$$\text{Average charging voltage} = \frac{146.4}{12} = 12.2\text{V}$$

$$\text{Average discharge voltage} = \frac{127.9}{11} = 11.62\text{V}$$

$$\text{Efficiency} = \frac{11.62 \times 16 \times 10}{12.20 \times 16 \times 11} = 0.867 \text{ or } 86.7\%$$

CHARGING PROCEDURE. British practice uses the 'constant current' method. This is also American practice but on the Continent the 'constant voltage' method is favoured. For this latter method the charging supply voltage is kept constant and is substantially higher than the battery e.m.f. for the discharged condition. The charging current is high initially but falls as the back e.m.f. of the battery rises. This method gives a lower charging time than the 'constant current' method, but due to the violent chemical action and heat generated in the battery there is danger of 'buckling' the plates, unless the battery is specially constructed!

For the 'constant current' method arrangements must be provided for increasing the voltage applied to the battery as charging proceeds and the back e.m.f. rises. If a generator is used and I is the charging current, R_i the internal resistance of the battery and E_b the battery e.m.f. at start of charge, then the applied voltage must be $V = E_b + IR_i$ ... start of charge (1).

If E_{b1} is the battery e.m.f. at the end of charge, the applied voltage would have to be $V_1 = E_{b1} + IR_i$ ... end of charge (2).

Thus subtracting (1) from (2) variation of voltage would be $V_1 - V = E_{b1} - E_b$ or the applied voltage must be increased by an amount equal to the rise of the battery e.m.f.

If a constant supply voltage is used for charging, then a variable resistor is required to obtain the necessary current control, and its value will be reduced as charging proceeds.

Let V be the supply voltage, I the charging current, R_i the internal resistance of the battery, E_b the battery e.m.f. at start of charge and R the control resistor. Then:

$$V = E_b + IR_i + IR \text{ ... start of charge (1)}$$

If E_{b1} is the battery e.m.f. at end of charge and R_1 the new value of the control resistor. Then:

$$V = E_{b1} + IR_i + IR_1 \text{ ... end of charge (2)}$$

Subtracting (1) from (2) $0 = E_{b1} - E_b + IR_1 - IR$ or $(E_{b1} - E_b) = I(R - R_1)$. Thus the control resistance must be reduced from R to R_1 as the battery voltage rises from E_b to E_{b1}.

Example 4.7. A 24V emergency battery is to be charged from the 110V ship's mains when the e.m.f. per cell has fallen to a minimum value of 1.8V. The battery consists of 12 cells in series, has a capacity of 100A h at a 10h rate and the internal resistance is 0.03Ω/cell. If charging continues until the voltage/cell rises to 2.2V, find the value of the variable resistor needed to control the charging (1 significant figure.)

The charging current can be assumed to be equal to the maximum allowable discharge current.

$$\text{Discharge current} = \frac{100}{10} = 10\text{A} = \text{charging current}$$

At start of charge, battery voltage = $12 \times 1.8 = 21.6$V

Battery internal resistance = $12 \times 0.03 = 0.36\Omega$

Then $110 = 21.6 + (10 \times 0.36) + (10 \times R)$

$$\text{or } R = \frac{110 - 25.2}{10} = \frac{8.48}{10} = 8.48\Omega$$

At end of charge, battery voltage = $12 \times 2.2 = 26.4$V

Then $110 = 26.4 + (10 \times 0.36) + (10 \times R_1)$

$$\text{or } R_1 = \frac{110 - 30}{10} = \frac{80}{10} = 8\Omega$$

Thus the variable resistor should have a value of 8.48Ω and be capable of being reduced to 8Ω. In practice a unit of 9Ω would be used which would be reduced by adjusting the sliding contact until the charging ammeter recorded the correct current. Further adjustments will be made periodically as charging proceeds. It is important to note that besides the ohmic value of the resistor, the wattage rating must be specified. For the unit in the example, a rating of $I^2R = 10^2 \times 9 = 900$W is required. The control resistor must be capable of dissipating up to this power as heat during the charging, although this waste of power will decrease slightly as charging proceeds. For example, at the end of the charge the power wasted in the resistor would be $10^2 \times 8 = 800$W.

The most important point to stress is the correct connection of the battery for charging, i.e. +ve terminal of battery to +ve of mains; −ve terminal of battery to −ve of mains. It is surprising how many times this requirement is overlooked through carelessness. For incorrect connection, no control of the current will be possible with the equipment provided and damage of the ammeter, control resistor or battery could result.

The Meaning of pH

Introductory ionic theory dealt with electrolytic dissociation which, for a solution, results in the formation of 2 separately charged ions (anions and cations). Such ionisation is assisted by a liquid which has a high 'dielectric constant' and thus can separate and support unlike charges. Pure water, being a poor conductor, serves as a good dielectric. The cations of an electrolyte are derived from the metallic part of the molecule, having +ve charge. Anions are from a non-metallic element or radical and have −ve charge. (A radical is a fundamental group of atoms, such as a sulphate (SO_4), a nitrate (NO_3) or a carbonate (CO_3) etc. that behave as individual entities and remain unchanged by most chemical reactions.) All acids produce hydrogen ions (H^+) and all alkalis produce hydroxyl ions (OH^-). Sulphuric acid (H_2SO_4) ionises to $H_2SO_4 \rightleftharpoons 2H^+ + SO_4^{--}$. Since each hydrogen ion (H^+) can carry only one +ve charge and as each H_2SO_4 molecule is electrically neutral, the sulphate ion must have 2 negative charges (SO_4^{--}). Similarly sodium chloride or common salt (NaCl) splits into $NaCl \rightleftharpoons Na^+ + Cl^-$. The double arrows indicate the splitting up is not complete and only a percentage of the solution is ionised; the amount depending on the physical conditions – strength of solution, temperature, type of salt, etc. The deionised portion of the solution is assumed to consist of neutral molecules. Caustic soda (NaOH) is an alkali and ionises thus: $NaOH \rightleftharpoons Na^+ + OH^-$. A hydroxyl ion ($OH^-$) is produced whereas the sulphuric acid molecule produces 2 hydrogen ions $2H^+$. Note that hydroxyl ions (OH^-) are in fact radicals.

For an electrolyte, the percentage ionisation is extremely high. It is known that the concentration of hydrogen ions present in the solution determines its properties. The greater this concentration the more acid the solution. Conversely the smaller the concentration the more alkaline the solution. Water is a special case since it ionises only slightly to give both hydrogen and hydroxyl ions. For many modern industrial processes, a knowledge of the acidity of the materials being used is most important. The pH value of a substance or solution is a measure of its acidity or alkalinity. A Swedish scientist Sorensen devised a scale to indicate the hydrogen ion content. It uses the pH symbol which stands for minus the logarithm to the base 10, for the ion concentration. This latter is expressed by [H^+].

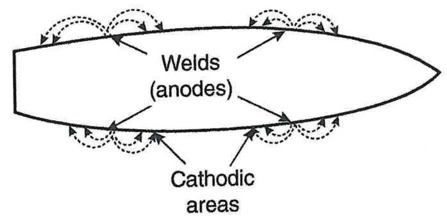
Note the letter 'p' comes from the German potenz, meaning 'a power', as the word is used in mathematics. For example, 100 = 10^2 – ten to the power two. The hydrogen ion concentration of a solution is the number of gram-equivalents of hydrogen contained in a litre of the solution. The actual extent of the ionisation of water has been determined and for pure water the ion concentration (H^+) is 0.000 000 1 gram-equivalents per litre or 10^{-7} normal. This power of 10 with the sign changed was taken by Sorensen to provide a scale up to 14 – neutrality being given as 7 (pure water). Thus by measuring its hydrogen ion concentration, it is possible to show where a solution lies on the acid/alkali scale. A strong acid will register near 1 while a strong alkali will register towards 14.

An electrical method for determining the pH value of a solution involves measuring the D.C. voltage between 2 special electrode assemblies which when immersed in solution, result in e.m.f.s being produced by voltaic action. The electrode assemblies are known as the *reference electrode* and the *measuring electrode*. The former is constructed so a constant potential is produced, irrespective of the pH of the solution under test. The measuring electrode assembly is constructed to allow its potential to vary with the solution under test and since the reference electrode potential is constant, the resulting potential variation between the 2 electrode assemblies is measured by a sensitive millivoltmeter. Such an instrument, suitably calibrated, provides a direct indication of the pH value of the sample or solution being tested.

Electrochemical Corrosion

The 2 main causes of electrochemical corrosion are due to (1) *galvanic* action and (2) *electrolytic* action. In each case the electrolyte may be water with impurities or moist earth. These corrosion causes are now considered separately.

(1) *Galvanic action.* This results in currents through the electrolyte from an anode, such as a metal structure, to some adjacent cathode. Metal is lost from the structure which can be both costly and/or dangerous. In the case of a merchant ship or naval vessel this results in rusting by oxidation of the hull when immersed in the conductive salt water environment. Galvanic action is caused by 'local cells' set up by slight differences in the surface composition of the hull metal, pitting of the plating, welds, rivets and millscale (figure 4.10). The corrosion occurs at local anodic areas from which currents flow through the sea to the local cathodic areas. The rate of corrosion is proportional to the currents which in turn are affected by metal composition, electrolyte temperature and even a ship's speed.



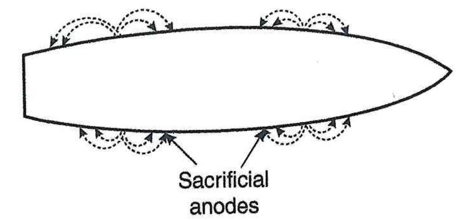
▲ Figure 4.10

(2) *Electrolytic action.* This is caused by stray currents due to leakage from electrical systems, such as a D.C. distribution network – examples being an electric dockside crane rail or tramway track. Such corrosion was a severe hazard in the early days of electric traction when the 'live rail' or cable followed a curve of the track and some conductor such as a metal pipe or different cable run was situated within the bend. This condition could provide a leakage current path, so a current could leave the live conductor, enter the pipe or adjacent cable sheath and then leave the latter at an appropriate point, to rejoin the live conductor. Where the current leaves the pipe or cable sheath, an anode is formed and as a result metal is dissolved at this point with possible serious results.

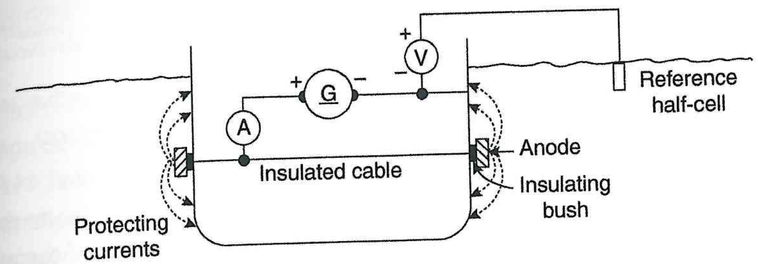
CATHODIC PROTECTION METHODS. In both the above cases, corrosion is prevented by introducing an anode adjacent to the structure, pipe or cable sheath. In this way current is *forced* to enter the original anode point so that this now becomes a cathode. This current being opposed to the original local currents neutralises or reverses them. For marine work 2 systems are in general use: (1) the 'sacrificial' anode method (2) the cathodic protection or 'impressed current' method. The latter is most favoured but both systems should be complementary to a good paint system.

(1) *The sacrificial anode method.* Counter currents are encouraged by galvanic action and the method is useful for protecting smaller structures or ships because the current adjustment range is restricted by constant potential effect. Also, as the name implies, the anodes waste away and require periodic replacement. Examples of materials which readily corrode away are magnesium or zinc. Magnesium gives a potential of some 1.8V positive with respect to iron and provided the resistance of the electrolyte path is sufficiently small, current enters the cathode (structure or hull). This is shown in figure 4.11.

(2) *The impressed current method.* For a ship, this is achieved by forcing a current through the hull, of a magnitude sufficient to nullify the effect of existing local cells. Such a current passes from the external anodes to the hull below the water-line so as to make the latter wholly cathodic. The anodes are supplied from a direct current source (generator or rectifier) and conduct the protective current into the water. The anodes



▲ Figure 4.11



▲ Figure 4.12

are connected to the positive terminal of the supply through insulating feeder cables, while the hull is connected to the negative terminal. For shore work, the structure to be protected is connected to the negative terminal. The anodes are buried in the ground and suitably sited to give the desired current distribution. For a ship, the current required to give protection varies from 20 to 300 amperes, variation being dependent on paint condition, ship speed, water temperature, etc. For shore work, variation depends mainly on soil moisture content. The anode material for ship work may be a metal such as raw lead or platinum plated titanium, which is affected only a little by the discharge current. Systems operate where some anode dissolving is inherent such as with the aluminium wire system, where a wire is trailed from the ship's stern. It is suitably insulated from the hull and the wastage compensated at regular intervals by paying out a suitable amount of wire. For shore work, anodes can be made of steel scrap, graphite or ferro-silicon.

If the protecting current is too high, electrolysis will cause excessive development of alkali and hydrogen resulting in paint blistering. If the current is too low, corrosion continues and in order to monitor and control protection at the correct value the potential between the electrolyte (sea) and the hull is measured with a sensitive voltmeter and reference electrode. In actual practice, a measuring half-cell is used for checking purposes, *silver–silver chloride* or *copper–copper sulphate* cells. Unprotected iron has a potential of –0.55V against the copper–copper sulphate measuring cell but, if reduced by the impressed current to −0.85V full protection is achieved. Control is effected manually or automatically by altering the potential of the motor-generator or transformer-rectifier unit. Modern automatic systems employ a hull-mounted reference electrode and an electronic amplifier to adjust the cathodic potential of the impressed protection current.

Practice Examples

4.1. An accumulator is charged at the rate of 6A for 18h and then discharged at the rate of 3.5A for 28h. Find the ampere hour efficiency (1 decimal place).

4.2. The mass of the cathode of a copper voltameter before deposit was 14.52g, and after a steady current was passed through the circuit for 50 min, its mass was 19.34g. The reading of the ammeter was 5.1A. Find the % error of the ammeter, taking the E.C.E. of copper as 330×10^{-9}kg/C (2 decimal places).

4.3. A 90V D.C. generator is used to charge a battery of 40 cells in series, each cell having an average e.m.f. of 1.9V and an internal resistance of 0.0025Ω. If the total resistance of the connecting leads is 1Ω, calculate the value of the charging current (2 decimal places).

4.4. Nickel is to be deposited on the curved surface of a shaft 100mm in diameter and of length 150mm. The thickness of deposit is to be 0.5mm. If the process takes 8h, calculate the current that must flow. The E.C.E. of nickel is 302×10^{-9}kg/C. The density of nickel is 8600kg/m^3 (1 decimal place).

4.5. A nickel–alkaline battery is discharged at a constant current of 6A for 12h at an average terminal voltage of 1.2V. A charging current of 4A for 22h, at an average terminal voltage of 1.5V is required to recharge the battery completely. Calculate the ampere hour and watt hour efficiencies (2 significant figures).

4.6. A battery of 80 lead–acid cells in series is to be charged at a constant rate of 5A from a 230V, D.C. supply. If the voltage per cell varies from 1.8 to 2.4V during the charge, calculate the maximum and minimum values of the required control resistor. If the ampere hour capacity of the cells is 60, state the probable charging time required, assuming that the cells were in a completely discharged condition at the commencement of the charge (nearest hour).

4.7. A metal plate measuring 50mm by 150mm is to be copper plated in 30min. Calculate the current required to deposit a thickness of 0.05mm on each side (ignore the edges). The E.C.E. of copper is 330×10^{-9}kg/C and its density is 8800kg/m^3 (1 decimal place).

4.8. A battery of 40 cells in series delivers a constant discharging current of 4A for 40h, the average P.D. per cell being 1.93V during the process. The battery is then completely recharged by a current of 8A flowing for 24h, the average P.D. per cell being 2.2V. Calculate the ampere hour and the watt hour % efficiencies for the battery (1 decimal place).

4.9. Thirty lead–acid accumulators are to be charged at a constant current of 10A, from a 200V D.C. supply, the e.m.f. per cell at the beginning and end of charge being 1.85V and 2.2V respectively. Calculate the values of the necessary external resistor required at the beginning and end of charge, assuming the resistance of the leads, connections, etc. to be 1Ω and that the internal resistance is 0.01Ω per cell (2 decimal places).

4.10. When a current of 3.5A was passed through a solution of copper sulphate, 4.2g of copper were deposited. If the E.C.E. of copper is 330×10^{-9}kg/C and the chemical equivalent of copper is 31.8, find the time for which the current was passed through the solution (nearest second) and also the mass of hydrogen in grammes liberated (4 decimal places).

5

MAGNETISM – ELECTROMAGNETISM

Gilbert shall live, till Load-stones cease to draw,
Or British Fleets the boundless Ocean awe.

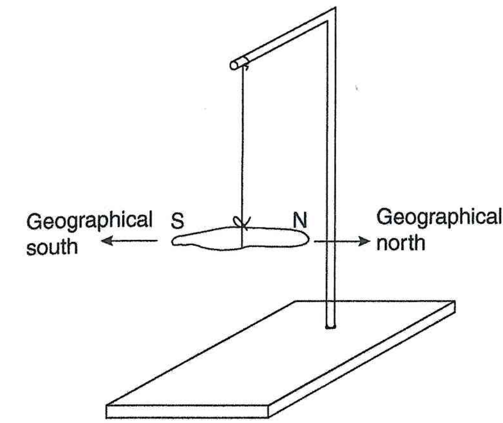
John Dryden

Magnets

Natural magnets

From ancient times it was known to civilisations such as the Greeks and Chinese, that certain types of iron ore have magnetic properties. Pieces of iron ore are able to attract and repel other such pieces but can also pass on their magnetism. Another property of this ore, called magnetite or lodestone, is that if it is freely suspended, as shown (figure 5.1), it will come to rest in a roughly geographical North–South direction. The end pointing north is called a north-seeking or North (N) Pole, while the other end is a South (S) Pole. The ore is a natural magnet and if brought into contact with a quantity of iron filings, the filings will stick mainly to its ends or poles.

Further investigations made with pieces of the magnetic ore show that, if 2 such magnets are each suspended as described above and their polarities determined and marked, when the *N* pole of one suspended magnet is brought near the *N* pole of the



▲ Figure 5.1

other suspended magnet, *repulsion* of poles results. Two *S* poles brought near each other will behave in a similar manner while, a *S* pole brought near the *N* pole of the other magnet produces an *attractive* effect. Thus every magnet is seen to have 2 poles of unlike polarity and that like poles repel while unlike poles attract.

Artificial magnets

A piece of iron can be converted into a magnet and exhibit properties similar to that of the iron ore. Such a piece of iron is then an artificial magnet and is said to be magnetised. A simple method of magnetising iron is to stroke it in one direction from end to end with one pole of an existing magnet. However, the most effective method is by use of electromagnetism.

Certain materials such as copper, aluminium, lead, brass, wood, glass, rubber, etc. *cannot* be magnetised. Thus all materials can be classified under the heading of magnetic or non-magnetic substances. A few metals such as nickel, cobalt and magnesium exhibit *slight* magnetic properties, but when alloyed with iron very strong magnetic properties result.

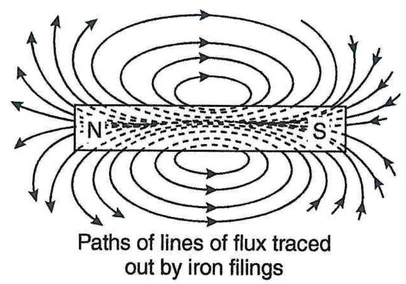
An artificial magnet is usually made in bar or horse-shoe form. When tested, the tips are found to constitute poles of opposite polarity and, if suspended, a bar magnet will again lie on an approximate *N–S* line. The magnetic compass makes use of this principle and consists of a short highly magnetised bar magnet which is pivoted at its centre. A card, calibrated in degrees and/or geographic points, is mounted below and is used with the magnet to obtain a 'bearing'. It is important to realise that the *N–S* direction indicated by such a compass is not exactly *geographic* N and S. The angle between the lines of magnetic and geographic N–S, is called the 'variation' and varies around the world. If the magnetic compass is used, due allowance must be made for the variation, before a map can be properly orientated and used correctly.

