \dfrac{500}{0.75} = 666.66\text{A}$

Twice full-load current = 20 × 2 = 40A

∴ Total resistance required in the armature circuit to limit starting current to

$40\text{A} = \dfrac{500}{40} = 12.5\Omega$

Series resistance will be $12.5 - 0.75 = 11.75\Omega$

39. Inductive reactance $X_L = 2\pi fL$

$= 2 \times \pi 50 \times 100 \times 10^{-3}$

or $X_L = 31.4\Omega$

Capacitive reactance $X_C = \dfrac{10^6}{2\pi fC} = \dfrac{10^6}{2 \times \pi \times 50 \times 20} = 159.23\Omega$

Total effective reactance $X = 159.23 + 31.4$

or $X = 127.83\Omega$ (capacitive)

Impedance of circuit $Z = \sqrt{R^2 + X^2}$

$= \sqrt{10^2 + 127.83^2} = 128.3\Omega$

40. On no load: $E_{b0} = V - I_aR_a = 200 - (2 \times 0.4)$

$= 200 - 0.8 = 199.2\text{V}$

On full load: $E_{b1} = 200 - (50 \times 0.4) = 200 - 20 = 180\text{V}$

As this is a shunt motor, constant field current and therefore the same constant flux is assumed for the no-load and full-load conditions.

Since $E_{b0} = k\Phi_0N_0$ and $E_{b1} = k\Phi_1N_1$ and $\Phi_0 = \Phi_1$

then $\dfrac{E_{b0}}{E_{b1}} = \dfrac{k\Phi_0N_0}{k\Phi_1N_1}$ or $N_0 = \dfrac{N_1E_{b0}}{E_{b1}}$

This gives $N_0 = 1200 \times \dfrac{199.2}{180} = 1328$ rev/min

SELECTION OF TYPICAL FIRST CLASS EXAMINATION QUESTIONS

1. The rudder motor of a steering gear (Ward-Leonard type) is a compound-wound machine, details of which are given below. From this information find: (a) armature current, (b) the torque developed by the motor and (c) output power of motor. Armature – volts 90, resistance 0.0288Ω, rev/min 370. *IR* drop over armature is taken as 8.5V. Shunt field – separately excited from 110V supply, resistance 65Ω, turns per pole 1000. Series field – separately excited by a line current of 325A, 11 turns per pole. Torque Nm = (armature current (A) × flux (kWb) per pole × 10^{-4} × 6.8) – 15.

M.m.f. per pole (At)	3000	3500	4000	4500	5000	5500	6000
Flux per pole (kWb)	2500	2700	2860	2990	3100	3190	3275

2. A 500V installation consists of a synchronous motor taking 50kW working in parallel with a load of 90kW having a power factor of 0.6 (lagging). If the power factor of the combined load is 0.8 (lagging), find the motor's power factor and reactive kVA.

3. A 175kVA, 6600/440V, single-phase transformer has an iron loss of 2.75kW. The primary and secondary windings have resistances of 0.4Ω and 0.0015Ω respectively. Calculate the efficiency on full load when the power factor is 0.9.

4. A shunt-wound generator has the following O.C. characteristic. If the actual field-resistance value is half that of the critical field resistance, above which the machine fails to excite, find the O.C. voltage. The e.m.f. when the generator is operating at a load of 200A falls to 135V. Find the terminal voltage and the armature resistance.

Field current (A)	0.5	1.0	2.0	3.0	4.0	5.0
O.C. voltage (V)	55	90	133	160	179	193

5. What are the units of length, mass and force in the SI system of units?

   A current of 12A produces a magnetic flux of 0.4mWb in a coil of 60 turns. What e.m.f. is induced in the coil if the current of 12A was reversed in 25ms? What is the inductance of the coil in Henrys?

6. A 2-wire ring main, 2km long, is supplied at a point 'X' with 220V. At a point 'Y' situated 400m from 'X' there is a load of 110A. The resistance of 1km of single, main conductor is 0.032Ω. Calculate the current in each section of the main and the voltage at the load.

7. In a supply, the voltage and current vary sinusoidally at a frequency of 50Hz, the r.m.s. values being 311.2V and 70.7A respectively, at a power factor of 0.866 (lagging). Plot the graphs, for a single cycle, of the voltage and current in their correct relative positions. From them, derive the instantaneous values of voltage and current at 3, 6, 11 and 18 milliseconds from the time that the voltage passed through zero, in the course of it increasing positively.

8. The moving coil of a permanent-magnet voltmeter is made of copper and has a resistance of 5Ω when at 20°C. The instrument is connected in series with a resistance of 995Ω at 20°C. Calculate the percentage error, high or low, of the reading at 50°C, if the series resistance is made of (a) copper and (b) manganin. Take the temperature coefficient of copper as 0.004 28/°C at 0°C and that of manganin as zero.

9. The field windings of a motor consists of 8 coils connected in series, each coil having 1200 turns. The flux linked with each coil is 0.05Wb when the current is 5A. Calculate the circuit's inductance and the value of the average e.m.f. induced, if the current was cut off in 50ms.

10. A 550kVA, 50Hz, single-phase transformer has 1875 and 75 turns in the primary and secondary windings respectively. If the secondary voltage is 220V, calculate (a) primary voltage, (b) primary and secondary currents and (c) maximum flux value.

11. The armature and field resistances of a 220V shunt motor are 0.25 and 110 ohms respectively and, when running on no load, the motor takes 6A. Calculate the losses attributable to iron, friction and windage and, assuming this value to remain constant on all loads, determine the efficiency when the current supplied is 62A.

12. Explain the principles underlying the necessity for the introduction of the term 'power factor' when considering A.C. machinery. An alternator is supplying a load of 560kW at a power factor of 0.7 (lagging). If apparatus is installed that raises the power factor to 0.8 (lagging), calculate the increase in power available for the same kVA loading.

13. A heater unit of negligible inductance has a resistance of 6.5Ω and is intended for use with 100V mains. For what 50Hz voltage would it be suitable when placed in series with an external device, of negligible resistance, having an inductance of 0.01H? If the frequency rises by 5% and the voltage remains constant, what would be the resulting change of voltage at the heater terminals?

14. The magnetic field in the air gap of a 2-pole motor has a flux density of 0.8T. The armature is wound with 246 conductors, each of 400mm effective length, mounted at 150mm effective radius, and at full load each conductor carries a current of 20A. Assuming that the actual torque produced is equivalent to that due to two-thirds of the number of conductors cutting the lines of force at right angles, find (a) the torque in newton metres and (b) the shaft power developed at 500 rev/min.

15. State briefly, the meaning of the expressions 'star-connected' and 'delta-connected' as applied to 3-phase A.C. practice. What is the ratio of the maximum line voltage to the maximum phase voltage in each case.

Determine the line current taken by a 440V, 3-phase, star-connected motor having an output of 45kW at 0.88 (lagging) power factor and an efficiency of 93%.

16. A battery is to consist of a number of cells connected in series. Each cell has an e.m.f. of 1.5V and an internal resistance of 0.5Ω. The external load has a resistance of 100Ω and requires approximately 2W for satisfactory operation. Determine how many cells will be required.

17. A battery comprises 6 cells each of e.m.f. 2.2V and internal resistance 0.1Ω. Determine the value of the load resistance connected when the battery delivers maximum power and evaluate this maximum power.

18. If an alternator supplies the following loads: (a) 200kW lighting load at unity power factor, (b) 400kW induction-motor load at 0.8 (lagging) power factor, (c) 200kW synchronous-motor load, find the power factor of the synchronous-motor load, to give an overall power factor of 0.97 (lagging).

19. A D.C. shunt-wound machine is run as a motor, being supplied with 55kW at 220V when its speed is 500 rev/min. Find the speed at which this machine must be driven to generate an output of 55kW with a terminal P.D. of 220V. The resistance of the armature is 0.02Ω and that of the field, which is the same for each case, is 110Ω.

20. A single-phase, 50Hz transformer, has a core with a square cross-section, each side being 270mm. The transformation ratio is 3500/440V and the maximum flux density in the core is not to exceed 1.4T. Find the number of turns of the windings required if the frequency is 50Hz.

21. A 500V, 3-phase alternator supplies a balanced delta-connected load in parallel with a balanced star-connected load. The delta load is 30kW at a power factor of 0.92 (leading) and the star load is 40kW at a power factor of 0.85 (lagging). Calculate the line current and the power factor of the supply.

22. A 440V shunt motor takes an armature current of 30A at 700 rev/min. The armature resistance is 0.7Ω. If the flux is suddenly reduced 20%, to what value will the armature current rise momentarily? Assuming unchanged resisting torque to motion, what will be the new steady values of speed and armature current? Sketch graphs showing armature current and speed as functions of time during the transition from the initial to the final steady state condition.