It is helpful to explain why a compass needle lies in the *N–S* direction. The earth itself behaves as though it contains a magnet having its *S* pole in the region of the geographic north and its *N* pole near the geographic south. A compass needle placed on the earth's surface will lie so that its *N* pole is attracted to the magnetic *S* (geographic *N*) pole of the earth and its *S* pole will be attracted to the magnetic *N* (geographic *S*). Summarising the facts so far about natural and artificial magnets, every magnet has 2 poles of unlike polarity and *like* poles repel while *unlike* poles attract.

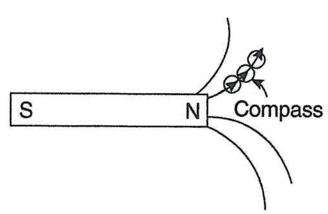
The magnetic field

This is the space around a magnet where its magnetic effects are felt. If a bar magnet is covered by a sheet of paper and iron filings sprinkled onto paper, after tapping the latter, the filings will align as shown (figure 5.2). The filings form a pattern which, if examined closely, shows that lines may be traced from the magnet's *N* pole to the *S* pole through the space outside and from the *S* to *N* poles inside the magnet.

The field can also be plotted using a small compass needle as shown (figure 5.3).



▲ Figure 5.2



▲ Figure 5.3

Field plotting with a compass needle is done as follows. Place the magnet on a sheet of paper and draw its outline. Set the compass needle against the *N* pole of the magnet and, with a pencil, mark a dot at the point in line with and adjacent to the *N* pole of the compass needle. Move the compass until the *S* pole of the needle coincides with the original dot.

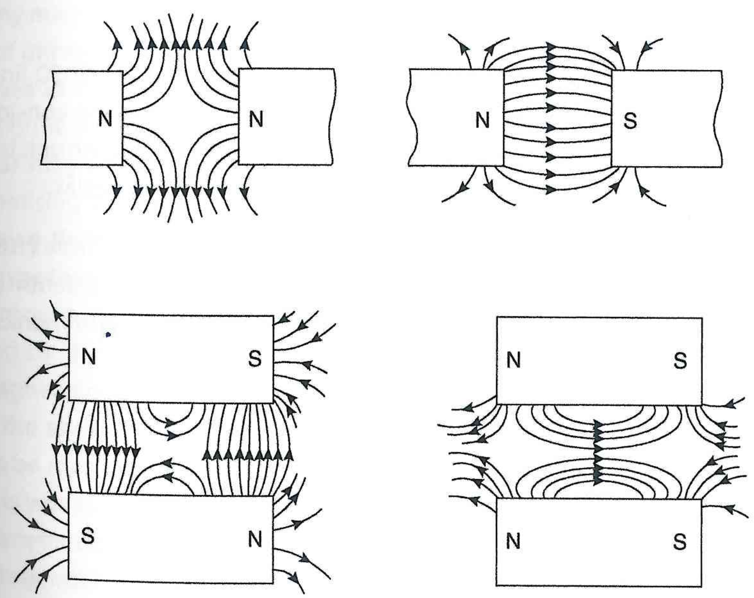
Mark the new point in line with and adjacent to the *N* pole of the needle. Repeat this procedure until the *S* pole of the magnet is reached. Join the dots together to give a *line of force* or a *line of flux* which can be described as the line which, when drawn through any point in a magnetic field, shows the direction of the magnetic force at that point. Using a compass needle the field can be mapped for a considerable distance around a magnet and the following deductions made:

(1) Lines of flux never cross.

(2) Lines of flux are always continuous.

If various magnetic field arrangements are plotted as shown (figure 5.4) then other conclusions can be deduced.

(3) Lines of flux are like stretched 'elastic bands' and will be as short as possible. This explains the attractive effect between 2 unlike magnetic poles, which if free to do so will move into contact, thereby reducing the length of the flux lines.



▲ Figure 5.4

(4) Lines of flux which are parallel and in the same directions repel each other, for example, when 2 magnets are brought together, with like poles adjacent to each other. There is a force of repulsion between the magnets and if the field is plotted between 2 like poles a neutral point is found where the effects of the 2 repulsive forces balance each other and the total effect is as shown by the absence of control on a compass needle placed at this neutral point.

The strength of the magnetic field around a magnet varies from point to point, but before this can be measured and methods devised for making such measurements, a system of magnetic units and terms must be introduced. Faraday conceived the idea of the line of flux, and further suggested the use of these lines to depict the strength of the magnetic field.

If a unit area at right angles to the lines of flux is considered definitions and terms can be made.

A number of lines of flux collectively are said to constitute the magnetic *Flux* (symbol Φ – Greek letter phi) which is passing through the area studied.

Another unit of importance is *Flux Density* – and the value, at any point, is obtained from the expression:

$$\text{Flux density} = \frac{\text{Flux}}{\text{Area}}$$

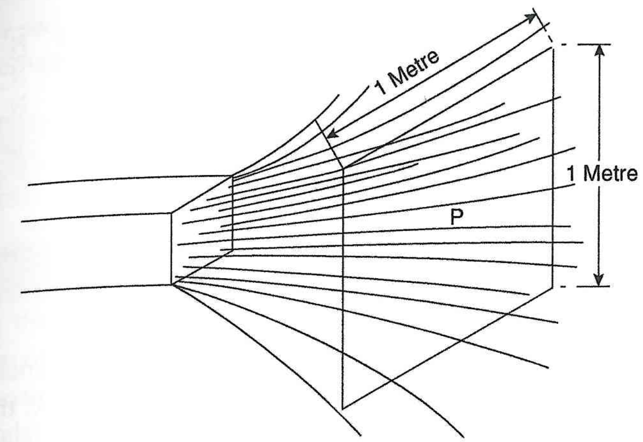
Figure 5.5 illustrates the SI unit of flux or the *Weber*. For example, if 50 lines of flux are shown passing through an area of 1 square metre, for the plane considered, the magnetic flux will be 50 Webers. The symbol for flux density is *B* and the unit is the *Tesla*. Thus for any point P in the plane considered, the flux density is 50 teslas.

Note. The tesla is a name introduced for the SI system after Nikola Tesla (1856–1943), an ethnic Serb. His revolutionary developments in the late nineteenth and early twentieth-century electromagnetism formed the basis of wireless communication and radio. The original unit was the weber per square metre, i.e. Wb/m^2.

We now have Flux = Flux density × Area

or Φ (Webers) = B (teslas) × A (square metres).

This relationship will be used throughout our study of electromagnetism and magnetic circuits and should be considered a basic and important formula. It is useful to emphasise that flux lines do not exist but the properties of magnets and magnetic fields can be assessed by assuming their existence and their having definite physical

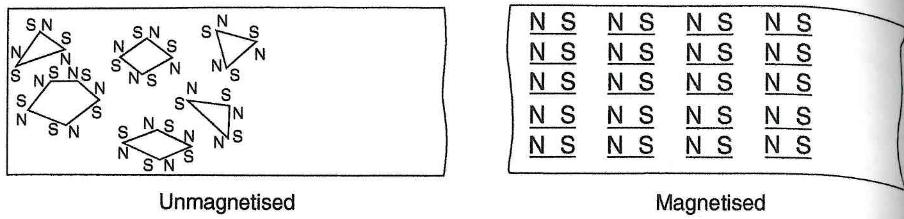


▲ Figure 5.5

properties. It must be remembered that the field of a magnet exists in all directions and is not confined to one plane.

Molecular theory of magnetism

A molecule is defined as the smallest particle of a substance that can exist separately and in any magnetic material every molecule is thought to be a complete magnet. In a piece of unmagnetised magnetic material the molecules are considered to arrange themselves in closed magnetic chains or circuits as shown (figure 5.6). Under this assumption it is considered that each molecular magnet is neutralised by adjacent molecular magnets so that no magnetism is apparent in the material. The process of magnetising a material is thought to be achieved by arranging the molecular magnets so their axes point in the direction of the magnetising force. The proof of this hypothesis is supported by the following observations. (1) There is a limit to the amount of magnetism that can be imparted to any material sample. This is explained by the supposition that, once all the molecules 'lined up', no amount of extra magnetising force can increase the magnet's strength. (2) When a magnet is broken, the ends of the molecular magnets are exposed and the broken pieces are found to be magnets themselves. (3) If heated to about 100°C and allowed to cool a magnet is weakened. If the magnet is heated until 'red hot', the magnetic properties are completely lost. Similarly if a magnetic material like hard steel, is cooled in a strong magnetic field then it will set as a permanent magnet. It is considered that during heating, energy is transferred to the magnet which causes oscillations of the molecular magnets which tend to break the 'lining up' and results in these magnets



▲ Figure 5.6

taking up random directions. Similarly in the cooling process, as energy is passed from the hot material, the oscillations decrease in magnitude and the molecular magnets settle in the direction of the magnetising field.

A modern theory of magnetism is based on electron theory and the concept of the atom such that an electron, the smallest known –ve charge, when rotating in an elliptical path, constitutes a circular current which sets up a magnetic force along the axis of gyration. In a molecule the magnetic effects of the electrons of the atoms may neutralise each other giving little resultant effect. A spinning electron also sets up a magnetic field along its spin axis. If the fields due to the effects of spin balance out, due to electrons spinning in opposite directions, the material is non-magnetic. A magnetic material is the result of the fields not balancing out, but to explain the overall apparent effect, it is thought that rather than single atoms or molecules being concerned, it is a group of molecules which act together. Such a group is called a 'domain' and is considered to function like the more molecular magnet already described.

Electromagnetism

Earlier theory has referred to an association between magnetism and electricity and this was more specifically mentioned in Chapter 2 when electrical units were defined. The discovery of a relation between an electric current and magnetism was made in 1820 by the Danish scientist Oersted (1777–1851), when he noticed that a wire arranged above and parallel to a compass needle caused deflection of the latter when a current was passed through the wire. Reversal of the current caused reversal of the deflection. Further experiments on the shape, direction and strength of magnetic fields associated with current-carrying conductors arranged in the form of loops and solenoids were the subject of much work by famous scientists such as Faraday, Maxwell and Gilbert. The result of their discoveries led to the deduction of certain fundamental relationships

which are now part of accepted basic theory. The shape of the magnetic fields due to simple arrangements of current-carrying conductors will now be considered.

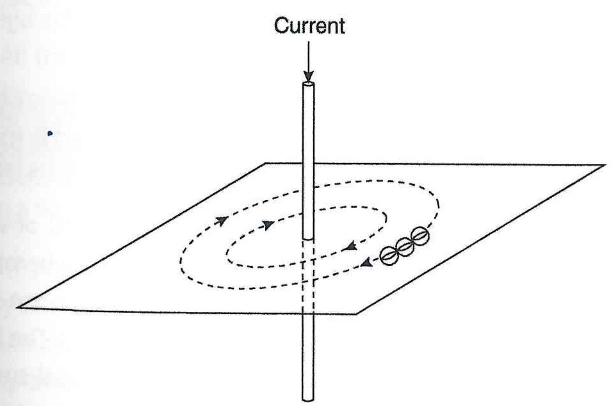
Field due to long straight current-carrying conductor

The field associated with such a conductor may be determined with iron filings or a compass needle as described earlier in the magnetism section – figures 5.2 and 5.3. Assuming the current is kept constant during such a test, a field consisting of concentric lines of flux is confirmed. Figure 5.7 shows a vertical wire passing through a sheet of cardboard. The directions of the current and lines should be noted as this is a fundamental relationship.

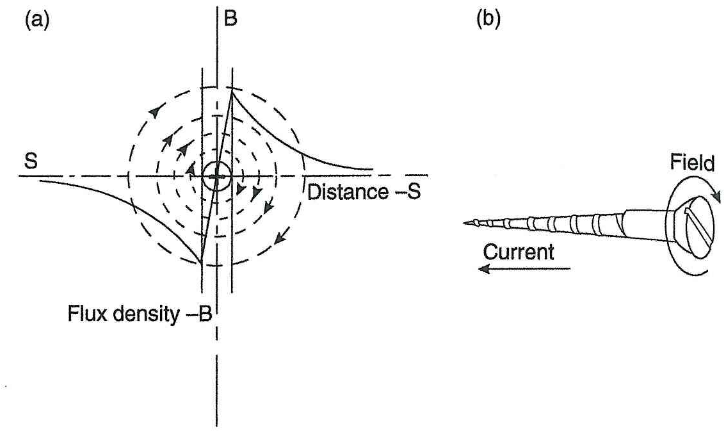
Further tests show that if the current is reversed, the field will reverse and if the strength of the field is measured with a sensitive instrument, the results will give a graph as illustrated (figure 5.8a), which shows flux density (*B*) plotted to a distance (*s*) from the centre of the conductor.

It is seen that inside the circular conductor, the strength of field or flux density varies from zero at the centre to a maximum on the circumference. Outside the wire flux density varies inversely as the distance from it.

Figures 5.8a and 5.8b, use the conventional method of indicating current direction. Consider an arrow, i.e. current entering the surface of the paper and receding from the viewer, with the feathered end seen as a cross. Similarly current flow towards the viewer is shown with the tip of the arrow, i.e. a point or dot. The relation between the direction of the lines of flux and the current is summarised by Maxwell's *Right-Hand*



▲ Figure 5.7



▲ Figure 5.8

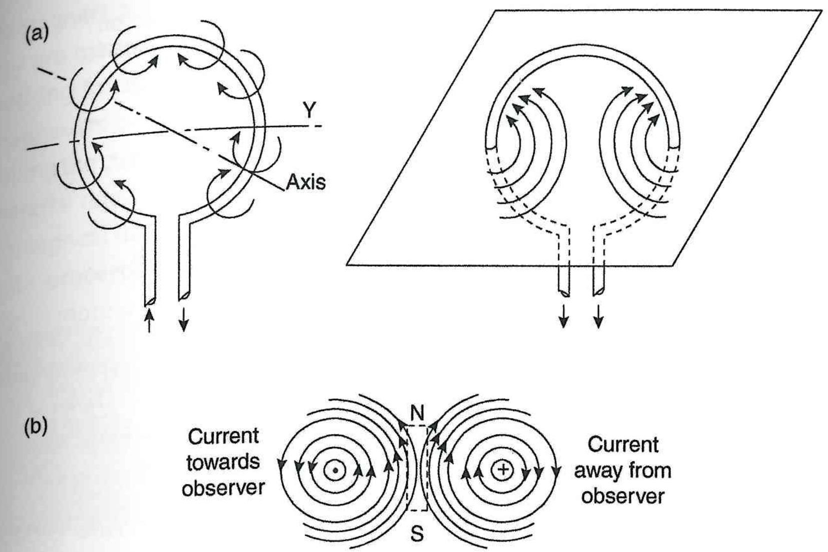
Screw Rule. This depicts that if current flows in the direction in which a right-handed screw moves forward when turned clockwise, then the resulting field will be in the direction of turning the screw. If the current is reversed, the screw will unscrew and the field reversed, or the direction of turning the screw is reversed, i.e. anticlockwise.

Field due to a current-carrying conductor bent to form a single loop

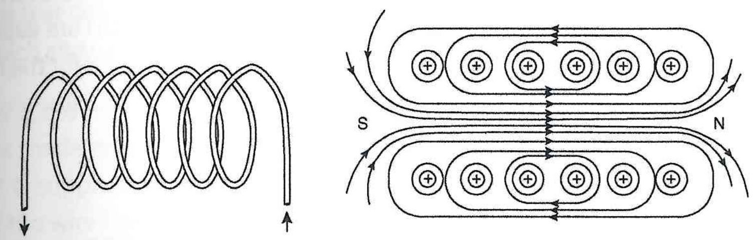
The diagrams (figures 5.9a and 5.9b) show the loop, the current and the lines of flux encircling the conductor as deduced from condition (1) above. The resulting field can be plotted by locating the loop in a sheet of cardboard as shown. The result is considered as the field taken through section XY of the loop and the similarity with the field of a short bar magnet will be noted. The loop is considered to set up a magnetic polarity determined from first principles.

Field due to a current-carrying conductor wound as a solenoid

The next step in electromagnetic field investigations is for a coil of wire, which is a collection of several loops. A *solenoid* is a form of a multi-turn coil where the axial length is much greater than its diameter. Turns of wire are wound in an open spiral or placed close together so that they touch, provided insulated wire is used. The insulation most commonly used is either a synthetic enamel or a fibrous material such as cotton or silk in the form of thread, tape or braid. The turns of a solenoid are arranged in several layers provided the current travels through the turns in the same direction. When the



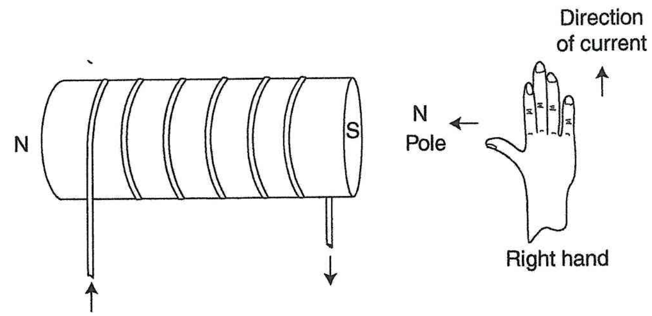
▲ Figure 5.9



▲ Figure 5.10

field is investigated by plotting with a compass, it is found to be as in the diagram (figure 5.10). All the turns tend to produce a magnetic field in the same direction, so that this can be deduced by considering the field of a single turn or loop. The turns unite to send a straight field up the centre which comes out at the ends, opens and spreads out to return at the other end, giving the same distribution of lines of flux as obtained from a bar magnet.

Again a definite polarity is attributed to the solenoid when carrying current. Polarity is determined by finding the direction of the lines for any one turn by applying the right-hand screw rule but additional aids are useful, the easiest of which is the *right-hand rule*. This is explained as follows, and is shown in the diagram (figure 5.11). Place the right hand on the coil with the fingers pointing in the direction in which current flows. Then the thumb will point in the direction of the *N* pole.

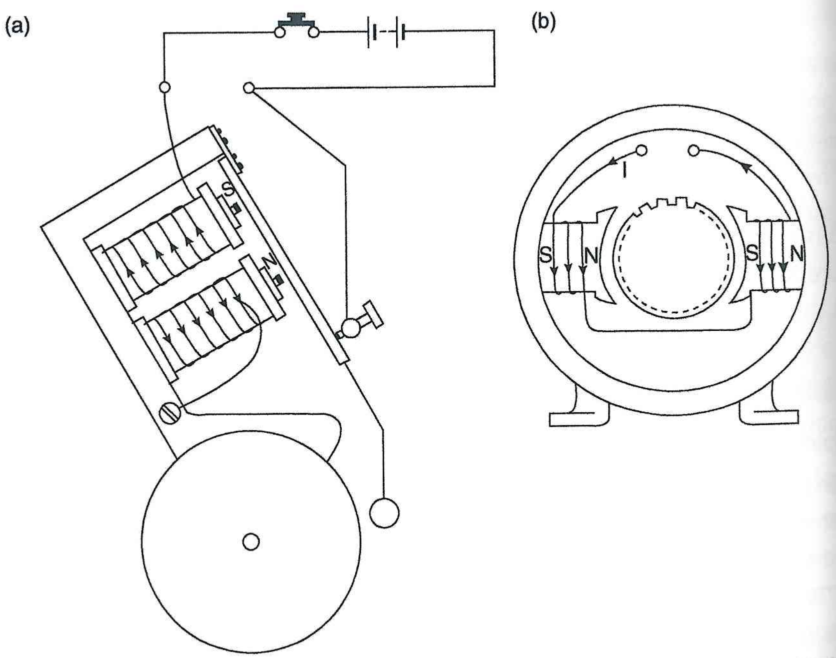


▲ Figure 5.11

Introduction of an Iron Core

The iron core of a solenoid, strengthens the field by concentrating flux and better defining the poles. A magnetic core allows the passage of flux more readily than air. Experiment shows that the best flux path is where the whole of the magnetic circuit is formed from magnetic material. Where this is not practical the air gaps or air paths are kept as short as possible and good examples are found in the electromagnetic paths for the flux in the electric bell and the electric motor or generator (figures 5.12a and 5.12b).

(a) (b)



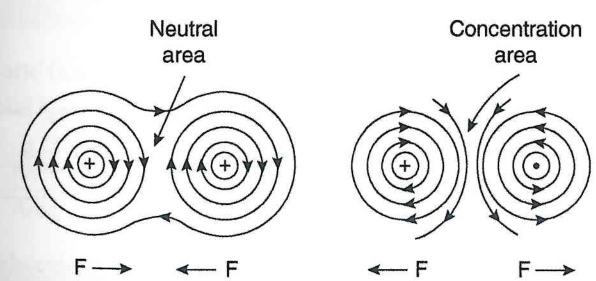
Electromagnets are preferred to permanent magnets in industry for 2 main reasons. (1) They are made more powerful than permanent magnets by providing the desired magnetising force, i.e. solenoid coils with sufficient turns and energising current. (2) The magnetism is controlled, i.e. it can be switched on and off or varied gradually by controlling the current. In summary permanent magnets are made of *hard steel* because this material retains its magnetism and the material is said to have a high 'retentivity'. Electromagnets have a core of *soft iron* which is more readily magnetised but loses its magnetic properties more quickly. The material is said to have a high 'susceptibility' and soft iron is more susceptible than steel.

Force on a current-carrying conductor in a magnetic field

Oersted's experiment with a compass needle and current-carrying wire show that, a force is produced when a current is switched on, bringing about a deflection of the needle. Similarly if a needle was fixed and the wire sufficiently flexible, wire movement will be noted when a current is switched on. Further investigations led to an accepted rule – that a force acts upon a conductor when it is carrying current and situated in a magnetic field provided it is at right angles to the lines of flux. Let us now consider the electromagnetic effects which allow the ampere to be defined as a fundamental unit of the SI system.

In Chapter 2, the phenomena leading to definition of the ampere were mentioned and the points made previously are revisited here in the light of electromagnetic theory. If a circuit is supplied through 2 wires laid together side by side, then if the current is large and the wire flexible, a mechanical effect is noted, especially when the current is switched on and off, as the wires will be seen to move. This action is explained with our knowledge of the field associated with a long straight conductor.

Consider the diagram (figure 5.13), which shows 2 conductors carrying current as shown. When the current in both conductors is in the same direction the resultant magnetic field is such as to enclose both conductors. If the current in each conductor is of the same magnitude then, by Maxwell's right-hand screw rule, the fields between



▲ Figure 5.13

the wires will cancel and the outside lines of flux unite to make a field which encircles both conductors. If flux lines are likened to elastic threads then the lines of flux are stretched and forces act to move the conductors together.

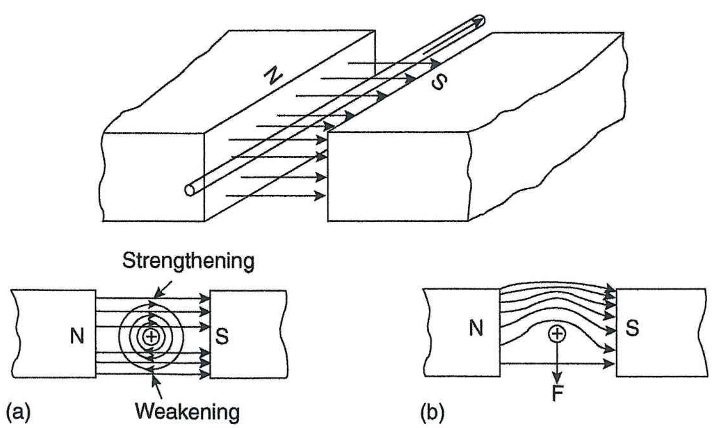
Lines of force through air will keep to the shortest possible paths. Figure 5.13 shows currents in opposite directions in the 2 parallel conductors. Here the resultant flux is concentrated between the conductors, forcing them apart.

The ampere

This is defined in accordance with the electromagnetic effects described and is the accepted definition for the unit of current. Thus the ampere is that value of current which, when flowing in each of 2 infinitely long parallel conductors, situated in a vacuum and spaced 1 metre between centres, causes each conductor to have acting on it a force of 2×10^{-7} newton per metre length of conductor.

Magnitude of force (on a current-carrying conductor in a magnetic field)

Figure 5.14 shows a conductor situated in and at right angles to a magnetic field. Assume that the conductor carries current in the direction shown and that the arrangement is illustrated (a) and (b). There is a magnetic field due to the current which interacts with the main field, which distorts it so that a strong field exists on one side of the conductor and a weak field on the other. Lines of flux first appear to stretch and then return to their shortest length, while a force is exerted on the conductor pushing it out of the way. This action forms the basis of operation of the electric motor and the



▲ Figure 5.14

reader should pay attention to the points made here. It can be shown that the force acting on the conductor varies directly with (1) the strength of the magnetic field, (2) the strength of the current in the conductor and (3) the length of the conductor in the magnetic field.

Summarising: Force ∝ Strength of field × Current in conductor × length of conductor in the magnetic field or $F \propto BI\ell$ where F is the force on the conductor in newtons, B is the flux density in teslas, I is the current in amperes, ℓ is the length of the conductor in the field in metres.

The above relationship is converted to the expression $F = BI\ell$ if the correct unit of flux density is chosen for this equality. This unit is the tesla and is defined below, giving the important formula:

$$F(\text{newtons}) = B(\text{teslas}) \times I(\text{amperes}) \times \ell(\text{metres}).$$

It is noted from now on that the expression *flux density* will be used in preference to the strength of magnetic field. This is because if lines of flux depict a magnetic field, the magnetic field strength is represented by the density of the lines. Lines well spaced apart create a weak field, while a strong field is represented by lines closely packed together. Field strength is measured by the *density* of these lines or by the flux density defined in SI units.

Unit of flux density

This unit is defined in accordance with the relationship $F = BI\ell$ because the units for F, I and ℓ are known. Thus B is defined in terms of the other 3 factors and the unit of flux density or the tesla is the density of magnetic field such that a conductor carrying 1ampere at right angles to the field experiences a force of 1 newton/metre length acting on it.

Unit of flux

The terms flux and flux density were introduced earlier with flux density determined by dividing the total flux by the area through which it passed. So:

$$\text{Flux density} = \frac{\text{Flux}}{\text{Area}}$$

Hence Flux = Flux density × Area. Using our definition for flux density, it follows that the weber is the unit of flux and it is the flux within an area of 1 square metre where the flux density has a value of 1 tesla. A more complex definition of the weber will follow in Chapter 6, emphasising the importance of this unit in the study of electromagnetism.

For the correct use of Φ (webers) = B (teslas) × A (square metres) see the example below.

Example 5.1. If the flux density inside a solenoid coil is measured to be 140mT and the inside diameter of a solenoid is 40mm, find the value of the total flux produced (4 significant figures).

Note. The main purpose of this example is to stress the importance of correct substitution in the formula, with attention given to the correct numerical magnitude. Thus 140mT is 140 milli teslas, or 140×10^{-3}T. Similarly 40mm must be converted to metres before substitution.

$$\text{Thus } A = \frac{\pi d^2}{4} = \frac{3.14 \times 40^2 \times 10^{-6}}{4}$$

$$= 12.56 \times 10^{-4}\ \text{m}^2$$

$$\Phi = 140 \times 10^{-3} \times 12.56 \times 10^{-4}$$

$$= 175.8 \times 10^{-6}\ \text{Wb}$$

$$\text{or } \Phi = 175.8\mu\text{Wb}.$$

Example 5.2. Find the force exerted on a conductor 160mm long when carrying 125A and placed at right angles to the lines of flux of a magnetic field of flux density 4×10^{-3} teslas (2 decimal places).

$$\text{Substituting in } F = BI\ell \text{ we have } F \text{ (newtons)} = 4 \times 10^{-3} \times 125 \times 160 \times 10^{-3}$$

$$= 0.08\text{N}$$

The force will be 0.08 newtons or 0.08N.

The Magnetic Circuit

Magnetising force, magnetic field strength or magnetic field intensity

As a magnetic field is produced by a coil of wire carrying a current, we must deduce a relationship which correlates the flux density at any point with the electromagnetic effort required to produce it. To allow this derivation, the electromagnetic effort is defined as the *magnetising force*, *magnetic field strength* or *magnetic field intensity* (symbol H) and measured in terms of the factors producing it: the current and number

Ampere-turns (symbol IN) of the coil acting over 1 metre length of the flux path and is considered to cause a flux density of B teslas. IN/ℓ ampere-turns/metre length of the magnetic circuit is a measure of H, but in SI units, it is considered that the same value of H is created by a current of IN amperes passing through 1 turn so the numerical value of ampere-turns may be replaced with a current A as an alternative to At. Thus H is measured in At/m or in A/m. In this book the original and common method is used, i.e. 200 ampere-turns, for example, appear as 200At rather than 200A. In line with this duality, a magnetising force value should be read as ampere-turns per metre although it may be given as amperes per metre. Summarising:

$$H \text{ (magnetising force)} = \frac{IN}{\ell} \text{ (ampere-turns/metre or amperes/metre).}$$

A magnetic circuit is taken as the complete length of the path through which the flux produced by the coil passes. We are concerned, in practical engineering or physics, with the flux path of machines and electromagnetic devices, so let us take a simple path for the simple 2-pole generator or motor shown in figure 5.12b.

We treat the field coils as the energising ampere-turns, spread over the poles of the machine and wound to produce a continuous solenoid effect. This practical arrangement gives a more symmetrical layout. Each coil has 2000 turns of thin wire and a coil current of 1.5 amperes. The *total* magnetising force producing the flux for this machine will be (2000 × 2) turns × 1.5 amperes or 6000 ampere-turns. By symmetry the flux through the poles and armature splits (figure 5.12b) and returns through both halves of the yoke of the machine.

So far it is noted that we consider a flux density to exist at a point by virtue of the magnetising force producing it. An analogy can be made with the electrical circuit and allows a clearer understanding of the magnetic circuit and associated problems.

Magnetomotive force or m.m.f.

A complete path is followed by a group of lines of magnetic flux and this path is the magnetic circuit. In an electric circuit current is due to an e.m.f. and in a magnetic circuit, flux is thought to be due to a *magnetomotive force* (*m.m.f.*) (symbol F) caused by current flowing through a coil of wire. Thus the *m.m.f.* is the total magnetising force produced by a solenoid coil and measured in ampere-turns (IN). From now on, the terms: magnetising force, magnetic field strength or magnetic field intensity are used for the force or m.m.f. acting over 1 metre length of a circuit and that the total force for

$$\text{or } H = \frac{F}{\ell} = \frac{IN}{\ell}$$

The passage of flux through a magnetic circuit is restricted by the circuit's *reluctance*. Reluctance (symbol *S*) is comparable with the resistance in an electrical circuit and is proportional to the length of the magnetic circuit, and inversely proportional to the area and the absolute *permeability* (symbol μ).

Table 5.1

Electric Circuit		**Magnetic Circuit**	
Quantity	Unit	Quantity	Unit
e.m.f. (*E*)	Volt	m.m.f. (*F*)	Ampere-turn
Current (*I*)	Ampere	Flux (Φ)	Weber
Resistance (*R*)	Ohm	Reluctance (*S*)	Amp-turn/ Weber or A/Wb
Also		Also	
$I = \frac{E}{R}$		$\Phi = \frac{F}{S}$	
Other Comparisons are:			
$R = \frac{\varrho l}{A}$		$S = \frac{l}{\mu A}$	
Electric force (*E*)	Volts/metre	Magnetising force (*H*)	Amp-turns/metre
$= \frac{V}{d}$		$= \frac{F}{\ell}$	
Current density (*J*)	Ampere/metre^2	Flux density (*B*)	Tesla
$= \frac{I}{A}$		$= \frac{\Phi}{A}$	

The above concept of a magnetic circuit allows formulae to be found, for the magnetising force in the fields of various current-carrying conductor arrangements, such as the long straight conductor, single loop and multi-turn coils like solenoids and toroids.

Permeability

We now can say that a magnetising force (*H*) produces a flux density (*B*), the magnitude of this flux density depends upon the type of material in the magnetic circuit (e.g. air, steel, soft iron, etc.). For any material the ratio of flux density to magnetising force is called the absolute permeability (μ) and measured in Henries or Henrys per metre (H/m), named after Joseph Henry (1797–1878).

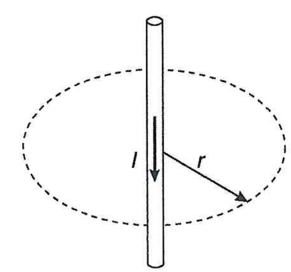
$$\text{Thus: } \mu = \frac{B}{H}$$

Permeability of free space (μ_0)

In vacuum and most non-magnetic materials the ratio between B and H is a constant. This can be shown by considering a long straight current-carrying conductor in a vacuum. Consider the diagram (figure 5.15) with a conductor of infinite length carrying a current of 1A.

The conductor forming the return path to the supply source is considered to be an infinite distance away so its current will not affect the magnetic field near the conductor.

The conductor arrangement constitutes a single turn and the m.m.f. F is then 1 turn × I amperes or $F = I$ ampere-turns. Consider any point on a line of flux distant r metres from the centre of the conductor. The magnetising force H at this point will be the m.m.f./metre length of flux or $H = \frac{F}{\ell} = \frac{F}{2\pi r}$ So



▲ Figure 5.15

$H = \frac{I}{2\pi r}$ ampere-turns/metre or amperes/metre, where ℓ is the circumference for a radius r.

This result helps us to find the flux density for a certain magnetising force and permeability (μ) of the medium in which the field is established.

Consider figure 5.16, the plan view of our previous diagram.

The conductor, in vacuum, is represented by A carrying a current of 1 ampere flowing *away* from the observer. The magnetising force, at any point P 1 metre from A is given by

$H = \frac{1}{2\pi}$ ampere-turns/metre as both I and r are unity in the formula derived earlier.

Next assume the flux density at point P is B tesla. Then (1) the force on a metre length of conductor placed at P, parallel to A and carrying a current of 1 ampere will be 2×10^{-7} newtons. This is known from the definition of the ampere. Also (2) the force on a metre length of this conductor is given by $BI\ell$ newtons, or is

$$B \text{ (teslas)} \times 1 \text{ (ampere)} \times 1 \text{ (metre)} = B \text{ (newtons)}.$$

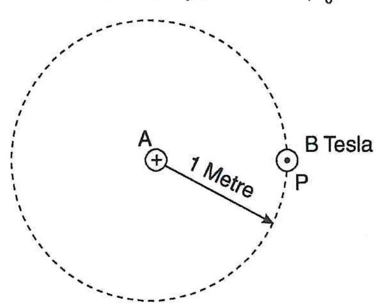
Thus equating expressions (1) and (2) for the force on the conductor we see that the value of B for the condition considered will be 2×10^{-7} newtons.

Hence:

$$\mu = \frac{\text{Flux density at point P}}{\text{Magnetising force at point P}} = \frac{B}{H} = \frac{2 \times 10^{-7}}{1/2\pi} \quad \therefore \ \mu = 4\pi \times 10^{-7}$$

In this case we consider a vacuum as the medium in which the field is established so:

Permeability of free space = $4\pi \times 10^{-7}$Henry/metre or $\mu_0 = 4\pi \times 10^{-7}$ H/m.



▲ Figure 5.16

Example 5.3. We must produce a flux of 0.018Wb across an air gap 2.54mm long with effective area 24×10^{-3}m². Find the ampere-turns required (3 significant figures).

Area of gap = 24×10^{-3} square metres

Required flux density $B = \frac{0.018}{24 \times 10^{-3}} = 0.75$T

Also $H = \frac{B}{\mu_0} = \frac{0.75}{4 \times \pi \times 10^{-7}}$

$= 59.7 \times 10^4$At/m

The length of the air gap = 2.54mm = 0.254×10^{-2}m.

So total ampere-turns needed = $59.7 \times 10^4 \times 0.254 \times 10^{-2} = 1515$At.

Example 5.4. A wooden ring with a mean diameter of 200mm and a cross-sectional area of 400mm² is wound uniformly with a coil of 300 turns. If the current passed through the coil is 5A calculate the value of flux produced in the coil (2 significant figures). The m.m.f. of the coil = $5 \times 300 = 1500$At.

The mean circumference = π D = $\pi \times 200$ = 628mm = 0.628m

The magnetising force H = At/m = $\frac{1500}{0.628}$ = 2380 At/m

The flux density $B = \mu_0 H$

$= 4 \times \pi \times 10^{-7} \times 2380$

$= 0.003$T

Total flux $\Phi = BA$

$= 0.003 \times 400 \times 10^{-6}$ Wb

$= 1.2\mu$Wb.

Example 5.5. The magnet system of a moving-coil instrument provides a flux density in the air gap of 0.25T. The moving coil, of 120 turns, is carried on a former of (active side) length 25mm and width 18mm (between air-gap centres). If the coil carries a current of 2mA, calculate the turning moment on it (2 significant figures).

$F = BI\ell$ newtons

$= 0.25 \times 2 \times 10^{-3} \times 120 \times 2 \times 25 \times 10^{-3}$

$= 3 \times 10^{-3}$ N

Torque = $F \times$ radius of coil

$= 3 \times 10^{-3} \times 9 \times 10^{-3}$

= 27µN m.

Practice Examples

5.1. A conductor carrying a current of 100A is situated in and lying at right angles to a magnetic field having a flux density of 0.25T. Calculate the force in newtons/metre length exerted on the conductor (2 significant figures).

5.2. A coil of 250 turns is wound uniformly over a wooden ring of mean circumference 500mm and uniform cross-sectional area of 400mm². If the current passed through the coil is 4A find (a) the magnetising force (1 significant figure) and (b) the total flux (3 decimal places).

5.3. A current of 1A is passed through a solenoid coil, wound with 3200 turns of wire. If the dimensions of the air core are length 800mm, diameter 20mm, find the value of the flux produced inside the coil (exactly in Webers).

5.4. Two long parallel bus bars, each carry 2000A and are spaced 0.8m apart between centres. Calculate the force per metre acting on the conductors (1 significant figure).

5.5. A moving-coil permanent-magnet instrument has a resistance of 10Ω and the flux density in the gap is 0.1T. The coil has 100 turns of wire, is of mean width 30mm and the axial length of the magnetic field is 25mm. If a P.D. of 50mV is required for f.s.d., calculate the controlling torque exerted by the spring (3 significant figures).

5.6. An air gap of length 3mm is cut in the iron magnetic circuit of a measuring device. If a flux of 0.05Wb is required in the air gap, which has an area of 650mm², find the ampere-turns required for the air gap to produce the necessary flux (4 significant figures).

5.7. A straight horizontal wire carries a steady current of 150A and is situated in a uniform magnetic field of 0.6T acting vertically downwards. Determine the magnitude of the force per metre length and the direction in which it acts (1 significant figure).

5.8. An armature conductor has an effective length of 400mm and carries a current of 25A. Assuming that the average flux density in the air gap under the poles is 0.5T, calculate the force in newtons exerted on the conductor (1 significant figure).

5.9. In an electric motor the armature has 800 conductors each carrying a current of 8A. The average flux density of the magnetic field is 0.6T. The armature core has an effective length of 250mm and all conductors may be taken as lying on an effective diameter of 200mm. Determine the torque and mechanical power developed when the armature is revolving at 1000 rev/min (4 significant figures).

5.10. Two long straight parallel bus bars have their centres 25mm apart. If each carries current of 250A, calculate the mutual force/metre run (1 decimal place).

6

ELECTROMAGNETIC CIRCUITS

Something is as little explained … as the attraction between iron and magnet is explained by means of the name magnetism.

Jacob Schlelden

Magnetising Force

In Chapter 5 the fundamental concepts, terminology and relationships of an electromagnetic circuit were introduced and developed. Before proceeding to consider further the effects of ferromagnetic materials it is useful to revise some of these basic relationships.