23. A load takes 250A at 240V and a power factor of 0.8 (lagging) from a 50Hz supply. The supply cable is then operating at its full rating. Find graphically, the additional power which might be supplied, without the cable exceeding its rating, when a 800μF capacitor is connected across the load.

24. Two shunt generators work in parallel. Each has an armature resistance of 0.015Ω and a shunt-field resistance of 85Ω. Machine A is excited so that its e.m.f. is 600V, while the other machine B is excited so that its e.m.f. is 620V. What is the output of each machine when they jointly supply a load of 2500A and what is the bus bar voltage?

25. A 230V motor, which develops 10kW at 1000 rev/min with an efficiency of 85%, is to be used as a generator. The armature resistance is 0.15Ω and the shunt-field resistance is 220Ω. If it is driven at 1080 rev/min and the field current adjusted to 1.1A, with a shunt regulator, what output in kW may be expected as a generator, if the armature copper loss was kept down to that when running as a motor?

26. Two coils are connected in parallel across a 220V, 60Hz supply. At the supply frequency, their impedances are 16Ω and 25Ω respectively, and their resistances are 3Ω and 7Ω respectively. Find the current in each coil, the total current and the total power. Draw a complete phasor diagram for the system.

27. A shunt-wound generator has a magnetisation curve given in the table below. The total resistance in the field circuit is 20Ω and the armature resistance is 0.02Ω. With the machine on load, estimate the e.m.f. generated and the armature current when the terminal voltage of the machine is 140V.

Field current (I_f) – amperes	1.2	2.8	5.0	7.0	7.7	9.0	11.0
Generated e.m.f. (E) – volts	46	88	126	149	154	162	168

28. A 12-pole, 3-phase, delta-connected alternator runs at 600 rev/min and supplies a balanced star-connected load. Each phase of the load is a coil of resistance 35Ω and inductive reactance 25Ω. The line terminal voltage of the alternator is 440V. Determine (a) frequency of supply, (b) current in each coil, (c) current in each phase of the alternator and (d) total power supplied to the load.

29. A coil of 100Ω resistance and 0.1H inductance is connected in series with a 0.1μF capacitor to a 230V variable frequency A.C. supply. Calculate the resonant frequency and the P.D. across the capacitor at resonance.

30. Three D.C. generators connected in parallel, each supply a load of 640A to a set of 220V bus bars. The e.m.f. of one generator is raised from 230V to 235V. If the load and the resistances are constant, determine the current supplied from each generator and the voltage at the bus bars.

31. A balanced delta-connected load and a balanced star-connected load are connected in parallel to a 220V, 3-phase supply. The delta-connected load takes a total power of 50kW at a power factor of 0.75 (lagging), and the star-connected load, 40kW at a power factor of 0.62 (leading). Calculate the power, volt amperes and power factor of the supply.

32. A section of supply cable AB 1km long has a fault to earth such that, when end B is disconnected, the resistance measurement from end A to earth is 5Ω. When end A is disconnected, the resistance reading from end B to earth is 3Ω. The length of the cable AB has a resistance of 4Ω when intact. Find the distance of the fault from end A.

33. A series-connected D.C. motor has a field and armature resistance of 0.1Ω and runs at 600 rev/min when taking a full-load current of 100A from a 210V supply. Calculate the speed of the motor when the torque is reduced 75%.

34. Each phase of a star-connected load consists of a resistor of 14Ω in parallel with a 400μF capacitor. Calculate the line current, power and power factor when the above load is connected to a 440V, 60Hz, 3-phase supply. What power would be dissipated in the load, if it is reconnected in delta?

35. A ring main, 900m long, is supplied at a point A at a P.D. of 220V. At a point R, 240m from A, a load of 45A is drawn from the main, and at a point C, 580m from A, measured in the same direction, a load of 78A is taken from the main. If the resistance of the main (lead and return) is 0.25Ω per kilometre, calculate the current which will flows in each direction round the main from the supply point A and the P.D. across the main, at the load where it is lowest.

36. A non-inductive coil of 6Ω resistance is connected in parallel with an inductive coil of 3Ω resistance and 9Ω impedance at 50Hz. If a P.D. of 110V is applied to the terminals, find the current in each coil and in the mains. If a capacitor of 600μF is connected in parallel with these coils, calculate the total current.

37. A 3-phase transformer has 560 turns on the primary and 42 turns on the secondary. The primary windings are connected to a line voltage of 6.6kV. Calculate the secondary line voltage when the transformer is connected (a) star-delta or (b) delta-star.

38. Two batteries, of e.m.f.s 220V and 225V and internal resistances of 0.2Ω and 0.3Ω respectively, are connected in parallel to supply a load resistor of 10Ω. Find the current supplied by each battery and the terminal voltage.

39. A kettle, when connected to a 220V D.C. supply, boils 1 litre of water initially at 11°C in 3.5min. Calculate the percentage time difference when the water is boiled, by connecting the kettle to a 220V, 50Hz A.C. supply. The inductance of the element of 0.05H. One litre of water has a mass of 1kg and its specific heat capacity as 4.2kJ/kg°C.

40. In a shunt motor the 4 field coils are connected in *series*. Each coil is wound to give 750 ampere-turns, the length of each turn being 450mm. At the safe working temperature, there are 45W dissipated at each coil. If the supply voltage is 220V, find (a) the field current, (b) the diameter of the wire and (c) the length of wire in each coil. Take the resistivity of copper as $2.0 \times 19^{-8}\Omega$m.

SOLUTIONS TO TYPICAL FIRST CLASS EXAMINATION QUESTIONS

1. This problem requires some understanding of the electrical connections of a Ward-Leonard (1861–1915) motor control system which enables a D.C. generator to drive at constant speed, ideal for lift systems and one which is still in use today.

Armature-resistance voltage drop 8.5V $R_a = 0.0288\Omega$

$$\therefore I_a = \frac{8.5}{0.0288} = 295.15\text{A}$$

$$\text{Shunt-filed current } I_{sh} = \frac{110}{65} = 1.692\text{A}$$

Shunt ampere-turns/pole = 1.692 × 1000 = 1692

Series ampere-turns/pole = 325 × 11 = 3575

Total ampere-turns/pole = 1692 + 3575 = 5267

Plot the graph. From this Φ = 3150 kilo webers per pole.

From the given expression:

$T = 295.15 \times 3150 \times 10^{-4} \times 6.8 - 15 = 617.2\text{Nm}$

$$\text{Output power} = \frac{2\pi NT}{60} = \frac{2\times\pi\times370\times617.2}{60} = 24\text{kW}$$

2. Total active power of combined load, P= 50 + 90 = 140kW

$$\text{Total apparent power of combined load, } S = \frac{140}{\cos\phi} = \frac{140}{0.8} = 175\text{kVA}$$

Total reactive power of combined load, Q = 175 × 0.6 = 105kVAr

Active power of original load, P_1 = 90kW

$$\text{Apparent power of original load, } S_1 = \frac{90}{\cos\phi} = \frac{90}{0.6} = 150\text{kVA}$$

Reactive power of original load, Q_1 = 150 × 0.8 = 120kVAr

Thus the reactive power is reduced by 120 – 105 = 15 kVAr and this kVAr figure is that of the motor, Q_2 operating at a leading power factor. Thus reactive power Q_2 of motor = 15kVAr

$$\text{Also apparent power of motor, } S_2 = \sqrt{50^2 + 15^2} = 52.2\text{kVA}$$

$$\text{Power factor of motor} = \frac{P_2}{S_2} = \frac{50}{52.2} = 0.96 \text{ (leading)}$$

3. Although the transformer has not been dealt with in any detail in this book, this problem can be worked from first principles.