The m.m.f. F is the force which causes magnetic flux Φ to be created in a magnetic circuit with reluctance S:

$$\text{i.e. } \Phi = \frac{F}{S} \text{ Wb.}$$

The m.m.f. is usually created by passing a current through a number of coil turns:

$$\text{i.e. } F = IN \text{ ampere-turns.}$$

Reluctance S depends upon the dimensions of the magnetic circuit and its permeability:

$$\text{i.e. } S = \frac{1}{\mu A} \text{ At/Wb.}$$

Flux density B is a measure of the magnetic flux Φ in a given area A:

$$B = \frac{\Phi}{A} \text{ Tesla.}$$

The magnetising force (magnetic field strength) is a measure of the m.m.f. per metre length of magnetic circuit required to maintain flux in that circuit:

$$H = \frac{IN}{\ell} \text{ At/m.}$$

Permeability is the ratio of flux density to the magnetising force producing it:

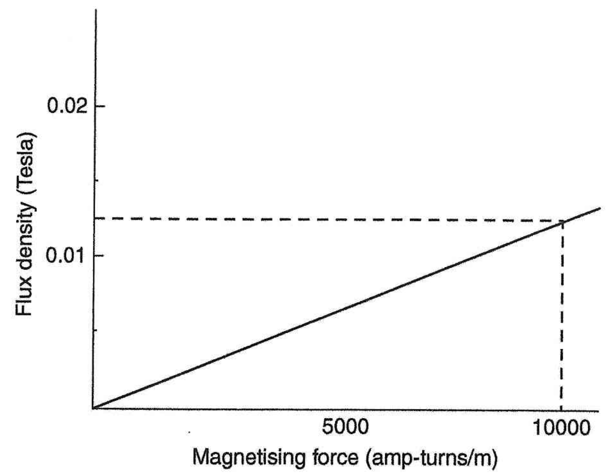
$$\mu = \frac{B}{H} \text{ H/m.}$$

For air, vacuum and most magnetic materials we use the permeability of free space μ_0, with a constant value:

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m,}$$

which for air $\mu_0 = \frac{B}{H}$ such that $B \propto H$.

If values of B against H are plotted for air a straight-line graph is obtained (figure 6.1). If measurements of flux density B are made, at a point outside, but near to, a long straight current-carrying conductor, for various values of magnetising force H, by changing the current (noting that $H = \frac{1}{2\pi r}$ where r is the radius from the point to the centre of the conductor), then the straight-line B/H relationship will be confirmed.



▲ Figure 6.1

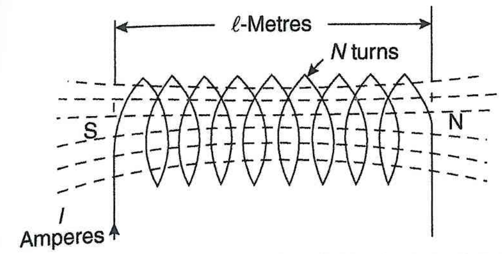
Magnetising force due to a long, straight current-carrying conductor

The magnetising force outside, and near to, a current-carrying conductor, is given by $H = \frac{1}{2\pi r}$. This expression was deduced in Chapter 5 and it should be remembered that, H is the m.m.f./metre length. M.m.f., F is measured in ampere-turns and the total m.m.f. for any magnetic circuit outside the conductor is found from $F = H\ell$.

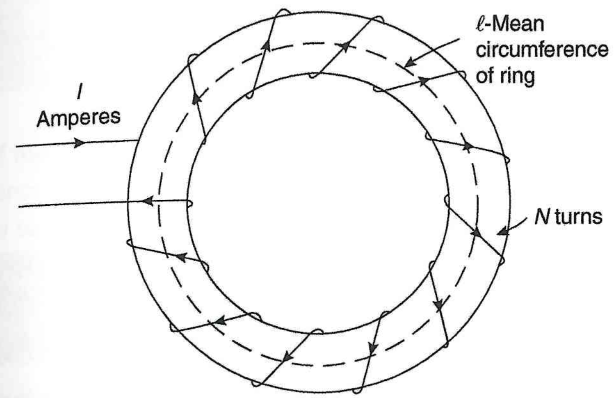
Magnetising force inside a solenoid

If a parallel field of flux lines is assumed inside a solenoid as illustrated (figure 6.2), its length can be taken as ℓ metres, the number of turns on the coil as N and the current passed as ℓ amperes. Lines of flux will spread out at the ends and for their return path they also spread out into space. This external return path has negligible magnetic reluctance and the whole m.m.f. of the coil 'sets up' the field inside the solenoid. Thus the m.m.f. per unit length is, by definition, H – the magnetising force.

$$\text{Thus } H = \frac{F}{\ell} = \frac{IN}{\ell} \text{ ampere-turns per metre.}$$



▲ Figure 6.2



▲ Figure 6.3

Magnetising force inside a toroid

Figure 6.3 shows a simple electromagnetic arrangement. It consists of a solenoid bent back upon itself so that the lines of flux are confined inside the coil. We consider a non-magnetic ring (or toroid) wound uniformly with a coil of N turns, carrying a current of ℓ amperes. The mean circumference is ℓ metres and as the flux is confined *inside* and the path is uniform, the magnetising force or m.m.f. per unit length is given by:

$$H = \frac{IN}{\ell} \text{ ampere-turns per metre.}$$

Example 6.1. A wooden ring with a mean circumference of 300mm and a uniform cross-sectional area of 400mm^2 is wound uniformly with 300 turns of insulated wire. If the current is 3A, calculate (a) the magnetising force (1 significant figure), (b) the flux density inside the toroid (4 significant figures) and (c) the total flux produced (2 significant figures).

(a) The total m.m.f. produced $F = 3 \times 300 = 900\text{At}$

The mean circumference is 300mm = 0.3 metres

$$\therefore \text{The magnetising force } H = \frac{F}{\ell} = \frac{900}{0.3}$$

$$= 3000\text{At/m}$$

(b) The flux density is given by $B = \mu_0 H$

$$= 4\pi \times 10^{-7} \times 3000 = 3.768 \times 10^{-3}\text{T}$$

$$= 3.768\text{mT}$$

(c) The total flux produced $\Phi = B \times A$

$$= 3.768 \times 10^{-3} \times 400 \times 10^{-6}\text{ Wb}$$

$$= 1.5 \times 10^{-6}\text{ Wb}$$

$$\text{or } \Phi = 1.5\mu\text{Wb}$$

Ferromagnetism

When iron is used as the core of an electromagnet, the field is *intensified* so that a greater flux than expected results from the magnetising ampere-turns of the energising coil.

As the only change in the relation $\Phi = \frac{F}{S}$ is due to the reluctance S, if the dimensions of the core l and A are kept the same as for the air path, it follows that the permeability of iron must be much greater than that of air. Thus we can refer to the permeability of a magnetic material, termed the *relative permeability.*

Relative permeability (μ_r)

This is the ratio of the flux density produced in a magnetic material to the flux density produced in air by the same m.m.f.

$$\therefore \text{Relative permeability} = \frac{\text{Absolute permeability}}{\text{Permeability of free space}}$$

$$\mu_r = \frac{\mu}{\mu_0}$$

$$\mu = \mu_0 \mu_r = \frac{B}{H}$$

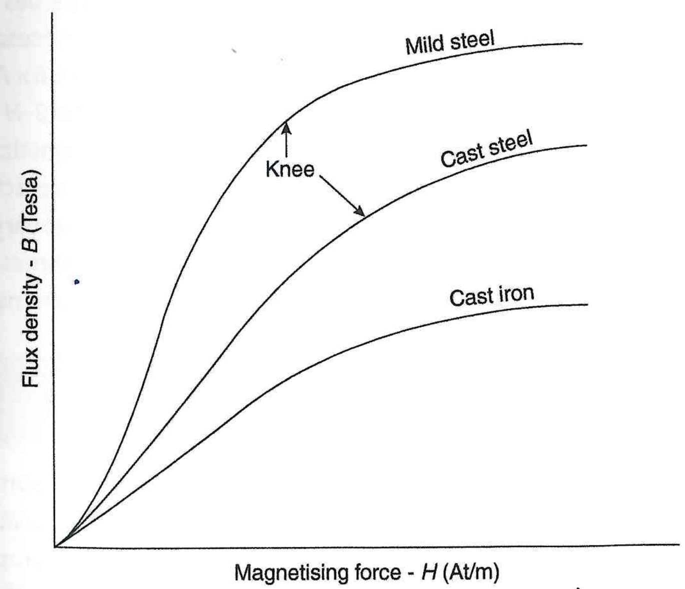
$$\therefore \text{B} = \mu_0 \mu_r \text{H}.$$

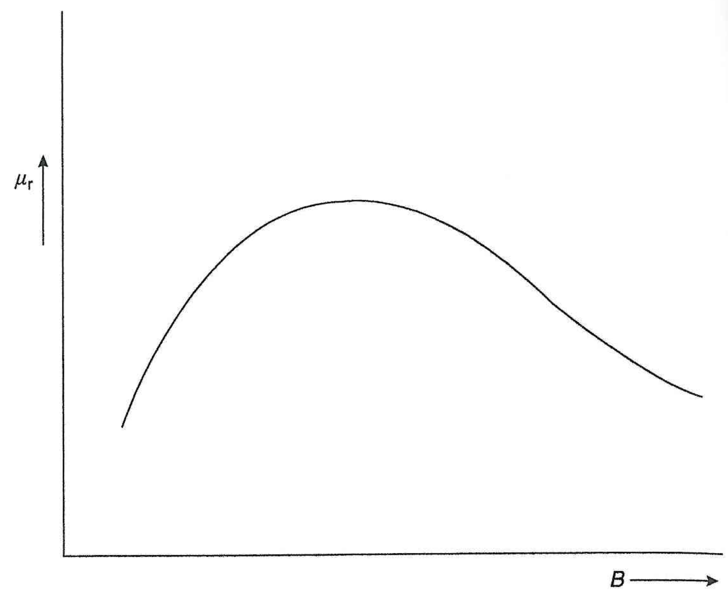
For materials such as iron, nickel, cobalt, etc., μ_r can be between 1000 and 2000 or even higher for some special electrical steels.

The *B–H* or Magnetisation Curve

If a specimen of magnetic material is made into a ring and wound with an energising coil, measurements of flux density for various values of magnetising force can be made by winding on a secondary coil and using the *principle of transformer action*. This is an accepted industrial method for determining the magnetic properties of various materials. It is observed that if flux density B is plotted against magnetising force H for air, a straight line is obtained, but for magnetic materials, typical curves result (figure 6.4).

At first all the graphs are approximately straight lines, with B proportional to H. Then the curves begin to flatten out forming a 'knee' and finally become horizontal, exhibiting little increase in B for a large increase in H. In this state, the material is said to 'saturate'.





▲ **Figure 6.5**

If permeability (μ_r) is plotted against *B*, a curve (figure 6.5) will result. The permeability curve has a peak corresponding close to the 'knee' on the *B–H* curve where the tangent goes through the origin. Beyond this peak, permeability drops off fairly rapidly.

An examination of the *B–H* and μ_r–*B* curves shows how the properties of various magnetic materials differ. For machine design, lower working *B* values necessitate larger section and therefore greater mass to obtain a required flux value as $\Phi = B \times A$. The effect of high permeability materials is also apparent and the shape of the *B–H* curve with the saturation effect shows the limits of machine field systems. Magnetic properties depend on the actual composition, for example, manganese-steel is practically non-magnetic, but small quantities of carbon or silicon when added to steel vary the shape of the *B–H* curve. Sheets of commercial steel marketed under trade names like Stalloy or Lohys (a transformer iron) are available to suit different design requirements.

Reluctance (symbol *S*)

This term is likened to the resistance of an electrical circuit. As flux is proportional to the m.m.f. and is restricted by the reluctance, further investigation shows that reluctance is proportional to the length ℓ of the magnetic circuit and inversely proportional to its area *A*. Furthermore it is inversely proportional to permeability, as the greater a

material's permeability the greater its flux and hence the smaller its reluctance. We thus write, $S = \frac{\ell}{\mu A}$ and with absolute permeability $(\mu = \mu_0 \mu_r)$ $S = \frac{\ell}{\mu_0 \mu_r A}$.

Calculations on magnetic circuits with magnetic materials are now possible, but unlike electrical circuit calculations which use $I = \frac{V}{R}$, it is not always necessary to use the comparable relationship of $\Phi = \frac{F}{S}$. The solution of most problems associated with a magnetic circuit can be made without determining the reluctance, and experience will show the best solution method. The following typical examples indicate the alternative way of treating simple problems.

Example 6.2. A solenoid is made up from a coil of 2000 turns, carries a current of 0.25A and is 1m long. An iron rod of diameter 20mm forms the core for the solenoid and is also 1m long. Calculate the total flux produced if the iron has a relative permeability of 1000 (4 significant figures).

$$\text{Coil m.m.f. is given by } F = H\ell = \frac{IN}{\ell} \times \ell = IN.$$

$$= 0.25 \times 2000 = 500\text{At}$$

$$\text{Area of iron} = \frac{\pi d^2}{4} = \frac{3.14 \times 400 \times 10^{-6}}{4}$$

$$= 3.14 \times 10^{-4}\ \text{m}^2$$

$$\text{Reluctance of iron, } S = \frac{\ell}{\mu A} = \frac{\ell}{\mu_0 \mu_r A}$$

$$\text{or } S = \frac{1}{4 \times \pi \times 10^{-7} \times 1000 \times 3.14 \times 10^{-4}}$$

$$= 2.533 \times 10^6\ \text{At/Wb}$$

$$\text{Flux } \Phi = \frac{F}{S} = \frac{500}{2.533 \times 10^6}$$

$$= 197.3 \times 10^{-6} = 197.3\mu\text{Wb}$$

Alternative solution

$$\text{m.m.f. of coil, } F = H\ell \text{ so magnetising force } H = \frac{F}{\ell}$$

$$\text{Thus } H = \frac{IN}{\ell} = \frac{0.25 \times 2000}{1} = 500\text{At/m}$$

$$\text{Also } B = \mu H = \mu_o \mu_r H = 4 \times \pi \times 10^{-7} \times 1000 \times 500$$

$$= 0.628\text{T}$$

$$\text{Total } \Phi = BA$$

$$= 0.628 \times 3.14 \times 10^{-4} \text{ webers}$$

$$= 197.3 \times 10^{-6} = 197.3\mu\text{Wb.}$$

Example 6.3. A cast-steel ring has a cross-section of 400mm^2 and a mean diameter of 240mm. It is wound with a coil having 200 turns. What current is required to produce a flux of 400μWb, if the relative permeability of the steel is 1000?

Area of steel = 400×10^{-6} m^2

$$\therefore B = \frac{\Phi}{A} = \frac{400 \times 10^{-6}}{400 \times 10^{-6}} = 1 \text{ telsa}$$

$$\text{Also } B = \mu_0 \mu_r H = 4 \times \pi \times 10^{-7} \times 1000 \times H$$

$$B = 4 \times \pi \times 10^{-4} \times H$$

$$\text{So } H = B/(\mu_0 \mu_r) = H = \frac{B}{4 \times \pi \times 10^{-4}} = \frac{1 \times 10^4}{4 \times \pi} \text{ At/m}$$

$$\text{m.m.f. of ring } F = H\ell = \frac{10^4}{4 \times \pi} \times \pi \times 240 \times 10^{-3}$$

$$\text{or F} = 600\text{At}$$

$$F = IN \quad \therefore I = \frac{F}{N} = \frac{600}{200} = 3A.$$

Alternative solution

$$\text{Reluctance of ring } S = \frac{\ell}{\mu_0 \mu_r A}$$

$$\text{or } S = \frac{\pi \times 240 \times 10^{-3}}{4 \times \pi \times 10^{-7} \times 1000 \times 400 \times 10^{-6}}$$

$$= 1.5 \times 10^6 \text{ At/Wb}$$

$$\text{then required m.m.f.} = \Phi S = 400 \times 10^{-6} \times 1.5 \times 10^6$$

$$= 600\text{At}$$

$$\text{Required current } I = \frac{600}{200} = 3\text{A.}$$

The Composite Magnetic Ring

The series arrangement

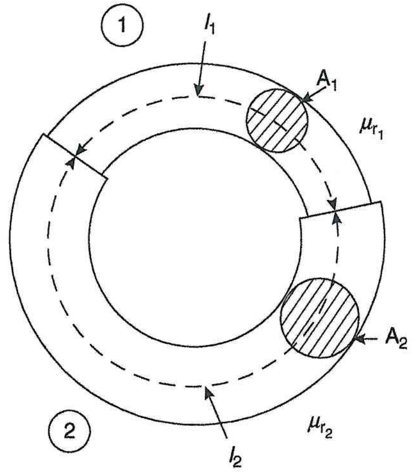
Consider a magnetic circuit built up as shown (figure 6.6). It is obvious that the toroid sections are in series with each other and that the same flux passes through them all.

The total m.m.f. = m.m.f. across section 1 + m.m.f. across section 2.

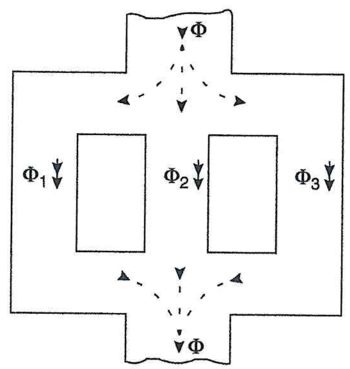
If the total flux is Φ then $\Phi S = \Phi S_1 + \Phi S_2$ where S is the reluctance of the composite circuit. Thus:

$$S = S_1 + S_2.$$

Summarising: Total reluctance = the sum of the individual reluctances of the sections for a series arrangement.



▲ **Figure 6.6**



▲ **Figure 6.7**

The parallel arrangement

Such a magnetic circuit is not often encountered but is considered here, as it is complementary to the series circuit. The arrangement is shown (figure 6.7).

If the different paths of the magnetic circuit are in parallel, then the m.m.f. is that which will produce the required flux in each part of the circuit separately. Let F = the m.m.f. required to produce fluxes Φ_1, Φ_2, Φ_3, etc. F also produces the total flux Φ.

$$So\ \Phi = \frac{F}{\text{Total reluctance of circuit}} = \frac{F}{S}$$

$$\text{But } \Phi_1 = \frac{F_1}{S_1} \qquad \Phi_2 = \frac{F_2}{S_2}$$

$$\text{and since } \Phi = \Phi_1 + \Phi_2 + \Phi_3$$

$$\therefore \Phi = \frac{F_1}{S_1} + \frac{F_2}{S_2} + \frac{F_3}{S_3}$$

But F_1, F_2, F_3 are the m.m.f.'s across the same points of the magnetic circuit and are equal to F.

$$\therefore \frac{F}{S} = F\left(\frac{1}{S_1} + \frac{1}{S_2} + \frac{1}{S_3}\right) \quad or \quad \frac{1}{S} = \frac{1}{S_1} + \frac{1}{S_2} + \frac{1}{S_3}$$

$\frac{1}{S}$ is referred to as the *permeance* of a magnetic circuit and the above is summarised by stating that the reluctance of a divided magnetic circuit (sections in parallel) is found by knowing that its permeance is equal to the sum of the permeances of the individual circuits.

Example 6.4. An iron ring has a mean diameter of 200mm and a cross-section of 300mm². An air gap of 0.4mm is made by a radial saw-cut across the ring. Assuming a relative permeability of 3000 for the iron, find the current needed to produce a flux of 250μWb, if the energising coil is wound with 600 turns (3 decimal places).

$$\text{Reluctance of iron } S_1 = \frac{(\pi \times 200 \times 10^{-3}) - (0.4 \times 10^{-3})}{4 \times \pi \times 10^{-7} \times 3000 \times 3 \times 10^{-4}}$$

$$= \frac{10^{-3}(628 - 0.4)}{4 \times \pi \times 9 \times 10^{-8}}$$

$$= \frac{62.76}{36\pi \times 10^{-6}}$$

$$= 555.2 \times 10^{3} \text{ ampere-turns/weber}$$

$$\text{Reluctance of air gap } S_A = \frac{0.4\times10^{-3}}{4\times\pi\times10^{-7}\times3\times10^{-4}}$$

$$= 1061.5\times10^{3}\text{ ampere-turns/weber}$$

$$\text{Total reluctance } S = S_1 + S_A = (555.2 + 1061.5)\times10^{3}$$

$$= 1616.7\times10^{3}\text{ At/Wb}$$

$$\text{Total m.m.f. F} = \Phi S = 2.5\times10^{-4}\times1616.7\times10^{3}$$

$$= 404.15\text{At}$$

$$\text{Current} = \frac{404.19}{600} = 0.674\text{A.}$$

Alternative solution

$$\text{Since } \Phi = 250\times10^{-6}\text{ weber} \quad \text{then } B = \frac{2.5\times10^{-4}}{3\times10^{-4}}$$

$$= 0.833\text{T}$$

Now H for air is given by:

$$H_A = \frac{0.833}{\mu_0} = \frac{0.833}{4\times\pi\times10^{-7}}$$

$$= 663.2\times10^{3}\text{ At/m}$$

$$\text{Length of air gap} = 0.4\times10^{-3}\text{ metre}$$

$$\text{Ampere-turns for air} = 663.2\times10^{3}\times0.4\times10^{-3}$$

$$= 265.28\text{At}$$

H for iron is given by:

$$H_1 = \frac{0.833}{\mu_0\mu_r} = \frac{0.833}{4\times\pi\times10^{-7}\times3\times10^{3}}\text{ At/m}$$

$$= 221.066\text{At/m}$$

$$\text{Now length of iron path} = (628 - 0.4)10^{-3}$$

$$= 627.6\times10^{-3}\text{metre}$$

$$\text{Ampere-turns for iron} = 221.066\times0.6276 = 138.74\text{At}$$

$$\text{Total ampere-turns} = 265.28 + 138.74 = 404.02\text{At}$$

$$\text{Current} = \frac{404.02}{600} = 0.673\text{A.}$$

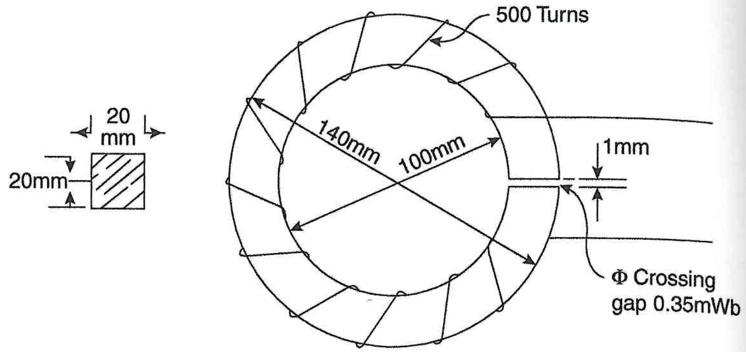
For the previous examples, **alternative solutions** were given in which the reluctances for the various sections of the magnetic circuit considered were not found. This **alternative solution** method is used when the relevant B and H data for a magnetic material is given in tabular or graphical form. The relative permeability is not given as a specific value and must be found before the reluctance is calculated. Obviously any such solution is tedious and the following example is recommended to the reader on how to solve this type of problem!

Example 6.5. An iron ring of square cross-section has an external diameter of 140mm, and an internal diameter of 100mm. A radial saw-cut through the cross-section of the ring forms an air gap of 1mm. If the ring is uniformly wound with 500 turns of wire, calculate the current required to produce a flux of 0.35mWb in the gap (1 decimal place). Magnetic data of the material of the ring is given (figure 6.8). Take μ_0 as $4\pi\times10^{-7}$ H/m.

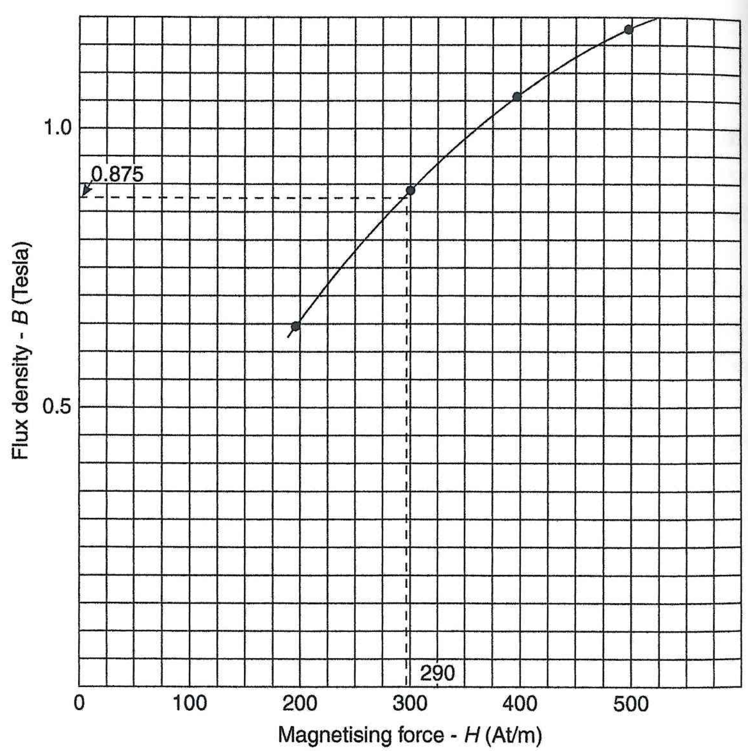
Solution uses the graph (figure 6.9) obtained from the above data.

Table 6.1

Flux density (T)	0.65	0.89	1.06	1.18
Magnetising force (At/m)	200	300	400	500



▲ **Figure 6.8**



Area of iron and air gap $= 20 \times 20 \times 10^{-6} = 4 \times 10^{-4}\ \text{m}^2$

Length of iron $= \pi \times (\text{mean diameter}) - \text{air gap}$

$$= (\pi \times 120 \times 10^{-3}) - (1 \times 10^{-3})\ \text{metre}$$

$$= 375.8 \times 10^{-3}\ \text{metre}$$

Length of air gap $= 1 \times 10^{-3}$ metre

Flux density for iron and air $\dfrac{\Phi}{A} = \dfrac{0.35 \times 10^{-3}}{4 \times 10^{-4}}\ \therefore\ B = 0.875\text{T}$

From graph for the iron $H = 290$At/m when $B = 0.875$T

But $H = \dfrac{IN}{\ell} = \dfrac{F}{\ell}\ \therefore F = H\ell$

So for iron, m.m.f. $F_{IRON} = 290 \times 375.8 \times 10^{-3} = 108.88$At

For air, since $H = \dfrac{B}{\mu_0} = \dfrac{0.875}{4 \times \pi \times 10^{-7}}$ At/m

and for air, m.m.f. $F_{AIR} = \dfrac{0.875}{4\pi \times 10^{-7}} \times 1 \times 10^{-3} = 696.7$At

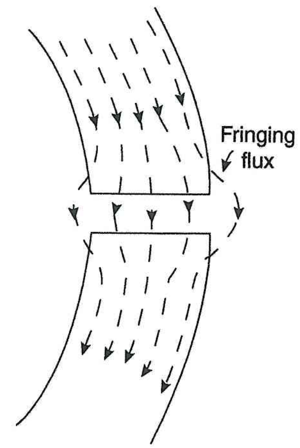
Total m.m.f. $F_{IRON} + F_{AIR} = 108.9 + 696.7 = 805.6$At

Current is deducted from $F/N = \dfrac{805.6}{500} = 1.6112$A

$\therefore$ Energising current = 1.6A (1 decimal place).

MAGNETIC FRINGING. Figure 6.10 illustrates how magnetic flux bridges an air gap, especially if the gap is comparatively large.

Due to spreading flux in air occupies a larger area than that of the iron, and the flux density is thus reduced. An allowance can be made for this effect in problems when required, but unless told otherwise the area of the air gap is taken as the area of the iron.



▲ Figure 6.10

MAGNETIC LEAKAGE. For some magnetic circuits, due to the shape of the iron core and placing of the energising coil, a small amount of flux leakage may occur (figure 6.11). Some flux lines are not confined to the iron and complete their paths through air. For practical purposes, a factor called the *leakage coefficient* is added which increases the required working flux value sufficiently to allow for this leakage.

Thus: the required total flux = the useful or working flux × leakage coefficient. The leakage coefficient is typically between 1.1 and 1.3. This leakage flux will result in reduced efficiency and increased energy losses.

Example 6.6. (a) A magnetic circuit has an iron path of length 500mm and an air gap of length 0.5mm, having uniform square cross-section, 1000mm^2 in area. Calculate the number of ampere-turns needed to produce a total flux of 1mWb in the air gap. Ignore fringing, and assume a leakage coefficient of 1.3. The *B–H* curve for the iron is given by the following (table 6.2):

(a) A conductor is passed through the air gap at a speed of 100m/s. If the length of the conductor is greater than the length of the side of the gap, calculate the e.m.f. induced.

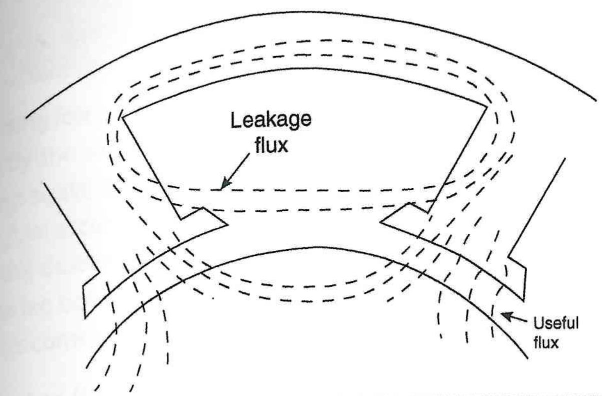
The solution uses the graph, obtained from the above data, as shown in figure 6.12.

Area of iron and air gap = $1000 \times 10^{-6} = 10^{-3}$ m^2

Length of iron = $500 \times 10^{-3} = 0.5$m

Flux density (*B*) for air $= \dfrac{1\times10^{-3}}{10^{-3}} = 1$T

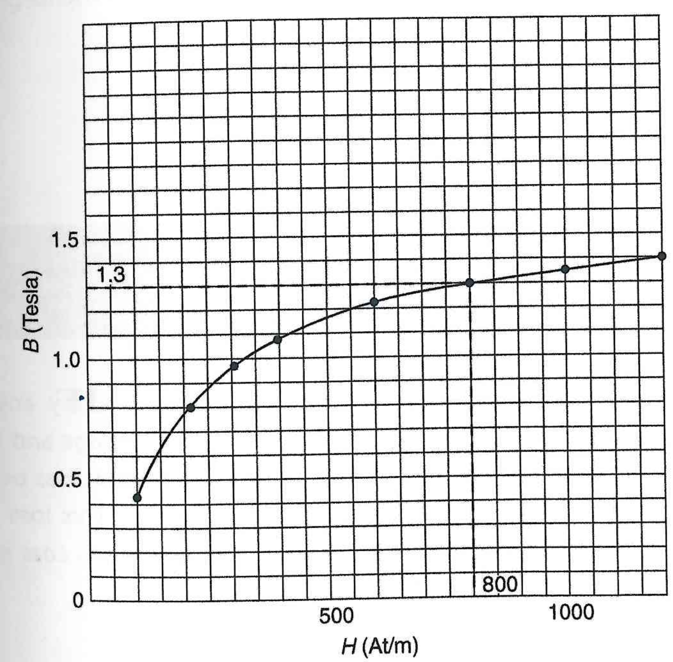
Flux in pole = Useful flux × Leakage coefficient



▲ Figure 6.11

Table 6.2

H (At/m)	100	200	300	400	600	800	1000	1200
B (T)	0.42	0.8	0.98	1.08	1.22	1.3	1.36	1.4



▲ Figure 6.12

Flux density (B) in iron $= \dfrac{1\times10^{-3}\times1.3}{10^{-3}} = 1.3\text{T}$

For air, since $H = \dfrac{B}{\mu_0} = \dfrac{1}{4\times\pi\times10^{-7}}$ At/m

and m.m.f. $F_a = \dfrac{1}{4\pi\times10^{-7}}\times0.5\times10^{-3} = 398.8\text{At}$

From graph for iron, $H = 800$At/m for a B value of 1.3T

and m.m.f., $F_1 = 800\times0.5 = 400$At

Total ampere-turns required for iron = 398.8 + 400 = 798.8At

(b) Induced e.m.f. given by $E = B\ell v$ volts

Since the area of the air gap is that of a square, the side of the square is $\sqrt{1000\times10^{-6}} = \sqrt{10\times10^{-4}}$

$= 3.162\times10^{-2}\ \text{m}^2$

Thus $E = 1\times3.162\times10^{-2}\times10^{2}$

$= 3.162$ volts

Note. In the above $B = 1$ tesla. $\ell = 3.162\times10^{-2}$ m and $v = 100$m/s.

Iron Losses

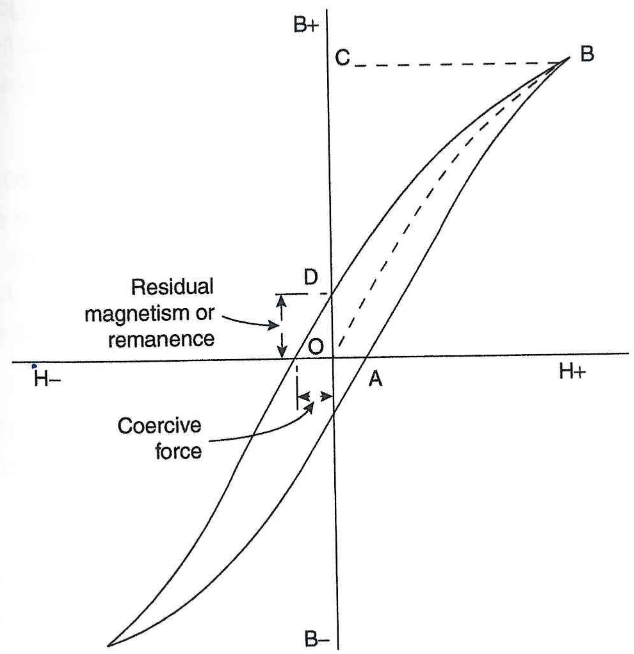
Electrical machine and transformer efficiencies are reduced by any losses in them. Apart from *Mechanical Losses* such as *Friction* and *Windage* and the *Copper Losses* (due to the resistance of the conductors), an additional loss occurs when a magnetic material is taken through cyclic magnetisation. This loss is the *Iron Loss* and is made up of 2 component losses: (1) the *Hysteresis Loss* and (2) the *Eddy-Current Loss.*

The hysteresis loop

If the magnetising force applied to an iron sample is increased from zero to its maximum value, in exactly the same way as when making the test for a B–H curve, and is then reduced to zero again, it is found that the new B–H curve, for decreasing values of H, lies *above* the original ascending curve, and that when H is zero B is left at some new value. The effect of the descending curve being above the ascending one is called 'hysteresis', as the B values lag behind those for the corresponding H strength when increasing. The word hysteresis comes from the Greek meaning 'to lag'. Figure 6.13 shows these effects.

The value of B when H is zero is the *remanence* and is a measure of the residual magnetism (distance OD). In order to demagnetise the iron and remove the residual magnetism it is necessary to apply a *negative* magnetising force known as the *coercive force*. If H is increased in the negative direction to its previous maximum value, the curve will reach a value equal to the previous maximum B and if H is next gradually reduced up to zero, then increased to its original maximum, a closed loop is traced. This is a hysteresis loop and is a measure of part of the iron loss.

To take the iron through the various stages represented by the loop, an alternating magnetising force is applied. One method of achieving this is to connect the energising



coil to an A.C. supply, when the iron continues to go through the same series of changes or magnetic cycles. To confirm energy is being expended, it is found that the iron core registers a temperature rise. The area of the loop is a measure of the power loss due to hysteresis. The energy absorbed per cubic metre per cycle, due to hysteresis, is given in joules by the area of the loop, provided the scales used for the graph are in appropriate SI units. The energy stored in the magnetic field in figure 6.13 can be represented by the area OABCDO in figure 6.14. When the field collapses, energy is returned to the supply which is represented by the area DBCB. The area of the loop OABDO represents the energy lost as heat through hysteresis and is the difference between the energy put into the magnetic circuit when setting up the field and that recovered when the field decays.

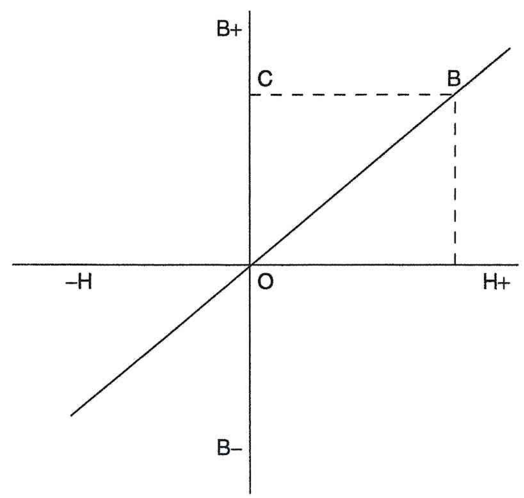
If the iron sample was non-magnetic, i.e. air, then the *B–H* curve will be a straight line, as shown (figure 6.14), and the energy stored in the field when it is set up, is represented by the area of the triangle OBC, will be recovered when the field collapses.

For air, the area of the right angle triangle OBC $= \frac{1}{2}$ height × base $= \frac{1}{2}$ OC × CB $= \frac{1}{2} B_m \times H$

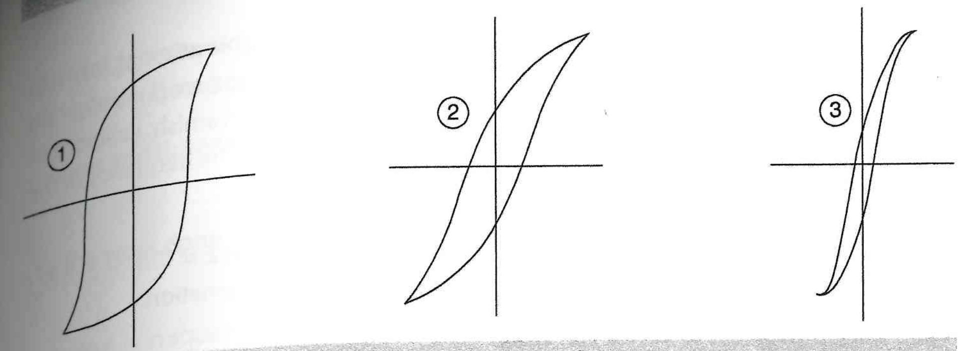
where B_m is the maximum flux density value, attained for the *H* value which was impressed.

As $B = \mu_0 H$. Therefore the area of the triangle $= \frac{1}{2} \times B_m \times \frac{B_m}{\mu_0} = \frac{B_{m^2}}{2\mu_0}$ and as the area of the triangle represents the energy stored in air per cubic metre (in joules), it follows that:

For air: Energy stored per cubic metre $= \frac{B_{m^2}}{2\mu_0}$ joules.



▲ Figure 6.14



▲ Figure 6.15

The types of hysteresis loop, as obtained from various magnetic materials, can be grouped into 3 classes as shown (figure 6.15).

Loop 1 is for hard steel. The large value of the coercive force indicates that the material is suitable for permanent magnets. The area however is large, showing that hard steel is not suitable for rapid reversals of magnetism.

Loop 2 rises sharply showing a high μ and a good retentivity (large intercept on *B* axis). The loop is typical of cast-steel and wrought iron, which are suitable materials for electromagnet cores and electrical machine yokes.

Loop 3 has a small area and a relatively high μ. The material (mainly alloyed sheet-steels) is suitable for rapid reversals of magnetism and is used for armatures, transformer-cores, etc.

1. HYSTERESIS LOSS. Since this is a function of loop area, the effect of varying *B* on the area must be considered. When the value of *H* is increased, for example, doubled, *B* is *not* doubled and thus the ratio of the loop area also is not quadrupled. It is found to increase about 3.1 times. If the Area of Loop is actually proportional to B_m^x with x somewhere between 1 and 2 – where B_m is the maximum value to which flux density has been taken.