In a transformer there are no rotational losses and the expression can be written:

$$\eta = \frac{\text{output (kW)}}{\text{output (kW)} + \text{copper loss (kW)} + \text{iron loss (kW)}}$$

$$\text{Thus } \eta = \frac{\text{kVA}\cos\phi}{\text{kVA}\cos\phi + P_c + P_{Fe}}$$

Here P_{Fe} = 2.75kW

$$\text{Now primary current} = \frac{175\,000}{6600} = 26.51\text{A}$$

$$\text{Primary copper loss} = \frac{26.51^2 \times 0.4}{1000} = 0.2812\text{kW}$$

$$\text{Similarly, secondary current} = \frac{175\,000}{440} = 397.7\text{A}$$

$$\text{Secondary copper loss} = \frac{397.7^2 \times 0.0015}{1000} = 0.237\text{kW}$$

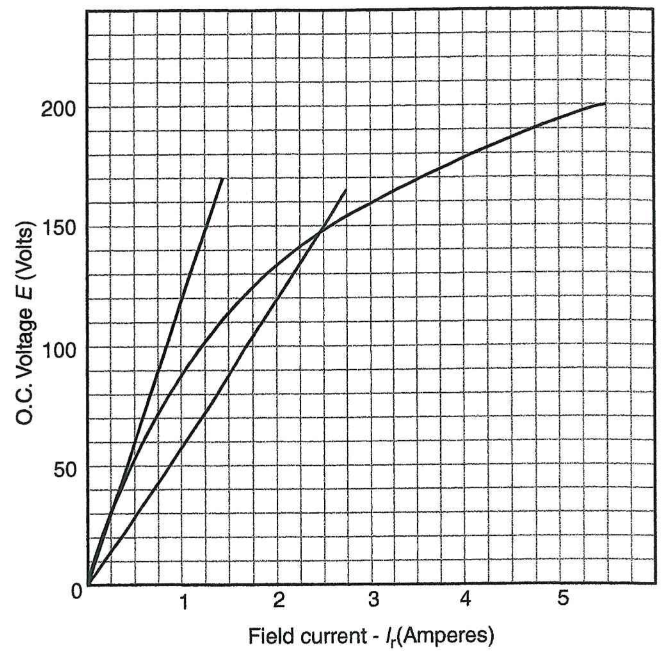
$$\text{Thus } \eta = \frac{175\times0.9}{175\times0.9 + (0.2812 + 0.237) + 2.75}$$

$$= 0.98$$

Efficiency on full load = 98%.

4. The O.C. characteristic is plotted as shown and a tangent drawn. The resistance value obtained from this tangent is the critical resistance. Consider a field current of 1A. The voltage value for this current, reading off the critical resistance line is 120V. The critical resistance is $= \frac{120}{1} = 120\Omega$.

According to the given data, the field resistance is $\frac{120}{2} = 60\Omega$ and the required O.C. voltage is obtained, by taking any field-current value, say 2A, and finding the voltage value for this current. Thus 2 × 60 = 120V. Plot this value and join the point to the origin to obtain the field voltage-drop line. The intersection with the O.C.C.



▲ **Figure 1**

Again, from the O.C.C., an e.m.f. of 135V requires a field current of 2.1A. It must be remembered that the field resistance has *not been altered* and is 60Ω. The voltage necessary to maintain 2.1A through this field resistance is the terminal voltage.

Terminal voltage = 2.1 × 60 = 126V

Armature voltage drop = 135 – 126 = 9V

Also $9 = I_a R_a$

$$\therefore R_a = \frac{9}{200 + 2.1} = 0.045\Omega$$

Note. I_a = line current + shunt-field current.

5. Using the expression $E_{av} = \dfrac{N(\Phi_1 - \Phi_1)}{t}$

We have $E_{av} = \dfrac{60\,[(0.4\times10^{-3}) - (-0.4\times10^{-3})]}{\ldots}$ volts

or $E_{av} = 1.92V$.

Also $E_{av} = L \times$ average rate of change of current

$$\text{or } 1.92 = L \times \frac{I}{t} \text{ Thus } 1.92 = L \times \frac{12}{12.5\times10^{-3}}$$

Note. The time taken for the current to fall to zero has been taken.

$$\text{so, } L = \frac{1.92\times12.5\times10^{-3}}{12} = 0.002\text{H}$$

The inductance of the coil is 2mH

L can also be found from first principles, as the inductance value is determined from the flux-linkages per ampere

$$\text{Thus } L = \frac{N\Phi}{I} = \frac{60\times0.4\times10^{-3}}{12} = 0.002\text{H}$$

6. Let *I* be the current in the short section of the ring, i.e. in the 400m length. Therefore (110 – *I*) is the current in the (2000 – 400) = 1600m length.

Resistance of 400m of cable (double conductor)

$$= \frac{0.032\times800}{1000} = 0.0256\Omega$$

Resistance of 1600m of cable

= 0.0256 × 4 = 0.1024Ω

Since points X and Y are connected by both sections of the ring, it follows that the voltage drop in the shorter section = the voltage drop in the longer section

or *I* × 0.0256 = (110 – *I*) × 0.1024.

and 0.0256*I* = 0.1024 × 110 – 0.1024*I*

or 0.128*I* = 0.1024 × 110

∴ *I* = 88A

Current in shorter section = 88A

Current in longer section = 110 – 88 = 22A

Voltage at load = 220 – (88 × 0.0256) = 217.75V

7. Cos ϕ = 0.866. ∴ ϕ = 30°C. Thus current lags voltage by 30°.

$$\text{Maximum value of voltage} = \frac{311.2}{0.707} = 440\text{V}$$

$$\text{Maximum value of current} = \frac{70.7}{0.77} = 100\text{A}$$

$$\text{Time for 1 cycle} = \frac{1}{50}\ \text{second} = 0.02\text{s}$$

$$\text{Time for } \frac{1}{2}\ \text{cycle} = 0.01\text{s}$$

Since voltage is sinusoidal, then $v = 440 \sin(2 \times 180 \times 50 \times t)$

When $t = 0.001$s, $v = 440 \sin(2 \times 180 \times 50 \times 0.001)$

$= 440 \sin 18° = 136$V

When $t = 0.002$s, $v = 440 \sin 36° = 259$V

Since voltage is sinusoidal, then t = 0.003s, $v = 440 \sin 54° = 356$V

When $t = 0.004$s, $v = 440 \sin 72° = 418$V

When $t = 0.005$s, $v = 440 \sin 90° = 440$V

When $t = 0.006$s, $v = 418$V, etc.

The voltage wave is plotted to a time base t, as shown.

Similarly for the current wave.

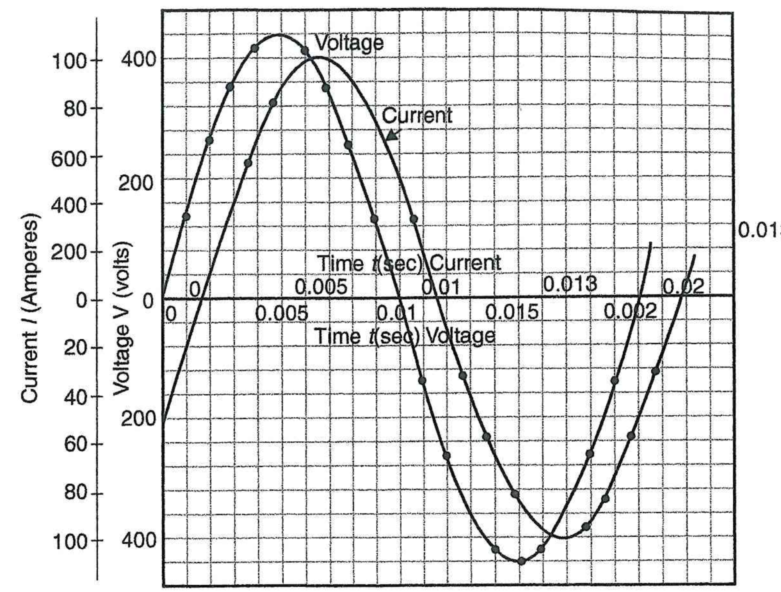
Since $i = 100 \sin(2 \times 180 \times 50 \times t)$

When $t = 0.001$s, $i = 100 \sin 18° = 30.9$A

When $t = 0.002$s, $i = 100 \sin 36° = 58.8$A

When $t = 0.003$s, $i = 100 \sin 54° = 80.9$A

When $t = 0.004$s, $i = 100 \sin 72° = 95.1$A



▲ Figure 2

When $t = 0.005$s, $i = 100 \sin 90° = 100$A

When $t = 0.006$s, $i = 95.1$A, etc.

Draw the current wave to a new scale with its zero displaced from that of the voltage-time scale by 0.001 66s. Use the ordinates derived above for this new time scale.

Note. If a time of 90° = 0.005s, 30° = 0.001 66s.

Read off the required answers from the original voltage-time scale on both the voltage and current waveforms.

Thus when

$t = 3 \times 10^{-3} = 0.003$s, $v = 355$V and $i = 38$A

$t = 0.006$s, $v = 415$V and $i = 96$A

$t = 0.011$s, $v = -140$V and $i = 25$A

$t = 0.018$s, $v = -260$V and $i = 93$A

8. (a) Assume the meter indicates correctly at 20°C when the resistance is 1000Ω. Let I_1 be the meter current at this temperature. Also let R_2 and T_2 be the resistance and temperature at 50°C and let R_1 and T_1 be the resistance and temperature at 20°C.