2. EDDY-CURRENT LOSS. When an armature rotates in a magnetic field, an e.m.f. is induced in the conductors. Since the conductors are let into slots, the armature teeth are considered as conductors with e.m.f.s. induced across them. Moreover, as the electrical circuit is complete for these e.m.f., currents will flow from one end of a tooth through the armature end-plate, along the shaft and back to the other end of the tooth through the opposite armature end-plate. Such 'eddy currents', produce a power loss, due to the resistance of the iron circuit, which is $\propto I^2R$ or $\frac{E^2}{R}$.

Eddy-current loss depends on several factors, with every reasonable attempt taken to minimise such losses. The principal methods by which this is achieved are (1) laminating the iron circuit and insulating the laminations from each other by varnish, cellulose or paper, (2) using iron with a high specific resistance and (3) keeping the frequency of the magnetic alternations or cycles to a minimum.

Since a generated voltage is proportional to flux and speed, then $E \propto \Phi N$ or $E \propto B_m f$ where B_m is the maximum flux density and f is the frequency of alternation.

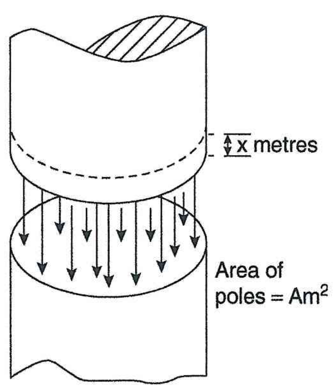
Again since power loss $\propto \frac{E^2}{R}$ we can write:

Power loss $\propto B_m{}^2 f^2$ or $P_E = K_E B_m{}^2 f^2$ watts per cubic metre, where K_E is an *eddy-current coefficient* which is dependent upon the type of material used, its thickness and other dimensions.

Pull of an Electromagnet

The energy stored in a magnetic field in air is given by $\frac{B^2}{2\mu_0}$ joules per cubic metre, where B is in teslas. Consider 2 poles arranged as shown (figure 6.16).

Each has an area A square metres and let F be the force of attraction (in newtons) between the poles.



▲ Figure 6.16

Let one pole move a small distance x (metres) against the force F, so the work done is Fx newton metres or joules. The volume of the magnetic field will increase by Ax cubic metres and the energy stored in the field is increased by $\frac{B^2}{2\mu_0} \times Ax$ joules. This is equal to the work done in separating the poles so that $Fx = \frac{B^2}{2\mu_0} Ax$ or $F = \frac{B^2 A}{2\mu_0}$ newtons,

where A is in square metres and B in teslas.

Example 6.7. An electromagnet is wound with 500 turns. The air gap has a length of 2mm and a cross-sectional area of 1000mm². Assuming the reluctance of the iron to be negligible compared with that of the air gap, and neglecting magnetic leakage and fringing, calculate the magnetic pull when the current is 3A.

M.m.f. of coil $F = N \times I = 500 \times 3 = 1500\text{At}$

This m.m.f. is used to pass the flux through the air gap, since the reluctance of the iron is negligible.

The magnetising force for the air, is given by 'the ampere-turns per metre' or

$$H = \frac{F}{L} = \frac{1500}{2 \times 10^{-3}}$$

Also the flux density in air is B where:

$$B = \mu_0 H$$

$$\therefore B = \frac{4 \times \pi \times 10^{-7} \times 1500}{2 \times 10^{-3}} \text{ tesla}$$

$$= 0.942\text{T}$$

$$\text{Now the pull } F = \frac{B^2 A}{2\mu_0} = \frac{0.942^2 \times 1000 \times 10^{-6}}{2 \times 4 \times \pi \times 10^{-7}} \text{ newtons}$$

Thus $F = 353.3\text{N}$.

Example 6.8. A 4-pole D.C. generator has a cast-steel yoke and poles and has a laminated steel armature. The dimensions of the component parts of the magnetic circuit are as follows in table 6.3:

Table 6.3

Yoke. Total mean circumference = 3.04m	CSA = $0.04m^2$
Pole. Total mean length = 0.24m	CSA = $0.065m^2$
Air gap. Total mean length = 2mm	CSA = $0.065m^2$
Armature. Total mean path between poles = 0.4m	CSA = $0.025m^2$

The magnetisation curves are given in table 6.4.

Table 6.4

	H (At/m)	400	800	1200	1600	2000	2400
Cast steel	*B* (T)	0.45	1	1.2	1.3	1.37	1.43
Laminated steel	*B* (T)	1	1.34	1.48	1.55	1.6	1.63

Calculate the ampere-turns per pole, for a flux per pole of 0.08Wb in the air gap. Figure 6.17 illustrates the problem and the appropriate magnetic characteristics are shown by the graphs for the diagram (figure 6.18).

AIR GAP

Length 2×10^{-3} m

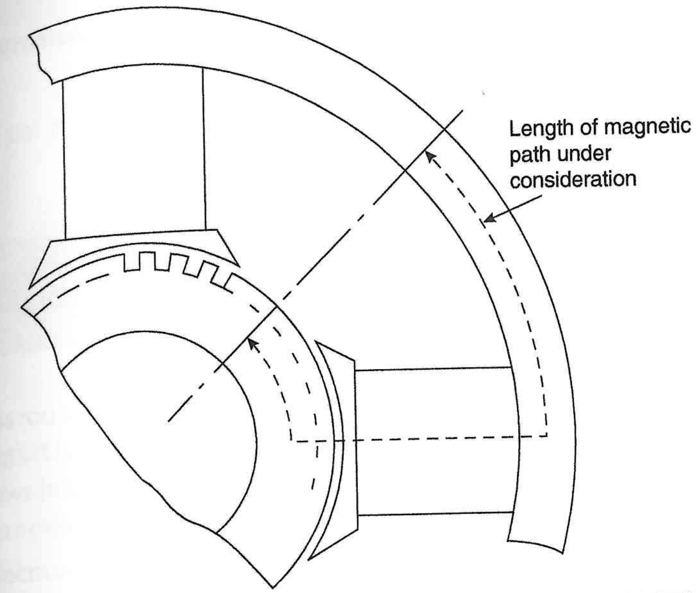
Area 0.065 = 6.5×10^{-2} m^2

$$\Phi = 0.08\text{Wb} \quad B_A = \frac{0.08}{6.5 \times 10^{-2}} = \frac{8}{6.5} = 1.23\text{T}$$

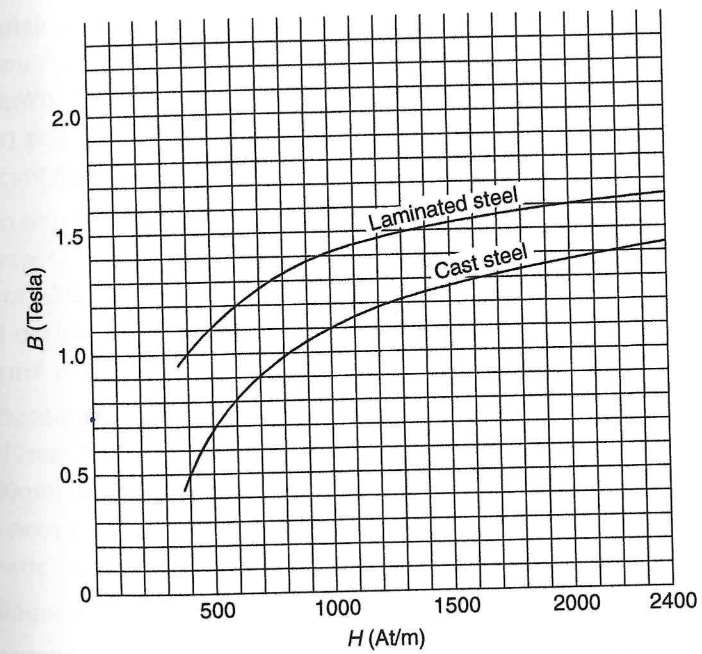
$$H_A = \frac{1.23}{\mu_0} = \frac{1.23}{4 \times \pi \times 10^{-7}} \text{ ampere-turns/metre}$$

Total ampere-turns or m.m.f. for air gap is given by:

$$F_A = \frac{1.23}{4 \times 3.14 \times 10^{-7}} \times 2 \times 10^{-3} \text{ampere-turns}$$



▲ Figure 6.17



▲ Figure 6.18

POLE (Cast steel)

Length 24×10^{-2}m

Area $0.065 = 6.5 \times 10^{-2} m^2$

$$\Phi = 0.08\text{Wb} = B_p = \frac{\Phi}{A} = \frac{0.08}{6.5 \times 10^{-2}} = 123\text{T}$$

From graph, $H_p = 1370$At/m or

Total $F_p = 1370 \times 24 \times 10^{-2} = 330$At

YOKE (Cast steel)

Length $\frac{3.04}{4} = 0.76 = 76 = \times 10^{-2}$m (between poles) or 0.38 magnetic length

Area $= 2 \times 0.04 = 0.08\ m^2$ but Area $= 8 \times 10^{-2}\ m^2$

(Note the doubling of area since full pole area is taken for the flux)

$$\Phi = 0.08\text{Wb} \quad B_Y = \frac{0.08}{8 \times 10^{-2}} = 1\text{T}$$

From graph $H_Y = 800$At/m

Total F_Y for yoke $800 \times 38 \times 10^{-2} = 304$At

ARMATURE (Laminates)

Length $\frac{0.4}{2} = 20 \times 10^{-2}$ metre (magnetic length)

Area $= 2 \times 0.025 = 0.05 = 5 \times 10^{-2}\ m^2$

$$\Phi = 0.08\text{Wb} \quad B_L = \frac{0.08}{5 \times 10^{-2}} = 1.6\text{T}$$

Total F_L for armature $= 2000 \times 20 \times 10^{-2}$
$= 400$At

Total m.m.f. per pole $= 1955 + 330 + 304 + 400$
$= 2989$At.

Practice Examples

6.1. A brass rod of cross-section 1000mm² is formed into a closed ring of mean diameter 300mm. It is wound uniformly with a coil of 500 turns. If a magnetising current of 5A flows in the coil, calculate (a) the magnetising force (4 significant figures), (b) the flux density (2 significant figures) and (c) the total flux (2 significant figures).

6.2. An electromagnetic contactor has a magnetic circuit of length 250mm and a uniform cross-sectional area of 400mm². Calculate the number of ampere-turns required to produce a flux of 500μWb, given that the relative permeability of the material under these conditions is 2500. Also $\mu_0 = 4\pi \times 10^{-7}$ H/m (nearest whole ampere-turns).

6.3. In a certain magnetic circuit having a length of 1m and a uniform cross-section of 500mm², a magnetising force of 500 ampere-turns produces a magnetic flux of 400μWb. Calculate (a) the relative permeability of the material (4 significant figures) and (b) the reluctance of the magnetic circuit, $\mu_0 = 4\pi \times 10^{-7}$ H/m (3 significant figures).

6.4. An iron ring having a mean circumference of 1.25m and a cross-sectional area of 1500mm² is wound with 400 turns of wire. An exciting current of 2.5A produces a flux of 0.75mWb in the iron ring. Calculate (a) the permeability (relative) of the iron (1 decimal place), (b) the reluctance of the iron (3 significant figures) and (c) the m.m.f. of the exciting winding (1 significant figure).

6.5. A U-shaped electromagnet has an armature separated from each pole by an air gap of 2mm. The cross-sectional area of both the electromagnet and the armature is 1200mm² and the total length of the iron path is 0.6m. Determine the ampere-turns necessary to produce a total flux in each air gap of 1.13mWb neglecting magnetic leakage and fringing (4 significant figures).

The magnetisation curve for the iron is given by:

B (T)	0.5	0.6	0.7	0.8	0.9	1.0	1.1
H (At/m)	520	585	660	740	820	910	1030

6.6. A circular ring of iron of mean diameter 0.2m and cross-sectional area 600mm² has a radial air gap of 2mm. It is magnetised by a coil having 500 turns of wire. Neglecting magnetic leakage, and fringing, estimate the flux density in the air gap, when a current of 3A flows through the coil. Use the magnetic characteristics as given by the graph of Q6.5 (4 significant figures).

6.7. A built-up magnetic circuit without an air gap, consists of 2 cores and 2 yokes. Each core is cylindrical, 50mm diameter and 160mm long. Each yoke is of square cross-section 47 × 47mm and is 180mm long. The distance between the centres of the cores is 130mm. Calculate the ampere-turns necessary to obtain a flux density of 1.2T in the cores (3 decimal places). Neglect magnetic leakage. The magnetic characteristics of the material are given:

B (T)	0.9	1.0	1.05	1.1	1.15	1.2
H (At/m)	200	260	310	380	470	650

6.8. An iron rod 15mm diameter, is bent into a semicircle of 50mm inside radius and is wound uniformly with 480 turns of wire so as to form a horse-shoe electromagnet. The poles are faced so as to make good magnetic contact with an iron armature 15 × 15mm cross-section and 130mm long. (a) Calculate the current required to produce a pull of 196.2N between the armature and pole-faces. Neglect magnetic leakage (3 decimal places). (b) Calculate the ampere-turns necessary to obtain a flux density of 1.15T in the air gap, if the armature is fixed so as to leave uniform air gaps 0.5mm wide at each pole-face. Neglect leakage and fringing. Use the magnetic characteristics as given by the graph of Q6.7 (2 decimal places).

6.9. Two coaxial magnetic poles each 100mm diameter are separated by an air gap of 2.5mm and the flux crossing the air gap is 0.004Wb. Neglecting fringing calculate (a) the energy in joules stored in the air gap (1 significant figure) and (b) the pull in newtons between the poles (3 significant figures).

6.10. Calculate the ampere-turns per field coil required for the air gap, the armature teeth and the pole, of a D.C. machine working with a useful flux of 0.05Wb/pole, having given: Effective area of air gap 60 000mm². Mean length of air gap 5mm. Effective area of pole 40 000mm². Mean length of pole 250mm. Effective area of teeth 25 000mm². Mean length of teeth 45mm (4 significant figures).

Magnetic leakage coefficient = 1.2. Magnetic characteristics of the materials are:

B (J)	1.3	1.4	1.5	1.6	1.8	2.0
H (At/m)	1200	1500	2000	3000	8500	24 000

7

ELECTROMAGNETIC INDUCTION

Sir Humphry Davy's greatest discovery was Michael Faraday.

Sir Henry Paul Harvey

Electrochemistry was the first branch of science to play a full part in electrical investigations of the early nineteenth century. At that time electricity research was of interest to scientists only and could not be put to use for real engineering processes. Chemical cells as they were then known were unable to produce sufficient energy or e.m.f. for practical purposes, nor had any electromagnetic devices been invented for engineering applications. As mentioned in Chapter 5, it was only after the relation between current and magnetism was discovered that this was to change. These discoveries revealed the related phenomena of electromagnetic induction and led to the development of machines which enabled engineers to produce electrical and mechanical energy.

The first electromagnetic *induction* experiments were attributed to Michael Faraday (1791–1867) who in 1821 showed that when magnetic flux linked with an electrical circuit is changing, an e.m.f. is induced in the circuit. This e.m.f. lasts only while the flux change takes place and the faster the change the greater the e.m.f. Michael Faraday proposed the first laws concerning induced e.m.f., winding the wires of the world's first electric generator himself and building the first ever transformer induction ring to change the voltage of an electric current (1831). These original devices are today on permanent display at the Royal Institute in London. Variation of both these revolutionary electric machines are now used in almost every power station: water, wind, gas or nuclear powered around the globe.

The magnetic flux linked with a circuit, usually a coil of insulated wire, is found to change in several different ways. Thus:

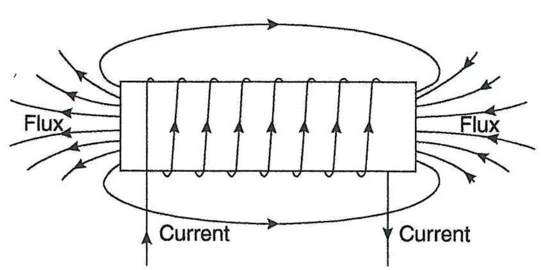
(1) A magnet could move near a coil of wire. This principle is used for the A.C. generator or alternator.

(2) A coil of wire could move near a magnet. This principle is used for the D.C. dynamo or generator.

(3) The flux could change by varying the current in the energising coil of wire. The ampere-turns are varied and the flux produced varies accordingly. This principle is essential to the operation of the transformer and familiar spark-coil of a petrol-engine ignition system. For these 3 methods of e.m.f. generation, it is seen that cases (1) and (2) involve relative physical movement between magnet and coil. Case (3) however, involves no such movement and the generated e.m.f. is achieved in a stationary coil with which only the linked flux changes. Thus there are 2 distinct forms of e.m.f. generation or induction referred to under 2 basic headings: (1) Dynamic Induction, and (2) Static Induction.

Before these 2 methods are considered let us look at what is meant by *flux-linkages*.

FLUX-LINKAGES. Earlier studies on magnetism showed that a magnet's field can be represented by lines of flux emanating from its poles. The strength of the flux is represented by the number of lines and is measured in webers, while flux density is measured in teslas. The flux lines make complete loops and the associated conductor or coil of wire in which the e.m.f. is induced is considered to consist of many turns. As the number of lines of flux associated with the turns are referred to as flux-linkages, a magnet with poles of flux strength 3.4μWb linked with a coil of 500 turns will result in $3.4 \times 10^{-6} \times 500 = 0.0017$ weber-turns. Figure 7.1 shows the basic concept.



▲ Figure 7.1

Laws of Electromagnetic Induction

Faraday's law

This summarises the known relationship deduced for generation of e.m.f. by electromagnetic induction and is stated: the magnitude of the e.m.f. produced, whenever there is a change of flux linked with a circuit, is proportional to the *rate of change* of flux-linkages.

Lenz's law

This identifies a phenomenon noted for e.m.f. produced by induction. The law is stated as the direction of the current due to the induced e.m.f. always set up an effect tending *to oppose* the change causing it.

Thus if flux-linkages increase, the field produced by the induced current resulting from the induced e.m.f. tends to oppose this effect, i.e. it opposes flux-linkage build-up. Similarly, if the flux-linkages reduce, as when the current in a coil is switched off, then the induced e.m.f. will induce a current which, if allowed to flow, will keep up the flux-linkages to their original value. The action of the induced current will not be able to prevent the change, but it will try to do so during the period the change occurs. Faraday's law can be expressed in mathematical form and formulae deduced for static and dynamic electromagnetic induction, which will be considered separately.

Static Induction

Consider a coil connected to a D.C. power supply. At the instant of switch-on the current produces flux which grows from the centre of the coil outwards, but this flux 'cuts' the coil turns and induces an e.m.f. which opposes the current growth (Lenz's law). Such a circuit in which a change of current causes a change of flux and therefore produces an induced e.m.f. is said to be inductive or to possess *self-inductance*. There is a 'resistance' or 'inertia' of the circuit to change taking place. A circuit has an inductance of 1 Henry if an e.m.f. of 1 volt is induced in a circuit when current changes at a rate of 1 amp/second.

Self-inductance

From this definition if a circuit has an inductance of L Henries and current changes from I_1 to I_2 amperes in t seconds, as shown in figure 7.2, then the average induced e.m.f.:

$$Eav = \frac{-L\,(I_2 - I_1)}{t} \text{ volts}$$

$$\text{or } Eav = -L \times \text{rate of increase of } I = -L\,\frac{dI}{dt}$$

Note: The negative sign indicates that the direction of induced e.m.f. *opposes* that of the current increase.

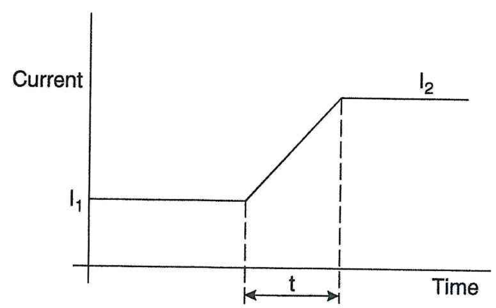
Example 7.1. The current through a coil having an inductance of 0.5H is reduced from 5A to 2A in 0.05s. Calculate the average e.m.f. induced in the coil (2 significant figures).

$$Eav = \frac{-L(I_2 - I_1)}{t} \text{ volts}$$

$$= \frac{-0.5(2-5)}{0.05} \text{ volts}$$

$$\therefore Eav = +30 \text{ volts.}$$

Note. The positive sign indicates that the induced e.m.f. tries to maintain the current flow.