Then from the resistance to temperature relation,

$$\frac{R_2}{R_1} = \frac{R_0(1+\alpha T_2)}{R_0(1+\alpha T_1)} \quad \therefore \quad R_2 = \frac{1000(1+\alpha 50)}{(1+\alpha 20)} \text{ ohms}$$

$$\text{or } R_2 = \frac{1000(1+0.004\ 28\times 50)}{1+0.004\ 28\times 20}$$

and $R_2 = 1118.2\Omega$

Meter current I_2 at 50°C $= \dfrac{V}{118.2}$ where $I_1 = \dfrac{V}{1000}$

The voltage across the meter can be assumed constant.

$$\therefore \frac{I_2}{I_1} = \frac{1000}{1118.2} \text{ or } I_2 = 0.894 I_1 \text{ amperes}$$

Now meter deflection is proportional to current

$$\therefore \text{Percentage error} = \frac{\text{difference in readings}}{\text{true reading}} \times 100$$

$$\text{or } \frac{I_1 - I_2}{I_1} \times 100 = \frac{I_1 - 0.894 I_1}{I_1} \times 100$$

$$\text{Thus percentage error } = \frac{0.106 I_1}{I_1} \times 100 = 10.6\% \text{ (low)}$$

(b) If the series resistor is made from manganin, its value at 50°C does not vary and the resistance of the 5Ω copper coil is given by R_2

$$\therefore \frac{R_2}{R_1} = \frac{R_0(1+0.004\ 28\times 50)}{R_0(1+0.004\ 28\times 20)}$$

or $R_2 = 5.58\Omega$

The resistance of the meter at t_2 is now 995 + 5.58 ohms = 1000.58Ω

$$\text{New current } I_2 = \frac{V}{1000.58} \text{ or as before } I_1 = \frac{V}{1000} \text{ A}$$

$$\therefore \frac{I_2}{I_1} = \frac{1000}{1000.58} \text{ So new current } I_2 = 0.9994 I_1$$

∴ Percentage error, as deduced before,

$$= \frac{I_1 - 0.9994 I_1}{I_1} \times 100 = \frac{1-0.9994}{1} \times 100$$

or percentage error = 0.0006 × 100 = 0.06% (low)

9. Using the expression:

Inductance (in henries) = flux-linkages per ampere

$$\text{Then } L = \frac{N\Phi}{I} = \frac{8\times 1200\times 50\times 10^{-3}}{5} = 96\text{H}$$

Also $E_{av} = L \times$ rate of change of current

$$= 96 \times \frac{5}{50\times 10^{-3}} = 9600\text{V}$$

Thus the value of induced e.m.f. is 9.6kV.

10. (a) Although no work has been done in this book on the transformer, it can be stated that the voltages induced in the primary and secondary windings are in direct proportion to their turns.

$$\text{Thus } \frac{V_1}{V_2} = \frac{N_1}{N_2} \text{ or } V_1 = V_2 \times \frac{N_1}{N_2}$$

$$\text{Hence } V_1 = 220 \times \frac{1875}{75} = 220 \times 25$$

and primary voltage = 5500V or 5.5kV

(b) The kVA rating applies equally to the primary and secondary sides.

Thus primary current $= \dfrac{\text{kVA rating} \times 1000}{V_1}$

$$= \frac{550 \times 1000}{5500} = 100\text{A}$$

Secondary current $= \dfrac{\text{kVA rating} \times 1000}{V_2}$

$$= \frac{550 \times 1000}{220} = 2500\text{A}$$

(c) This part of the question is answered by a knowledge of the e.m.f. formula for the transformer. This is developed in Volume 7 but can be memorised.

Thus $V_1 = 4.44\Phi_m f N_1$ volts.

or $5500 = 4.44 \times \Phi_m \times 50 \times 1875$

Thus $\Phi_m = \dfrac{5500}{2.22 \times 100 \times 1875}$

$= 13.2 \times 10^{-3}$ webers

Maximum value of flux = 13.2mWb

11. Since $I_f = \dfrac{220}{110} = 2\text{A}$, then $I_a = I_L - I_f = 6 - 2 = 4\text{A}$

Input power = 220 × 6 = 1320W

Copper loss (armature), $I_a^2 R_a = 4^2 \times 0.25 = 16 \times$

Copper loss (field), $I_f^2 R_f = 2^2 \times 110 = 440\text{W}$

Total copper loss = 444W

Rotational loss = 1320 − 444 = 876W. These are the losses attributable to iron, friction and windage.

When the current is 62A, the input is 220 × 62 = 13 640W

The output = input − losses (all values in watts)

= 13 640 − copper losses − rotational losses

$= 13\,640 - (60^2 \times 0.25 + 2^2 \times 110) - 876$

= 13 640 − (900 + 440) − 876

= 13 640 − 2216 = 11 424W

So efficiency $= \dfrac{11\,424}{13\,640} = 0.837$ or 83.7%

12. A basic definition of power factor is that it is the ratio of the 'active power' to the 'apparent power' being expended in a circuit.

Thus power factor, $\cos\phi = \dfrac{\text{active power}}{\text{apparent power}} = \dfrac{P}{S}$

or active power = apparent power × power factor

For the problem:

$P = S\cos\phi$ or kW = kVA × $\cos\phi$

Hence 560 = kVA × 0.7

Thus apparent power supplied $= \dfrac{560}{0.7} = 800\text{kVA}$

With the power factor increased to 0.8 and the apparent power kept constant, the new 'active power' = 800 × 0.8 = 640kW.

The increase in active power would be:

640 − 560 = 80kW

13. Resistance of heater = 6.5Ω Working voltage 100V

$\therefore$ Rated current $= \dfrac{100}{6.5} = 15.38\text{A}$

Reactance of coil, $X_L = 2\pi f L$

or $X_L = 2 \times \pi \times 50 \times 0.01 = 3.14\Omega$

Impedance of circuit, $Z = \sqrt{6.5^2 + 3.14^2}$

or $Z = 7.22\Omega$

Circuit voltage would be = 15.38 × 7.22 = 111.04V

Applied voltage should be 111.04V to give 100V on heater.

If frequency rose to $50 + \left(\frac{5}{100} \times 50\right) = 52.5\text{Hz}$, X_L would rise in proportion.

$\therefore$ New reactance $X_{L1} = 3.14 \times \frac{52.5}{50} = 3.297\Omega$

New impedance $Z_1 = \sqrt{6.5^2 + 3.297^2}$

or $Z_1 = 7.288\Omega$

New heater current $= \frac{V}{Z_1} = \frac{111.04}{7.288} = 15.23\text{A}$

New voltage across heater terminals = 15.23 × 6.5 = 98.995V

Change of voltage = 100 × 98.995 = 1.005V

14. Force on 1 conductor:

$F = BI\ell$ newtons

$= 0.8 \times 20 \times 400 \times 10^{-3}$ newtons = 6.4N

Force due to all *active* conductors

$= \frac{2}{3} \times 246 \times 6.4 = 164 \times 6.4 = 1050\text{N}$

(a) Torque produced = $1050 \times 150 \times 10^{-3} = 157.5$Nm

(b) Shaft power developed $= \frac{2\pi NT}{60}$ Watts

$\frac{2 \times \pi \times 500 \times 157.5}{60} = 8.24\text{kW}$

15. Output from motor = 45kW

Input to motor $= \frac{45 \times 1000 \times 100}{93}$ watts $= 48.4\text{kW}$

But 3-phase power (watts) $= \sqrt{3}VI\cos\phi$

$\therefore 48.4 \times 10^3 = \sqrt{3} \times 440 \times I \times 0.88$

$I = \frac{48.4 \times 10^3}{1.732 \times 440 \times 0.88} = 72\text{A}$

16. Let n = the no. of cells in series.

Battery e.m.f. = $n \times 1.5$ volts

Battery internal resistance = $n \times 0.5$ ohms

Also since $P = I^2R$, then for the load: $2 = I^2 \times 100$

or $I = \frac{\sqrt{2}}{10} = 0.1414\text{A}$

Also for the circuit, current I is given by:

$I = V/R = \frac{1.5n}{100 + 0.5n}$ or $\frac{1.5n}{100 + 0.5n} = 0.1414$

$\therefore 1.5n = 14.14 + 0.0707n$

or $1.4293n = 14.14$

giving $n = 10$ (approx.)

As a check: $I = \frac{1.5 \times 10}{100 + (0.5 \times 10)} = 0.143\text{A}$

So power dissipated = $0.143^2 \times 100$

= 2.045W

or the given rating, 2W (approx.)

17. Total internal resistance = 6 × 0.1Ω = 0.6Ω. For maximum power; external load resistance and internal battery resistance are equal.

Thus load resistance = 0.6Ω

$$\text{Load current } I = \frac{E}{R+R_i} = \frac{6\times 2.2}{0.6+0.6}$$

$$= 11\text{A}$$

$\therefore$ Maximum power $P = I^2R$

$$= 11^2 \times 0.6$$

$$= 72.6\text{W}$$

An **alternative solution** is possible using calculus.

$$I = \frac{E}{R+R_i} \text{ and } P = I^2R$$

$$\therefore P = \left(\frac{E}{R+R_i}\right)^2 R = \frac{E^2R}{R^2+2RR_i+R_i^2}$$

$$= \frac{E^2}{R+2R_i+R_i^2R^{-1}}$$

$$= \frac{13.2^2}{R+1.2+0.6^2R^{-1}}$$

$$\therefore P = \frac{174.24}{R+1.2+0.36R^{-1}}$$

P is maximum when the denominator is minimum. Differentiate denominator with respect to R.

$$\text{Thus } \frac{d(R+1.2+0.36R^{-1})}{dR} = 1+0-0.36R^{-2}$$

$$= 0 \text{ for min. value}$$

$$1+0-0.36R^{-2} = 0$$

$$\therefore R = 0.6\Omega$$

The second differential determines whether R is maximum or minimum value.

$$\frac{d^2(R+1.2+0.36R^{-1})}{dR^2} = \frac{d(1+0-0.36R^{-2})}{dR}$$

$$= 0.72R^{-3}$$

Since the second differential is positive, R is a minimum value.

Thus when $R = 0.6\Omega$ P is a maximum value.

Calculate power as shown previously.

18. (a) Apparent power $S_a = 200\text{kVA}$ Reactive power $Q_a = S_a \sin\phi_a = 0\text{kVAr}$

(b) Active power $P_b = 400\text{kW}$ $\cos\phi_b = 0.8$ $\sin\phi_b = 0.6$

$$S_b = P_b/\cos\phi_b = \frac{400}{0.8} = 500\text{kVA}$$

$$Q_b = S_b \sin\phi_b = 500 \times 0.6 = 300\text{kVAr}$$

(c) $P_c = 200\text{kW}$

Total active power of loads, $P = 200 + 400 + 200 = 800\text{kW}$

$$\text{Total apparent power of all loads, } S = \frac{P}{\cos\phi}$$

$$= \frac{800}{0.97} = 824.74\text{kVA}$$

Also $\cos\phi = 0.97$. $\therefore \phi = 14°4'$ and $\sin\phi = 0.234$

So reactive power of all loads, $Q = S\sin\phi = 824.74 \times 0.243$

$$= 200.4\text{ kVAr}$$

Thus lagging kVAr value is reduced by 300 – 200.4 = 99.6. This must therefore, be the leading reactive power Q_c of the synchronous motor. Apparent power rating of motor

$S_C = \sqrt{200^2 - 99.6^2}$

$= 223.3\text{kVA}$

Power factor of motor, $\cos\phi_C = \dfrac{200}{223.3} = 0.89$ (leading)

19. Motor condition

Input power $= 55\text{kW}$ Line current $= \dfrac{55\,000}{220} = 250\text{A}$

Field current $= \dfrac{220}{110} = 2\text{A}$

Armature current = 250 – 2 = 248A

Back e.m.f. = 220 – (248 × 0.02) = 220 – 4.96

= 215.04V

Generator condition

Output power $= 55\text{kW}$ Line current $= \dfrac{55\,000}{220} = 250\text{A}$

Field current $= \dfrac{220}{110} = 2\text{A}$

Armature current = 250 + 2

= 252A

Generated e.m.f. = 220 + (252 × 0.02)

= 225.04V

As e.m.f. is proportional to speed, then $\dfrac{225.04}{215.04} = \dfrac{N_2}{500}$

or $N_2 = \dfrac{225.04}{215.04} \times 500$

Thus generator speed = 1.0464 × 500

= 523.3 rev/min

20. If $B_m = 1.4\text{T}$ and area of core $= 0.27 \times 0.27 = 0.0729\text{m}^2$

Then $\Phi_m = 1.4 \times 0.0729 = 0.102\text{Wb}$

Substituting in the formula:

$V_1 = 4.44\Phi_m f N_1$ or $N_1 = \dfrac{3500}{4.44 \times 0.102 \times 50}$ turns

$\therefore N_1 = \dfrac{3500}{2.22 \times 10.2} = 154.7$ turns

or primary turns = 155 (approx.)

Secondary turns $= \dfrac{440}{3500} = \dfrac{N_2}{155}$

or $N_2 = 155 \times \dfrac{44}{350} = 19.5$ turns

Thus secondary turns = 20 (approx.)

21. Delta-connected load

Active power, $P_1 = 30\text{kW}$ at a power factor or 0.92 (leading)

$\therefore$ Apparent power, $S_1 = \dfrac{30}{0.92} = 32.61\text{kVA}$

$\cos\phi = 0.92 \therefore \phi = 22°56'$ $\sin\phi = 0.3896$

So the reactive power, $Q_1 = 32.61 \times 0.3896 = 12.7\text{kVAr}$

Star-connected load

Active power, $P_2 = 40\text{kW}$ at a power factor of 0.85 (lagging)

Apparent power, $S_2 = \dfrac{40}{0.85} = 47.1\text{kVA}$

$\cos\phi = 0.85 \therefore \phi = 31°37' \sin\phi = 0.5242$

Thus reactive power, $Q_2 = 47.1 \times 0.5242 = -24.7$kVAr

Note. –ve sign given to the lagging reactive power value to distinguish it from the leading reactive power value of the other load.

Total active power on alternator, $P = P_1 + P_2$

$$= 30 + 40 = 70\text{kW}$$

Total reactive power loading, $Q = Q_1 + Q_2$

$$= 12.7 - 24.7$$

$$= -12\text{kVAr}$$

So apparent power, loading on alternator is:

$$S = \sqrt{70^2 + 12^2}$$

$$= 71.06\text{kVA}$$

Again 3-phase kilovolt amperes is given by $\frac{\sqrt{3}VI}{1000}$

$$\text{Hence } \frac{1.732 \times 500 \times I}{1000} = 71.06$$

$$\text{or } I = \frac{71.06 \times 2}{1.732} = 82.1\text{A}$$

Line current = 82.1A

$$\text{Supply power factor} = \frac{P}{S} = \frac{70}{71.06} = 0.98 \text{ (lagging)}$$

The lagging condition is determined from the resultant –ve sign of the total reactive power value.

22. Original conditions:

$E_b = 440 - (30 \times 0.7) = 419$V

Original flux condition Φ_1

Final flux condition: $\Phi_2 = 0.8\Phi_1$.

Assume no speed change then since generated e.m.f. is proportional to flux:

new E_b value = $0.8 \times 419 = 335.2$V

and momentary current is given by $\frac{V - E_b}{R_a}$ amperes

$$\text{or } I_a = \frac{440 - 335.2}{0.7} = 149.7\text{A}$$

If the final torque condition T_2 = original torque T_1 and since torque is proportional to flux *and* armature current, then

$T = k\Phi I_a$

Thus we can write $T_1 = T_2$ or $k\Phi_1 I_{a1} = k\Phi_2 I_{a2}$

$$\text{and } I_a = \frac{\Phi_1 I_{a1}}{\Phi_2} = \frac{\Phi_1 I_{a1}}{0.8\Phi_1} = \frac{3.0}{0.8} = 37.5\text{A}$$

New armature current will be 37.5A

New back e.m.f. = $440 - 37.5 \times 0.7$

$$= 413.75\text{V}$$

Also since $E_b \propto \Phi N$

$$\text{We can wirte } \frac{E_{b2}}{E_{b1}} = \frac{k\Phi_2 N_2}{k\Phi_1 N_1} \text{ or } \frac{413.75}{419} = \frac{0.8\Phi_1 N_2}{\Phi_1 700}$$

$$\text{So } N_2 = \frac{413.75 \times 70}{419 \times 0.8} = 864.4 \text{ rev/min}$$

23. Original power transmitted by supply cable is given by:

$$P_1 = VI\cos\phi = \frac{240 \times 250 \times 0.8}{1000} = 48\text{kW}$$

Current taken by a 800μF capacitor is given by:

$$I_C = \frac{V}{X_C} \text{ where } X_C = \frac{1}{2\pi fC} = \frac{10^6}{2\times\pi\times 50\times 800}$$

$$\text{Thus } X_C = \frac{10^2}{25.12} = 3.98 = 4\Omega \text{ (approx.)}$$

$$\text{and capacitor current} = \frac{240}{4} = 60\text{A}$$

There are now 2 currents, I_L = 250A, *lagging* the voltage by 36°44′ (Note cos 36°44′ = 0.8) and I_C = 60A, *leading* the voltage by 90°. The problem calls for a graphical solution. Draw a voltage ordinate horizontally. Choose a suitable current scale and, from the origin, i.e. left-hand point of the voltage ordinate, draw I_C, to scale, vertically up. Next from the origin draw I_L to scale below the voltage ordinate by 36°44′. Complete the parallelogram for the current vectors, draw and measure the resultant current for magnitude and phase. This part of the problem is worked out here mathematically.