▲ **Figure 7.2**

As a change of current produces a change of flux-linkages, the e.m.f. induced can be expressed in terms of the number of coil turns and the rate of change of flux.

$$Eav = \frac{-N\,(\Phi_2 - \Phi_1)}{t} \text{ volts} = -N\frac{d\Phi}{dt}$$

Example 7.2. When a D.C. current passes through an iron-cored coil of 2000 turns a magnetic flux of 30mWb is produced. The supply switch is *opened* and the current falls to zero amperes in 0.12s leaving a residual flux of 2mWb. Find the average value of induced e.m.f. (1 decimal place).

$$Eav = \frac{-N(\Phi_2 - \Phi_1)}{t}$$

$$= \frac{-2000(2-30)\times 10^{-3}}{0.12}$$

$$Eav = +466.6 \text{ volts}$$

Note. Again the +ve sign indicates that the induced e.m.f. tries to maintain the current flow.

From these equations:

$$Eav = \frac{-L(I_2 - I_1)}{t}$$

$$\text{and } Eav = \frac{-N(\Phi_2 - \Phi_1)}{t}$$

and setting these 2 equations equal:

$$\frac{-L(I_2 - I_1)}{t} = \frac{-N(\Phi_2 - \Phi_1)}{t}$$

$$\therefore \text{Inductance } L = \frac{N(\Phi_2 - \Phi_1)}{(I_2 - I_1)} = \frac{N d\Phi}{dI}$$

$$\text{or Inductance } L = \frac{N\Phi}{I}$$

Hence Inductance L = Flux-linkages/current.

In Chapter 6 flux was expressed in terms of m.m.f. (F) and reluctance

$$F = \Phi S \quad \text{or} \quad IN = \Phi S \quad \text{and} \quad \Phi = \frac{IN}{S}$$

$$\text{Thus } L = \frac{N}{I} \times \frac{IN}{S}$$

$$\therefore \text{ Inductance } L = \frac{N^2}{S} \text{ Henries}$$

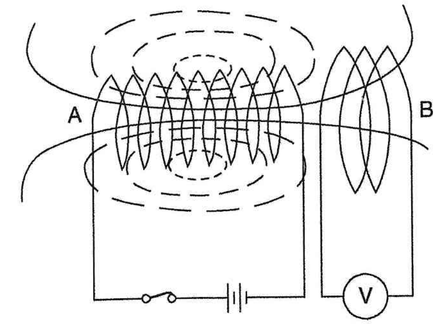
E.m.f. due to static induction

Consider the diagram (figure 7.3) which shows 2 coils A and B of insulated copper wire. Coil A can be connected to a battery through a switch, while B is wound over or adjacent to coil A and connected to a sensitive centre-zero voltmeter. This instrument is used because, as the pointer is positioned at the centre of the scale, deflection to the left or right depends on the polarity of the supply.

At the instant of switching on the current in coil A, flux is imagined to grow out and cut the turns of coil B. The initial growth is shown by the dotted flux lines becoming fuller until the final condition (full lines) is reached. The cutting of coil B by the flux of A results in an induced e.m.f., its magnitude and direction governed by Faraday's and Lenz's laws. The flux-linkages, i.e. flux linking with the turns (N_B) of coil B increase and if the linking flux grows to a value of Φ webers from its original zero value, then the rate of change of flux-linkages will be the flux-linkages divided by the time (t_1) taken for them to grow, i.e. the time taken for the current to reach its final value.

$$\text{Thus e.m.f. induced in coil B} = \frac{\text{flux-linkages}}{\text{time}} = \frac{-N_B\Phi}{t_1} \text{ volts.}$$

For this equation N_B = turns of coil B, Φ is the flux in webers linking with it and t_1 is the time taken for the energising current to reach its final value I. It might be assumed that value I is reached immediately the switch is closed, because the electricity flow is considered to be instantaneous, but the current takes an appreciable time to reach its full value – due to the circuit 'inertia' or inductance. It is seen that when the switch



▲ Figure 7.3

left, showing an e.m.f. is induced in coil B – the *secondary* circuit. The value of e.m.f. $E_B = \frac{-N_B\Phi}{t_1}$ and the voltmeter will show the polarity of coil B to be such that the current, flowing through the instrument, is in a direction through the coil as to set up a secondary flux, opposite to the original flux Φ and will try to stop it growing. Although a kick of the voltmeter pointer is seen, it returns to the zero position even though current in coil A is allowed to flow indefinitely. Thus an e.m.f. is induced only during the time when the flux-linkages change. Further experiments with coil B show that if the number of turns of wire were doubled, then the induced e.m.f. will be twice as large, even though the flux Φ of coil A is the same. The flux-linkages have increased and the induced e.m.f. rises proportionately.

Consider next the instant of switching off the current in coil A. The voltmeter kicks to the right this time, showing an induced e.m.f. of reversed polarity. The flow of current in coil B is such as to try and maintain the flux to its original value Φ and again $E_B = \frac{-N_B\Phi}{t_2}$ where t_2 is the time taken for 'switching off'. It is noted here that t_2 need not equal t_1. If the switch is opened quickly, the current of A will be interrupted quickly and E_B can be larger at switching off than at switching on, i.e. the rate of growth of the flux is controlled by the inductance and resistance R of the circuit.

Up to now we have only considered the induced e.m.f. in coil B and this is said to be due to *Mutual Induction*, i.e. the mutual action of coil A on B. We now turn our attention to *Self-Induction*, i.e. the conditions within coil A itself. At the instant of switching on, the flux grows outwards and in so doing cuts the turns of coil A – the primary circuit. An e.m.f. is induced given by $E_A = \frac{-N_A\Phi}{t_2}$. Here N_A is the turns of coil A, Φ is the linked flux and t_1 the time taken for the current to reach its full value. As before, the direction of the self-induced e.m.f. E_A will be such as to cause a current to flow in the opposite direction through the battery to produce a flux opposing the build-up of flux Φ. We can now see the reason for the opposition to the growth of current in coil A at the instant of switching on and why the current *I* takes a little time to reach its full value. As before when the switch is opened, flux collapses and in doing so, again cuts the turns of coil A, inducing a voltage of reversed polarity, which tries to keep the current flowing. This self-induced e.m.f. at 'switching off' can be extremely large in some instances where a large number of turns of an energising winding are associated with a strong magnetic flux. For example, the opening of the field circuit of a large alternator or D.C. generator. Special arrangements help to limit the e.m.f. to a safe value and prevent 'breakdown' of insulation by large induced voltages.

Mutual inductance

Two coils have a mutual inductance of 1 Henry when a change of current at the rate of 1 ampere/second in 1 coil produces an e.m.f. of 1 volt in the other.

Consider once again figure 7.3. If coils A and B have a mutual inductance of M Henries, and the current in coil A increases from I_1 to I_2 amperes in t seconds then

$$\text{Average e.m.f. induced in B} = \frac{-M(I_2 - I_1)}{t}$$

$$= -M \times \text{Rate of increase of current in coil A}$$

The e.m.f. induced into coil B can be expressed in terms of flux-linkages in the same way as was applied to self-inductance.

Let the flux change from Φ_1 to Φ_2 Webers in t seconds due to a change of current from I_1 to I_2 amperes in the primary, and let coil B have N_B turns.

Thus average e.m.f. induced is:

$$E_{e.m.f.}\,B = \frac{-N_B(\Phi_2 - \Phi_1)}{t}$$

$$\text{Hence} = \frac{-M(I_2 - I_1)}{t} = \frac{-N_B(\Phi_2 - \Phi_1)}{t}$$

$$\therefore\ M = \frac{N_B(\Phi_2 - \Phi_1)}{(I_2 - I_1)} = 5V$$

$$\text{So mutual inductance} = \frac{\text{Change in flux linkages on secondary}}{\text{Change in current in primary}}$$

Example 7.3. (Self-induction). A coil of 800 turns is wound on a wooden former and a current of 5A is passed through it to produce a magnetic flux of 200μWb. Calculate the average value of e.m.f. induced in the coil when the current is (a) switched off in 0.08 seconds (1 significant figure) and (b) reversed in 0.2 seconds (1 decimal place).

$$\text{(a)}\quad Eav = \frac{-N(\Phi_2 - \Phi_1)}{t} = \frac{-800 \times (0 - 200 \times 10^{-6})}{0.08}$$

$$Eav = 2V$$

(b) $Eav = \dfrac{-N(\Phi_2 - \Phi_1)}{t}$ here $\Phi_2 = \Phi_1$ *numerically* but is in the reverse direction, or $\Phi_2 = -\Phi_1$

$$\text{This } Eav = \frac{2N\Phi_1}{t} = \frac{2 \times 800 \times 200 \times 10^{-6}}{0.2}$$

$$Eav = 1.6V$$

Example 7.4. (Mutual induction). If the coil of the above example has a secondary coil of 2000 turns wound onto it, find the e.m.f. induced in this second coil when the current of 5A is switched off in 0.08 seconds (1 significant figure.). It can be assumed that all the flux of 200μWb created by the 5A current in the primary links with the secondary coil.

$$\therefore\ M = \frac{N_B(\Phi_2 - \Phi_1)}{(I_2 - I_1)} = 5V$$

Note. The e.m.f. of the secondary is $\frac{5}{2} = 2.5$ times the induced e.m.f. in the primary and is proportional to the turns ratio $\frac{2000}{800} = 2.5$. This is the basic principle of the transformer and the ignition system spark-coil. It shows how a large voltage is induced in a secondary coil by the flux associated with a low-voltage primary coil. For a petrol-engine ignition system, the e.m.f. in the secondary may be close to 8000V compared with 12V applied to the primary coil. This is achieved by using a coil of appropriate turns ratio between the primary and secondary coils, by providing an iron magnetic circuit to concentrate the flux for maximum linkage and by interrupting the primary circuit quickly by an engine-driven cam-operated switch.

Example 7.5. The ignition coil of a petrol engine has an inductance of 4.5H and carries a current of 4A. If, when the distributor points close, the circuit current collapses uniformly to zero in 2ms, find the average e.m.f. induced in the coil (1 decimal place).

$$Eav = \frac{-L(I_2 - I_1)}{t}$$

$$= \frac{-4.5(0-4)}{2\times10^{-3}}$$

$$Eav = 9000V = 9kV$$

Coupling factor

There is a relationship between the mutual induction of 2 coupled coils and their individual self-inductances, depending upon the magnetic coupling between them. The mutual inductance of the 2 coils is expressed as the change in flux-linkages of one coil to the change in current in the other.

$$M = N_B \frac{d\Phi}{dI_A} \text{ while } M = N_A \frac{d\Phi}{dI_B}$$

$$\text{Hence: } M^2 = N_B \frac{d\Phi}{dI_A} \times N_A \frac{d\Phi}{dI_B}$$

and rearranging so,

$$M^2 = N_A \frac{d\Phi}{dI_A} \times N_B \frac{d\Phi}{dI_B}$$

$$\text{But as } L_A = N_B \frac{d\Phi}{dI_A} \text{ and } L_B = N_B \frac{d\Phi}{dI_B}$$

L_A and L_B will give thesame product M^2

$$\text{Thus } M^2 = L_A \times L_B$$

$$\text{or } M = \sqrt{L_A\, L_B}$$

This is the *maximum* value of mutual inductance available between the 2 coils, but this is difficult to achieve due to magnetic leakage and fringing effects. The magnetic coupling between the coils is also affected by their separation and the angular displacement between them. In general, mutual inductance is given by:

$M = k\sqrt{L_A L_B}$ where *k* is the coupling factor, where for perfect coupling k = 1.

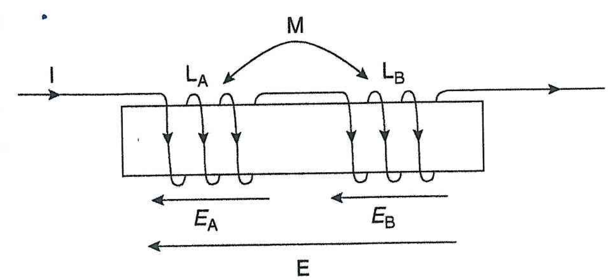
Inductance of 2 coils in series

In our study of mutual inductance, we have only considered the effect of 2 magnetically coupled but electrically separated coils. However, the effects of mutual- and self-inductance can be applied to electrically connected coils. Consider first the effects of 2 coils carrying the same current, and wound so that their magnetic fields *assists* one another (figure 7.4a).

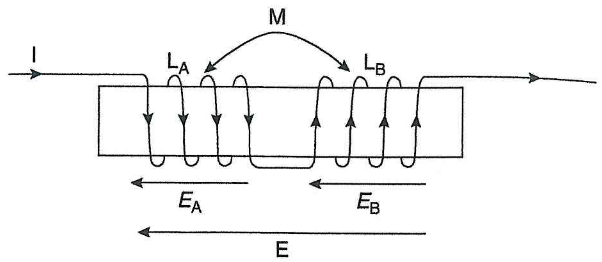
From our previous work on mutually coupled coils, we saw that a coil has an e.m.f. induced into it due to (1) the self-inductance of the coil and (2) the mutual inductance of the other coil.

$$\text{i.e. } E_A = \left(-L_A \frac{dI}{dt}\right) + \left(-M\frac{dI}{dt}\right) = -(L_A + M)\frac{dI}{dt}$$

$$\text{and } E_B = \left(-L_B \frac{dI}{dt}\right) + \left(-M\frac{dI}{dt}\right) = -(L_B + M)\frac{dI}{dt}$$



▲ Figure 7.4a



▲ **Figure 7.4b**

However, E_A and E_B are in series assisting each other

$$\therefore E = E_A + E_B$$

$$E = \left[-(L_A + M)\frac{dI}{dt}\right] + \left[-(L_B + M)\frac{dI}{dt}\right]$$

$$E = -(L_A + L_B + 2M)\frac{dI}{dt}$$

So that $E = -L\frac{dI}{dt}$ Where L is the total inductance.

$$\therefore L = L_A + L_B + 2M \text{ Henries (for coils assisting)}$$

Similarly we may consider 2 coils wound in *opposition* (figure 7.4b).

In this case the induced e.m.f. due to mutual inductance *opposes* that due to self-inductance. Hence, by similar proof to that shown for coils assisting, it is shown that when the coils oppose each other:

$$L = L_A + L_B - 2M \text{ Henries.}$$

Example 7.6. Two coils of inductances 10mH and 15mH respectively have a coupling factor of 0.8 between them. What is their combined inductance when they are connected in series (a) assisting and (3 significant figures) (b) opposing (2 significant figures)?

$$M = k\sqrt{L_A L_B} = 0.8\sqrt{10 \times 15 \text{ mH}}$$

$$= 9.8\text{mH}$$

(a) $L = L_A + L_B + 2M$

$= 10 + 15 + (2 \times 9.8)$ mH

$L = 44.6$mH

(b) $L = L_A + L_B - 2M$

$= 10 + 15 - (2 \times 9.8)$ mH

$L = 5.4$ mH

Magnetic Induction

It is worth considering the magnetic energy stored within the inductance component of the circuit, or its 'inertia'. If we assume that at time t = 0 a coil of inductance L Henries and resistance R is connected across the terminals of a battery of e.m.f. V. The circuit equation will be:

$$V - L\frac{dI}{dt} - IR = 0$$

Or rephrased as: $V = L\frac{dI}{dt} + RI.$ (1)

If the total power output of the battery is *VI* then the total work done by the battery in raising the current in the circuit from zero at time t = 0 to I_T at a later time t = *T* is:

$$W = \int_0^T VI \, dt. \quad (2)$$

Putting equation (1) into equation (2).

$$W = L\int_0^T I\frac{dI}{dt}dt + R\int_0^T I^2 dt,$$

giving the result

$$W = \frac{1}{2}LI_T^2 + R\int_0^T I^2\,dt.$$

The first term is the amount of energy stored in the inductor at time T, while the second term represents the irreversible conversion of electrical energy into heat in the resistor. The energy stored in the inductor can be recovered after the inductor is disconnected from the battery. For example, the energy in an inductance of 3 Henries/m with a current of 2A is given by:

$$W = \frac{1}{2}\times I_T^2 = \frac{1}{2}\times 3\times 2\times 2 = 6J$$

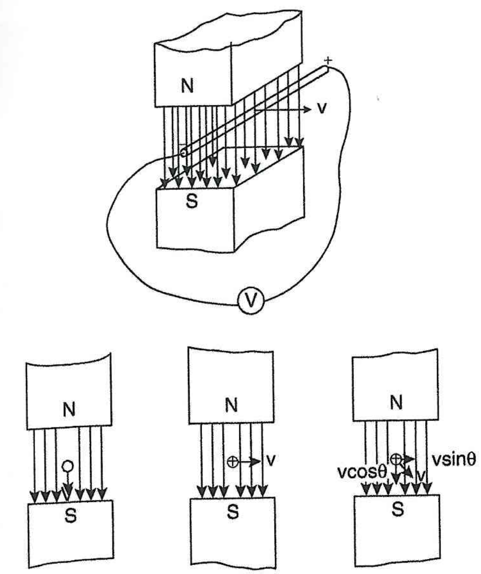
Dynamic Induction

As mentioned earlier, this condition covers cases where there is relative movement between a magnetic field and a conductor. Obviously this concerns either a stationary conductor and a moving field or a stationary field and a moving conductor, or even both moving together but relatively! To avoid repetition of basic theory, the immediate explanations and diagrams will refer to a fixed field and moving conductor.

The magnetic diagrams (figure 7.5) show a field produced by 2 permanent magnets and a conductor moved so as to cut the field, thus altering the flux-linkages.

Three cases are shown.

For case (a) there is no change of flux-linkages, i.e. no cutting of the magnetic field. The conductor moves at a velocity of v metres/second in the same direction of the lines of flux and no e.m.f. is recorded on the voltmeter. For case (b) the conductor moves at right angles to the field of flux density B teslas and the voltmeter shows a constant deflection. Flux-linkages are considered to change since the flux lines are 'cut' as a conductor passes through. If the conductor is moved from left to right, a polarity is noted, which reverses if the conductor is moved from right to left. Alternatively, if the field is reversed so that the flux lines pass from a bottom N pole to a S pole at the top of the diagram, and the conductor moved left to right, a reversed polarity is again indicated. The investigation will show further deductions. Thus:



▲ Figure 7.5

The magnitude of the induced e.m.f. varies with the speed of cutting the field or rate of change of flux-linkages. Hence $E \propto v$. Again, if the field cut is varied by altering the flux density, the e.m.f. will vary as B or $E \propto B$. Also, the longer the conductor cutting a field, the greater will be the magnitude of the e.m.f. and $E \propto \ell$. Summarising these 3 conditions we see that $E \propto B\ell v$, where ℓ is the length of the conductor in metres.

Case (c) of the diagram shows the conductor cutting the field at an angle θ. It is an intermediate condition between cases (a) and (b) and is best treated by resolving v into 2 component velocities at right angles to each other. Consider $v \cos\theta$ to be the component velocity in the direction of the flux lines, then $v \sin\theta$ will be the component of velocity at right angles to the field.

In accordance with the reasoning for cases (a) and (b) we see that velocity $v \cos\theta$ is responsible for no induced e.m.f. while velocity $v \sin\theta$ is responsible the induced e.m.f. and $E \propto v \sin\theta$.

$E \propto B\ell v \sin\theta$ is a more general expression than that already deduced and will cover all possible conditions.

For instance, for the condition of case (a) $\theta = 0°$ and as $\sin 0° = 0$ $\therefore$ $B\ell v \sin 0° = 0$ or $E = 0$ as stated.

Again for case (b), if $\theta = 90°$ then $\sin 90° = 1$ and $B\ell v \sin 90° = B\ell v$ giving $E = B\ell v$.