$$\text{Thus } I = \sqrt{I_L^2 - I_C^2 - 2I_L I_C \cos\theta}$$

$$= \sqrt{250^2 + 60^2 - (2\times 250\times 60\times \cos 53°16')}$$

$$= 219.4\text{A}$$

Assuming the same power of 48kW was being supplied, the new power factor of the supply would be obtained from:

$$P_2 = VI\cos\phi_2 \text{ or } \cos\phi_2 = \frac{480\ 000}{240\times 219.4}$$

$$= 0.91 \text{ (lagging)}$$

The question is not clear as to the final loading conditions but it is assumed that this power-factor condition of 0.91 (lagging) is maintained. The maximum current would be limited to 250A and the power transmitted would be P_2.

$$\text{Thus } P_2 = \frac{240\times 250\times 0.91}{1000} \text{ kilowatts}$$

$$= 54.6\text{kW}$$

Additional power, $P_2 - P_1 = 54.6 - 48 = 6.6$kW

24. Although the parallel working of generators has not been considered in this book, this more testing problem can be solved by a direct application of Kirchhoff's laws. Thus, let I_A be the current output from machine A and I_B the current output from machine B. Let V = the common bus bar or terminal voltage.

Then $I_A + I_B$ = 2500A and 2 voltage equations can be built up as follows, since

$$I_{aA} = I_A + \frac{V}{85} \text{ and } I_{aB} = I_B + \frac{V}{85}$$

Thus $V = 600 - 0.015 I_{aA}$

and $V = 620 - 0.015 I_{aB}$

$$\text{or } V = 600 - 0.015\left(I_A + \frac{V}{85}\right)$$

$$\text{or } V = 620 - 0.015\left(I_B + \frac{V}{85}\right)$$

$$\text{giving } V = 600 - 0.015 I_A - \frac{0.015V}{85} \ldots \text{(i)}$$

$$V = 620 - 0.015 I_B - \frac{0.015V}{85} \ldots \text{(ii)}$$

Subtracting (i) from (ii),

$0 = 20 - 0.015 I_B + 0.015 I_A$ or $0 = 20 - 0.015(I_B - I_A)$

and $0 = 20 - 0.015\,[I_B - (2500 - I_B)]$

$= 20 - 0.015\,[I_B - 2500 + I_B]$

$= 20 - 0.03 I_B + 37.5$

$$\text{Thus } I_B = \frac{57.5}{0.03} = 1916.66\text{A}$$

$I_A = 2500 - 1916.66 = 583.34$A

From (i) $V - 600 - (583.34 \times 0.015) - \dfrac{0.015V}{85}$

$$= 600 - (5.8334 \times 1.5) - \frac{0.015V}{85}$$

or $V = 600 - 8.75 - \dfrac{0.015V}{85}$

Whence $V + \dfrac{0.015V}{85} = 591.25$

giving $\dfrac{85V + 0.015V}{85} = 591.25$

or $85.015V = 591.25 \times 85$

and $V = \dfrac{591.25 \times 85}{85.015}$

Thus $V = 590.75\text{V}$

Output of machine A $= \dfrac{583.34 \times 590.75}{1000} = 344.6\text{kW}$

Output of machine B $= \dfrac{1916.66 \times 590.75}{1000} = 11.32\text{kW}$

Note. Although the kW rating of B is correct for the values given in the question, it is high for the accepted sizes of D.C. machine. The bus bar is a common term used in electrical power distribution. It is a strip or bar of metal (usually aluminium, brass or copper) that conducts electricity in switchboards, substations, distribution boards, battery banks or other such electrical system. Its main purpose is to conduct high current electricity, in some cases tens of thousands of amperes. Their flat or hollow tube shape allows heat to dissipate more efficiently due to their high surface area. Their one purpose is to conduct electricity, and they do not function as structural members.

25. Motor output = 10kW

Motor input $= 10 \times \dfrac{100}{85} = 11.76\text{kW} = 11\,760\text{W}$

Current taken from supply $= \dfrac{11\,760}{230} = 51.31\text{A}$

Shunt-field current, $I_f = \dfrac{230}{220} = 1.045\text{A}$

Armature current, $I_a = 51.13 - 1.045 = 50.085\text{A}$

Back e.m.f., $E_b = 230 - (50.1 \times 0.15) = 222.5\text{V}$

As a generator, speed is increased and flux is increased in proportion to the shunt-field current.

$\therefore E = 222.5 \times \dfrac{1080}{1000} \times \dfrac{1.1}{1.045} = 252.98\text{V}$

Also since armature copper loss is the same as for the motor, armature current must be the same = 50.1A.

$\therefore$ Terminal voltage, $V = 252.98 - (50.1 \times 0.15) = 245.47\text{V}$

Output current = 50.1 − 1.1 = 49A

So output $= \dfrac{245.47 \times 49}{1000} = 12.03\text{kW}$

26. Let I_A = the current in the first coil, then $I_A = \dfrac{220}{16} = 13.75\text{A}$

$\cos\phi_A = \dfrac{3}{16} = 0.187 \quad \phi_A = 79°13' \quad \sin\phi_A = 0.9824$

I_B = the current in the second coil, then $I_B = \dfrac{220}{25} = 8.8\text{A}$

$\cos\phi_A = \dfrac{7}{25} = 0.28 \quad \phi_B = 73°44' \quad \sin\phi_B = 0.96$

Total active components, $I_a = (13.75 \times 0.187) + (8.8 \times 0.28)$ amperes

$= 2.57 + 2.464 = 5.034\text{A}$

Total reactive components, $I_r = (-13.75 \times 0.9824)$

$= -(8.8 \times 0.96)$ amperes

$= -13.51 - 8.448 = -21.96$A

Resultant current, $I = \sqrt{5.034^2 + 21.96^2}$ amperes

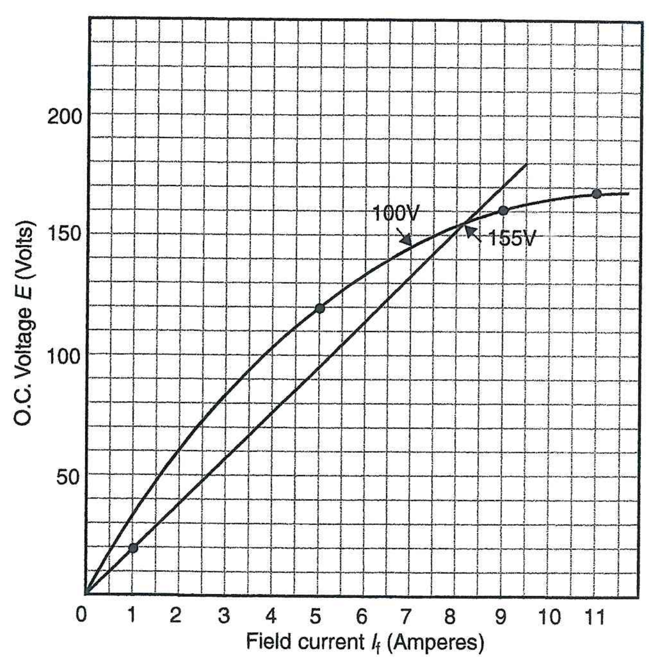
$= 22.53$A

$$\cos\phi = \frac{5.034}{22.53} = 0.223 \text{ (lagging)}$$

$$\text{Power, } P = \frac{220 \times 22.53 \times 0.233}{1000} \text{ kilowatts} = 1.105\text{kW}$$

The phasor diagram for this problem can be considered as one of the basics for a parallel circuit and has already been illustrated several times.

27. The magnitude of the e.m.f. generated on O.C. is determined from the point where the field voltage-drop line intersects the O.C.C.



▲ Figure 3

Plot the O.C.C. and the field voltage-drop line by obtaining a typical value. Thus, for a field current of 4A, the voltage drop would be 4 × 20 = 80V. Plot this point. Draw a line from the origin through this point and produce the line to cut the O.C.C. The induced e.m.f. is then 155V.

The voltage drop across the field is also the terminal voltage of the generator and from a graph a terminal voltage of 140V is obtained when the field current is 7A, i.e. read horizontally for 140V on the voltage scale to determine the corresponding field-current value on the field voltage line. This value is 7A. For this value of field current however, the e.m.f. generated is 149V.

Also $V = E - I_a R_a$ ∴ $140 = 149 - (I_a \times 0.02)$ or $0.02 I_a = 149 - 140$

Giving an armature current, $I_a = \frac{9}{0.02} = 450$A

28. A fundamental formula for the alternator and an A.C. supply is:

$f = \frac{PN}{120}$ where P = the number of poles.

(a) Therefore $f = \frac{12 \times 600}{120} = 60$Hz

(b) Since the load is balanced, the voltage across each phase, $V_{ph} = \frac{440}{\sqrt{3}}$ volts

Impedance of 1 phase of load, $Z_{ph} = \sqrt{35^2 + 25^2}$ ohms $= 43.01$W

For a star-connected load, current in 1 phase of load,

I_{ph} = Line current, I

or $I_{ph} = \frac{440}{\sqrt{3} \times 43.01} = 5.91$A

Thus current in a coil = 5.91A

(c) Current in each phase of alternator,

or $I_{ph} = \frac{I}{\sqrt{3}} = \frac{5.91}{\sqrt{3}} = 3.41$A

(d) Power factor of load $= \cos\phi = \frac{R}{Z} = \frac{35}{43.01}$

$= 0.81$ (lagging).

and total power of load $= \sqrt{3}VI\cos\phi$

$$= \frac{\sqrt{3}\times 440\times 5.91\times 0.81}{1000} \text{ kilowatts} = 3.65\text{kW}$$

29. At resonance $2\pi fL = \frac{1}{2\pi fC}$

$$\therefore f = \frac{1}{2\pi\sqrt{LC}}$$

$$= \frac{1}{2\pi\sqrt{0.1\times 0.1\times 10^{-6}}}$$

Resonant frequency = 1592Hz. At resonance there is *no* resultant reactance, i.e. R = Z

$$\therefore I = \frac{V}{R} = \frac{230}{100}$$

$= 2.3$A

$$X_c = \frac{1}{2\pi fC} = \frac{1}{2\pi\times 1592\times 0.1\times 10^{-6}} = 1000\Omega$$

P.D. across capacitor $= IX_c = 2.3 \times 1000 = 2300$V.

30. Before adjusting the e.m.f. of one of the machines, the e.m.f. of each machine is considered to be 230V. The bus bar voltage is 220V. Voltage drop due to resistance *R* of one machine and cables up to bus bars = 230 – 220 = 10V.

Current = 640A.

$\therefore$ Resistance *R* of machine and cables $= \frac{10}{640} = \frac{1}{64}$ ohms

Let V = the bus bar voltage under the new condition.

Then current supplied by generator No 1 $= \frac{235 - V}{\frac{1}{64}}$ A

$= 64\,(235 - V)$

Current supplied by generator No. 2 = (230 – V) 64 A

Current supplied by generator No 3 = (230 – V) 64 A

Power supplied by the 3 machines = V (235 – V) 64 + V (230 – V) 64 + V (230 – V) 64 W

$= 64\,(235V - V^2) + 2 \times 64(230V - V^2)$

Now the original power supplied by 3 machines

= 3 × 640 × 220 watts = 422 400W or 422.4kW

$\therefore 3 \times 640 \times 220 = 64\,(235V - V^2) + 2 \times 64\,(230V - V^2)$

and $6600 = 235V - V^2 + 460V - 2V^2$

or $-3V^2 + 695V = 6600$

Thus $V^2 - 231.66V + 2200 = 0$

Solving for V using the quadratic formula:

$$V = \frac{231.66 \pm \sqrt{231.66^2 - 4\times 2200}}{2}$$

$$= \frac{231.66 \pm 211.9}{2} = \frac{443.56}{2} = 221.78\text{V}$$

Thus bus bar voltage will be 221.8V

Current of machine No. 1 $= \frac{231 - 221.78}{\frac{1}{64}}$

$= 13.22 \times 64 = 846$A

$$\text{Current of machine No. 2} = \frac{230 - 221.78}{\frac{1}{64}}$$

$$= 8.22 \times 64 = 526\text{A}$$

$$\text{Current of machine No. 3} = \frac{230 - 221.78}{\frac{1}{64}}$$

$$= 8.22 \times 64 = 526\text{A}$$

Check. Power supplied = (846 + 526 + 526) 221.78

$$= 1898 \times 221.78\text{ W}$$

$$= 421\text{kW (approx.).}$$

Using the other root of the quadratic equation we have:

$$V = \frac{231.66 - 211.9}{2} = \frac{19.76}{2} = 9.88\text{V}$$

Current of machine No. 1 = (235 – 9.88) × 64

$$= 225.12 \times 64 = 14\,408\text{A}$$

Current of machine No. 2 = (230 – 9.88) × 64

$$= 220.12 \times 64 = 14\,088\text{A}$$

Current of machine No. 3 as for No. 2.

The above conditions though theoretical, would relate to a bus bar voltage of 9.88V and could be imagined as the result of a 'short-circuit' at the bus bars, where the power of 422kW could be assumed to be dissipated.

31. Delta load

Active power, $P_1 = 50\text{kW}$ $\cos\phi = 0.75$

Hence $\phi = 41°25'$ and $\sin\phi = 0.6615$

$$\text{Apparent power, } S_1 = \frac{50}{0.75}$$

$$= 66.66\text{kVA}$$

Reactive power, $Q_1 = 66.66 \times 0.6615$

$$= -44.1\text{kVAr (lagging)}$$

Star load

Active power, $P_2 = 40\text{kW}$

$\cos\phi = 0.62$ $\phi = 51°41'$ and $\sin\phi = 0.7846$

$$\text{Apparent power, } S_2 = \frac{40}{0.62} = 64.5\text{kVA}$$

Reactive power, $Q_2 = 64.5 \times 0.7846 = 50.6\text{kVAr}$ (leading)

Total power, $P = P_1 + P_2 = 50 + 40 = 90\text{kW}$

Total reactive power, $Q = Q_1 + Q_2 = -44.1 + 50.6 = 6.5\text{kVAr}$ (leading)

$$\text{Apparent power, } S = \sqrt{90^2 + 6.5^2} = 90.23\text{kVA}$$

$$\text{Power factor} = \frac{P}{S} = \frac{90}{90.23} = 0.988\text{ (leading)}$$

32. Let the resistance of the fault be R ohms

Since the cable resistance itself is 4 ohms, then (Resistance of end A to earth – R) + (Resistance of end B to earth – R) = 4

Or $5 - R + 3 - R = 4$ or $8 - 2R = 4$

giving $2R = 4$ or $R = 2\Omega$

It can be assumed that for a cable, the resistance is proportional to length.

Then from end A to fault = 5Ω and the fault resistance is 2Ω

∴ cable length must have a resistance of 3Ω.

Thus fault must be $\frac{3}{4} \times 1000 = 750\text{m}$ from end A. Practically this is a useful technique to know!

33. The torque of a motor $T \propto \Phi I_a$ and since, for a series motor, $\Phi \propto I_f$ and $I_f = I_a = I_L$ then

$T \propto I_L^2$ or $T = kI_L^2$

On full load

$E_{bi} = V - I_a R = 210 - (100 \times 0.1) = 200V$

When load torque is reduced to 25% of full-load value, then $T_2 = T_1 \times \frac{1}{4}$ where, T_2 is the new torque and T_1 the original torque.

Then $\frac{T_2}{T_1} = \frac{kI_{L2}^2}{kI_{L1}^2}$ or $\frac{I_{L2}^2}{I_{L1}^2} = \frac{1}{4}$

$\therefore I_{L2}^2 = \frac{I_{L1}^2}{4}$ or $I_{L2} = \frac{I_{L1}}{2} = \frac{100}{2} = 50A$

On new load condition

$E_{b2} = 210 - (50 \times 0.1) = 205V$

As $E_b \propto \Phi N$ then $= \frac{E_{b2}}{E_{b1}} = \frac{k\Phi_2 N_2}{k\Phi_1 N_1}$ Thus $\frac{205}{200} = \frac{50 \times N_2}{100 \times 600}$.

Note that field current $I_f = I_a = I_L$ is now substituted for flux, since $\Phi \propto I_f$.

Thus $N_2 = \frac{205}{200} \times 600 \times 2 = 1230$ rev/min.

Speed of motor on 75% load torque = 1230 rev/min

34. Reactance X_C of capacitor $= \frac{1}{2\pi fC} = \frac{10^6}{2 \times \pi \times 60 \times 400} = 6.63\Omega$

Since the load is balanced:

The voltage across a phase, $V_{ph} = \frac{440}{\sqrt{3}} = 254.5V$

Current in the resistor $I_R = \frac{254.5}{14}$

$= 18.18A$, in-phase with V_{ph}

Current in the capacitor $I_C = \frac{254.5}{6.63}$

$= 38.38A$, leading V_{ph} by 90°

Let I_{ph} = the resultant of 18.18A and 38.38A which are in quadrature

$\therefore I_{ph} = \sqrt{18.18^2 + 38.38^2}$

= 42.45A. This is also the line current since the load is star connected $\therefore I = 42.45A$

Power factor of load $= \cos\phi = \frac{I_R}{I_{ph}} = \frac{18.18}{42.45}$

= 0.43 (leading)

Power of load, $P = \sqrt{3} VI \cos\phi$

$= \sqrt{3} \times 440 \times 42.45 \times 0.43$ watts $= 13.9kW$

If the load is in delta, the current per phase would rise in the proportion of

$\frac{440}{254.5}$ or $= \sqrt{3} \times$ original I_{ph}

$= \sqrt{3} \times 42.45$ amperes

The line current would be $\sqrt{3}$ times this new phase current.

$\therefore$ New $I = \sqrt{3} \times \sqrt{3} \times 42.45 = 3 \times 42.45 = 127.35A$

The power factor of the load will remain the same

So new power, $P = \sqrt{3}VI\cos\phi = \sqrt{3}\times 440 \times 127.35 \times 0.43$ watts $= 41.7$kW

35. Total load = 45 + 78 = 123A. Length BC is (580 – 240) = 340m. Let the current in the remaining 320m section AC be I amperes:

Then current in section AC = I amperes

Then current in section AB = $(123 - I)$ amperes

Then current in section BC = $(123 - I - 45)$ amperes

$= (78 - I)$ amperes

Resistance of section AC $= \frac{0.25}{1000}\times 320 = 0.08\Omega$

Resistance of section AB $= \frac{0.25}{1000}\times 240 = 0.06\Omega$

Resistance of section BC $= \frac{0.25\times 340}{1000} = 0.085\Omega$

By Kirchhoff's laws, the voltage drops in either section of the main feeding the load at C are equal.

$\therefore I\times 0.08 = (123 - I)\,0.06 + (78 - I)\,0.085$

or $8I = 6\,(123 - I) + 8.5\,(78 - I)$

giving $8I = 738 - 6I + 663 - 8.5I$

or $22.5I = 738 + 663 = 1401$

Thus current I in section AC $= \frac{1401}{22.5} = 62.27$A

Current in section AB = 123 – 62.27 = 60.73A

Current in section BC = 78 – 62.27 = 15.73A

P.D. at point C = 220 – (62.27 × 0.08)

= 215V

P.D. at point B = 220 – (60.73 × 0.06)

= 216.36V

P.D. at load C is lowest = 215V

36. Let non-inductive coil of 6Ω be designated branch A.

Then

$I_A = \frac{110}{6} = 18.33\text{A} \quad \cos\phi_A = 1 \quad \sin\phi_A = 0$

Let inductive coil of impedance 9Ω be designated branch B

Then $I_B = \frac{110}{9} = 12.22\text{A}$

$\cos\phi_B = \frac{3}{9} = 0.33$ (lagging) $\sin\phi_B = 0.943$

Resolving in active and reactive components

$I_a = I_A\cos\phi_A + I_B\cos\phi_B$

$= 18.33\times 1 + 12.22\times 0.33 = 18.33 + 4.033 = 22.363\text{A}$

and $I_r = -I_A\sin\phi_A - I_B\sin\phi_B$

$= -18.33\times 0 - 12.22\times 0.943 = -0 - 11.52 = -11.52\text{A}$

$\therefore I = \sqrt{22.36^2 + 11.52^2} = 25.2\text{A}$

Thus current taken from mains is 25.2A

With capacitor of 600μF connected in parallel:

$X_C = \frac{1}{2\pi fC} = \frac{10^6}{2\times\pi\times 50\times 600} = 5.3\Omega$

Current $I_C = \frac{110}{5.3} = 20.75\text{A}$

Resolving as before, active component I_a remains the same but the reactive component $I_r = 20.75 - 11.52 = 9.23$A and is now vertically upwards, i.e. leading V by 90°.

Resultant current $I = \sqrt{22.36^2 + 9.23^2} = 24.2$A

37. The turns ratio per phase of the transformer are 560 to 42 or $\frac{560}{42} = \frac{13.33}{1}$. The voltages per phase will be in the same proportion.

(a) With the transformer connected, primary in star and secondary in delta, then:

Primary voltage per phase $= \frac{6600}{\sqrt{3}}$ volts

Secondary voltage per phase $= \frac{6600}{\sqrt{3} \times 13.33} = 286$V

But for a delta connection, line voltage = phase voltage

or $V = V_{ph} = 286$V

(b) With the transformer connected, primary in delta and secondary in star, then:

Primary voltage per phase = 6600V

Secondary voltage perphase $= \frac{6600}{13.3} = 495.4$V

But for a star connection, line voltage $= \sqrt{3}$ phase voltage or $V = \sqrt{3} V_{ph} = \sqrt{3} \times 495.4 = 858$V

38. Let V = the terminal voltage, I_1, the current given by the 220V battery and I_2 the current given by the 225V battery. The following equations can then be set down.

$220 - 0.2I_1 = V \ldots$ (i)

$225 - 0.3I_2 = V \ldots$ (ii)

and $10(I_1 + I_2) = V \ldots$ (iii)

Subtracting (i) from (ii) we have:

$225 - 220 - 0.3I_2 + 0.2I_1 = 0$

Thus $5 = 0.3I_2 - 0.2I_1$, or $I_2 = \frac{5 + 0.2I_1}{0.3}$

Using (ii) and (iii) we can write:

$225 - 0.3I_2 = 10I_1 + 10I_2$ and substituting for I_2

$$225 - 0.3\left(\frac{5 + 0.2I_1}{0.3}\right) = 10I_1 + 10\left(\frac{5 + 0.2I_1}{0.3}\right)$$

Thus $225 - 5 - 0.2I_1 = 10I_1 + \frac{50 + 2I_1}{0.3}$

and $(220 \times 0.3) - 0.06I_1 = 3I_1 + 50 + 2I_1$

or $66 - 50 = 3I_1 + 2I_1 + 0.06I_1$

Hence $16 = 5.06I_1$, $I_1 = \frac{16}{5.06} = 3.16$A

Also $I_2 = \frac{5 + 0.2I_1}{0.3} = \frac{5 + 0.632}{0.3} = 18.77$A

and $V = 10(I_1 + I_2) = 10(3.16 + 18.77) = 219.3$V

39. Heat received by the water = mass × temperature rise × specific heat capacity

= 1 × (100 − 11) × 4.2 kilojoules = 373.8kJ

∴ Energy put into water = 373.8kJ and assuming an efficiency of 100% for the kettle!

Then the energy taken from mains = 373 800J

With a D.C. supply, the current taken would be $\frac{373\,800}{220 \times 3.5 \times 60}$

or $I = 8.09$A

and the resistance of the kettle element $= \dfrac{220}{8.09} = 27.19\Omega$

On A.C., the reactance X of the kettle becomes effective.

$\therefore X = 2\pi fL = 2 \times \pi \times 50 \times 0.05 = 15.7\Omega$

Hence $Z = \sqrt{27.19^2 + 15.7^2} = 31.4\Omega$

Thus with an A.C. supply, current taken will be $\dfrac{220}{31.4}$

or $I = \dfrac{22}{3.14} = 7.06\text{A}$

The input power will be given by $P = I^2R$

or $P = 7.06^2 \times 27.19 = 1355\text{W}$

The time taken to produce 373 800J is given by:

$$t = \frac{373\,800}{1355} = 275.7\text{s}$$

The percentage time difference between heating with A.C. instead of D.C. would be:

$$\frac{276 - 210}{210} = 0.314 = 31.4\%$$

40. (a) Total power dissipation of coils $= 4 \times 45 = 180\text{W}$

If applied voltage = 220V, then field current t $= \dfrac{180}{220}$

or $I_f = 0.818\text{A}$

(c) No. of turns per field coil $= \dfrac{750}{0.818} = 916.86$

Length of turn = 450mm

$\therefore$ Length of wire $= 450 \times 916.86 = 412\,587\text{mm}$

$= 412.6\text{m}$

(b) Since $R = \dfrac{\rho\ell}{A}$ and $R = \dfrac{220}{4} \times \dfrac{11}{9}$

$= 67.22\Omega$

Then $A = \dfrac{\rho\ell}{R} = \dfrac{2 \times 10^{-8} \times 412.6}{67.22} = 12.3 \times 10^{-8}\ \text{m}^2$

or $12.3 \times 10^{-8} = \dfrac{\pi d^2}{4}$ where d = diameter of wire

Then $d^2 = \dfrac{12.3 \times 10^{-8} \times 4}{\pi}$

giving $d^2 = \dfrac{15.62}{10^8}$ or $d = \dfrac{\sqrt{15.62}}{10^4} = \dfrac{3.95}{10^4}$ metres

or diameter of wire $= 3.95 \times 10^{-4} \times 10^3\text{mm}$

$= 0.395\text{mm}$

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