E.m.f. due to dynamic induction

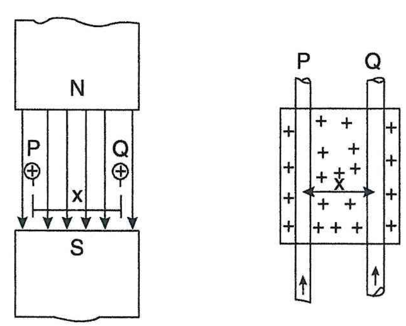
As explained, the induced e.m.f. is proportional to B, ℓ, v and the sine of the angle made by the direction of cutting and the field direction. The actual magnitude of such an e.m.f. can be deduced in more definite terms thus:

Consider case (b) of figure 7.5. In 1 second, the area cut by a conductor of length ℓ metres and *moving* at a velocity of v metres/second is ℓv square metres. If the flux density in this area is B teslas, then the flux cut per second by the conductor = $B\ell v$ webers. Using Faraday's law, we see that $B\ell v$ acts as a measure of the magnitude of induced e.m.f. in volts or induced e.m.f. $E = B\ell v$ volts.

If case (c) is considered the flux cut is proportional to the component of velocity perpendicular to the field or the induced e.m.f. $E = B\ell v \sin\theta$ volts. The above formula is deduced as follows: figure 7.6 shows a conductor Q, carrying a current of ℓ amperes in the direction shown. As before the flux density of the field is taken as B teslas and the length of the conductor is ℓ metres. A force is exerted on the current-carrying conductor in a magnetic field so the conductor in the diagram experiences a force $BI\ell$ newtons to the left. Accordingly a force of $BI\ell$ newtons must be applied in the opposite direction to oppose movement of the conductor.

Consider the conductor to move from position Q to position P x metres away. The work done by the conductor in moving from Q to P = Force × distance = $BI\ell x$ newton metres or joules.

Let E volts be the e.m.f. induced in the conductor as a result of cutting the magnetic field and t seconds the time taken to do this work.



▲ Figure 7.6

Then mechanical power expended = $\frac{BI\ell x}{t}$ watts and if this appears as electrical power, it will be EI Watts (e.m.f. × current) or $EI = \frac{BI\ell x}{t}$ and $E = \frac{B\ell x}{t}$ volts

As $\frac{x}{t}$ = velocity of the cutting v then as before, $E = B\ell v$ volts.

From above, $B\ell x$ is (the flux density × area) or the flux Φ cut by a conductor moving from position Q to P in time t seconds, and since

$$E = \frac{B\ell v}{t} \quad \text{then } E = \frac{B \times \text{area}}{t} = \frac{\Phi}{t} \quad \text{or} \quad E = \frac{\Phi}{t}$$

Thus $E(\text{volts}) = \frac{\Phi(\text{webers})}{t(\text{seconds})}$ and we have an alternative formula for the e.m.f. generated in a conductor cutting a magnetic field. It is similar to that deduced for the statically induced e.m.f. namely $E = \frac{N\Phi}{t}$ where N is the number of turns of the coil and Φ is the change of flux.

Thus the formula for $E = \frac{\text{flux-linkages}}{\text{time}}$ or $E = \frac{\Phi}{t}$ as the flux-linkages are numerically equal to Φ, there being only one conductor.

Important Note. $\frac{\Phi}{t}$ is $\frac{\text{Flux cut}}{\text{time}}$ and we have an alternative way of stating Faraday's law, now expressed as: 'The e.m.f. generated in a conductor is proportional to the rate of cutting lines of flux or is proportional to the flux cut/second.' This form of Faraday's law is more applicable to dynamic induction and will be used in connection with the generator, motor and alternator.

Example 7.7. A conductor is moved to cut a magnetic field at right angles. Find the e.m.f. induced in it, if the average density of the field is 0.45 teslas, the length of conductor is 80mm and the speed of cutting is 8.88 metres/second (2 decimal places).

In the Formula $E = B\ell v$ we have

$$E = 0.45 \times 0.08 \times 8.88 = 0.32\text{V}.$$

An alternate solution could be:

Area swept by the conductor/second = 0.08 × 8.88 m^2. The flux in this area would be $\Phi = 0.45 \times 0.08 \times 8.88$ webers, and e.m.f. = flux cut per second

$$\text{or } E = \frac{0.45 \times 0.08 \times 0.88}{1} = 0.32\text{V}.$$

Example 7.8. A 4-pole generator has a flux of 12mWb/pole. Calculate the value of e.m.f. generated in one of the armature conductors, if the armature is driven at 900 rev/min (2 decimal places).

In 1 revolution a conductor cuts $4 \times 12 \times 10^{-3} = 0.048$Wb (4 poles at 12mWb per pole).

$$\text{Time of 1 revolution of the armature} = \frac{1}{900} \text{ minutes} = \frac{60}{900} \text{ or } \frac{1}{15}\text{s}$$

$$\therefore \text{Rate of cutting flux} = \frac{\text{Flux cut per revolution}}{\text{time taken to complete a revolution}}$$

$$\text{Thus } E = \frac{0.048}{\frac{1}{15}} = 0.048 \times 15 = 0.72 \text{ volts/conductor.}$$

THE WEBER. 'The weber is that magnetic flux which, when cut by a conductor in one second, generates in the conductor an e.m.f. of value equal to one volt.'

Alternative ways of defining the weber or SI unit of flux are: 'An e.m.f. of one volt is generated when a conductor cuts flux at the rate of one weber/second', or 'an e.m.f. of one volt is generated when the flux linked with one turn changes at the rate of one weber/second.'

Direction of induced e.m.f. (hand rules)

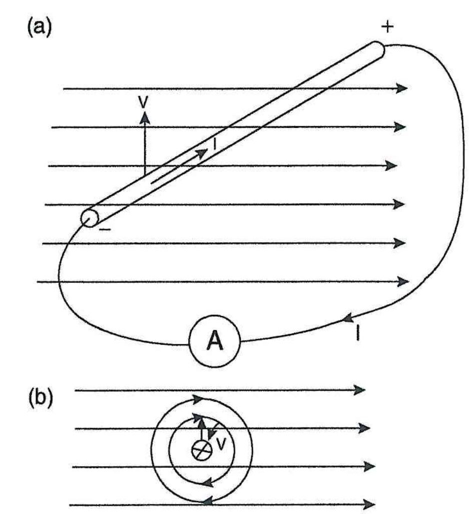
The direction of the induced e.m.f. is deduced from first principles, with Lenz's law or by application of a rule first enunciated by Professor T. A. Fleming and commonly known as FLEMING'S RIGHT-HAND RULE. It is noted that there is also a left-hand rule and to avoid confusion, the following is suggested for memorising the appropriate rule. The generator is studied *before* the motor and for the average person, use of the right hand is preferred *before* the left. Hence use the right-hand rule for the generator and the

in armature conductors through electrodynamic induction. The right-hand rule is now explained.

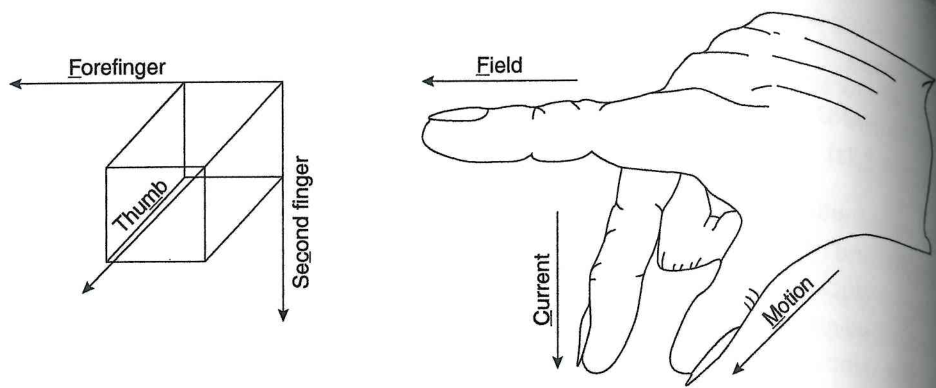
RIGHT-HAND RULE (Fleming's). Consider a conductor in a magnetic field as shown (figure 7.7a).

The magnetic field is in the direction left to right and the conductor moves at right angles and upwards with a velocity of *v* metres/second. The e.m.f. across the ends of the conductor is as shown, i.e. the polarity is such that, if the ends of the conductor are joined externally through an ammeter, current will flow as indicated. Its direction in the conductor is seen and if attention is given to figure 7.7b, it is observed that the field due to the conductor current is clockwise, to strengthen the field at the top and weaken the field at the bottom. According to Lenz's law, opposition is offered to the motion of the conductor as with field lines concentrating before the conductor. A force opposed to the direction of movement is apparent. The right hand can be drawn and used to find the induced current direction and thus the induced polarity. This is shown (figure 7.8).

To use the rule, place the thumb, index finger and second finger of the right hand at right angles to each other. Point the index finger in the direction of the flux lines and the thumb in the direction of moving the conductor. The current in the conductor, due to the induced e.m.f. will be in the direction indicated by the second finger. For the example considered (figure 7.6), current will be into the paper. The flux lines/index finger is to the right and the thumb/conductor up.



▲ Figure 7.7



▲ Figure 7.8

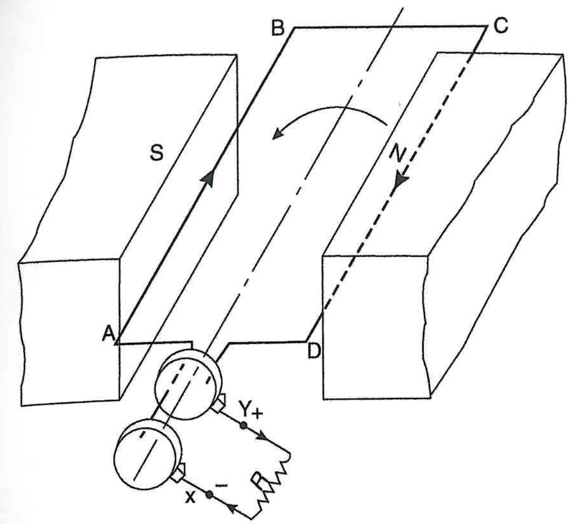
The Simple Magneto-Dynamo

Once the principles of electromagnetic induction were discovered, it was quickly realised that it was possible to construct a machine converting mechanical energy into electrical energy and generate electricity as a result of being driven by a steam engine or water turbine. The idea of making insulated conductors move through a stationary magnetic field presented no difficulties for a small machine and so the construction of such a magneto-dynamo followed fundamental requirements. A typical machine is illustrated (figure 7.9), and consists of permanent magnets to provide a field and a simple coil mounted on, yet insulated from, a shaft which can be rotated. To allow contact to be made with moving conductors, they are connected to slip-rings also mounted on but insulated from the shaft. Fixed *brushes* in turn, contact slip-rings to make sliding connections and allow an external circuit to be completed.

It is seen that the coil consists of 2 'active' conductors: AB and CD, connected in series by the connection BC (which, together with the front connections to the slip-rings, plays no part in the generation of e.m.f. but merely carry current to an external circuit). The load resistance of the external circuit is shown concentrated in *R*, connected to machine terminals: X and Y.

Consider the machine operation as follows:

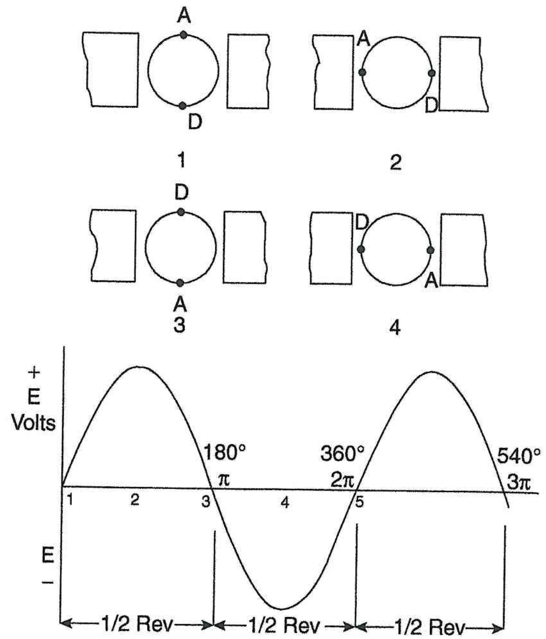
As one conductor AB moves down through the field, the other CD moves up and the induced e.m.f.s will be such that A is +ve relative to B and C is +ve relative to D. The induced current, if allowed to flow, will be as shown by the arrows and, as it is



▲ Figure 7.9

from terminal Y to terminal X through the external circuit, Y is +ve with respect to X. The student can check the right-hand rule for the polarity of the terminals in each half revolution. It is noted that the right-hand rule as described, can be applied to conductor AB, the condition being that AB is moving from the top vertical position round past the centre of the magnet pole and then to the bottom vertical position, moving anticlockwise. The position where it moves past the pole at right angles is of first importance, being a condition of maximum e.m.f.

After the coil has rotated a half revolution, conductor DC begins to move down and conductor AB up. The polarity induced is the reverse of that for the first half revolution, D being +ve relative to C and B is +ve relative to A. Terminal X is now the +ve terminal and Y is the negative. An alternating e.m.f. is generated (figure 7.10), which illustrates 4 positions of the coil viewed from the slip-ring end. For position 1, A and D are moving horizontally along the field and no e.m.f. is being generated. A similar condition exists for position 3, but for positions 2 and 4 maximum e.m.f. is being generated, as field-cutting at right angles is taking place. For intermediate positions of e.m.f. generation, as represented by $E \propto B\ell v \sin\theta$, is followed, as the conductors cut a uniform field at an angle but move at a uniform velocity. Thus e.m.f. generated at any instant is not constant but varies and it is customary to use a small letter for what is termed the *instantaneous* value. The expression $e = B\ell v \sin\theta$ volts gives the magnitude of the voltage generated. If voltage is plotted against revolutions in degrees or radians, a waveform such as that illustrated is obtained.

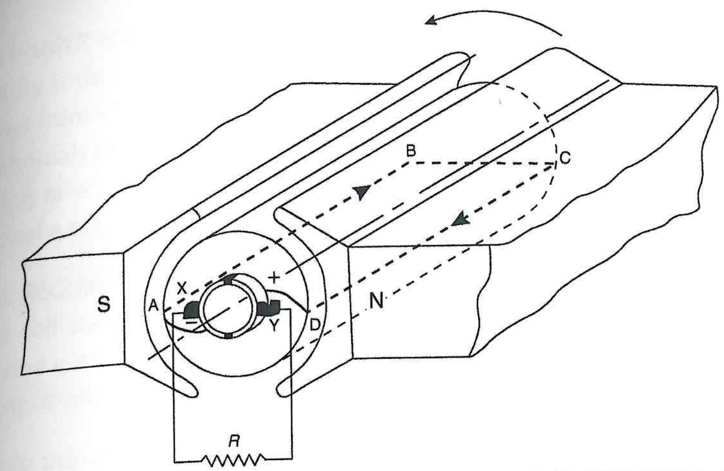


▲ **Figure 7.10**

The simple D.C. generator

The simple magneto-dynamo machine, as described (figure 7.9), or electrical generator, as it is now called has a uniform field arranged to be cut by the conductors. It provides an e.m.f. whose magnitude varies sinusoidally, that is, the e.m.f. polarity and value follows a sine wave. A sinusoidal waveform is desired for A.C. working but for a D.C. generator, modifications are needed to achieve a near constant unidirectional voltage magnitude and polarity. It is apparent that a distinction is being made between the generation of D.C. and A.C. and from here on the division between the 2 methods of generating, transmitting and utilising electrical energy will become marked. In this book, both D.C. and A.C. theory is discussed. The major portion of electrical theory is concerned with A.C. circuits and machines and if later study difficulties are to be minimised, attention must be given to A.C. theory at the start.

The first of the modifications referred to for the D.C. machine involves introduction of an iron or magnetic material into the armature or moving-coil part of the assembly. The coil made up from insulated wire is wound onto an iron armature carried on the shaft. The magnet system is provided with iron pole-shoes or arc extensions as shown (figure 7.11). As the length of the flux path through air is reduced to 2 small air gaps, the remainder



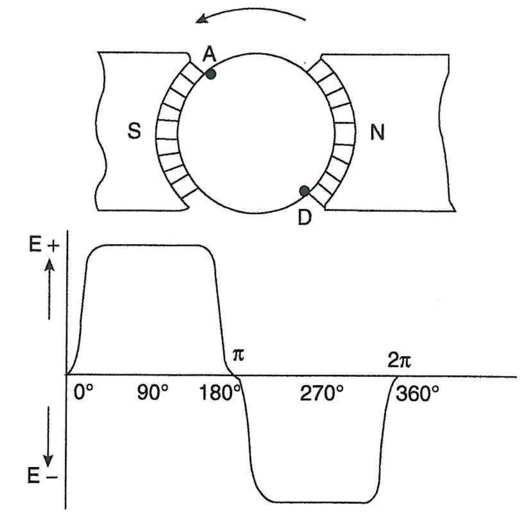
▲ **Figure 7.11**

gaps increases and the conductors cut a stronger field. As the air gaps are now small and of constant width, the flux lines will cross them as shown and the field will be uniform over the pole-faces. Moving conductors thus pass from a small arc with substantially no magnetic flux into a large arc of constant flux density. Flux lines are radial in the gaps and cut at right angles for most of the distance under the pole. The e.m.f. waveform is as shown (figure 7.12), i.e. proportional to the flux density through which it passes.

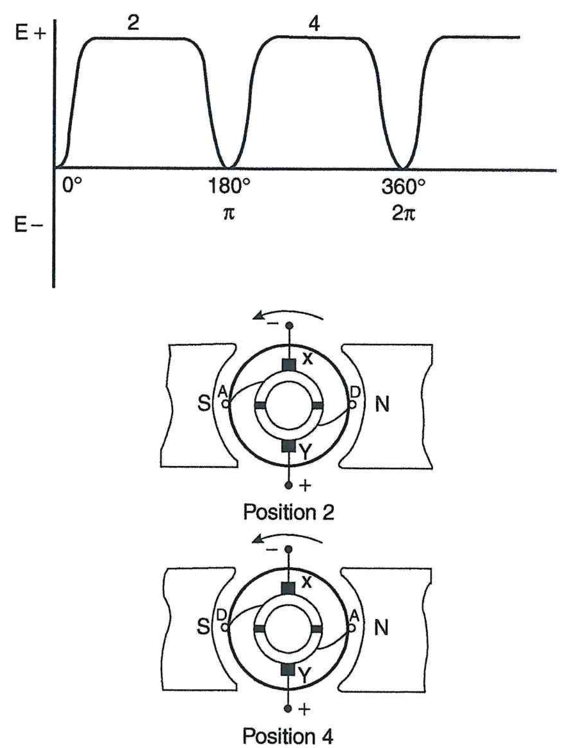
COMMUTATION. To obtain a constant unidirectional e.m.f. or true D.C. generator, the next modifying step is to fit a form of automatic reversing switch or *commutator* which, even though the moving coil continues to generate an alternating e.m.f., ensures that a unidirectional or 'rectified' e.m.f. appears at the machine terminals. Figure 7.11 shows how a commutator is fitted. It consists of a metal slip-ring which is split into 2 parts, each insulated from the other and from the shaft. The ends of the coil are connected to each half or segment of the commutator. The stationary brushes are adjusted that they bridge the gap in the slip-rings at the instant when the e.m.f. induced in the coil has zero value and is due to reverse.

Figure 7.13 shows the side view of the commutator and the reversing action of the switching arrangement is seen more clearly. The diagrams are considered to be complementary to those of figure 7.10 although only conditions for positions 2 and 4 are shown. The obvious position for the brushes is on the 'magnetic neutral axis' and that the brush Y will always be the +ve and brush X the −ve terminal. The new shape of the waveform is shown.

For position 2 it is seen that −ve end D of conductor CD is connected to the +ve brush Y, while the + ve end A of AB is connected to the −ve brush X. For position 4 when the



▲ Figure 7.12

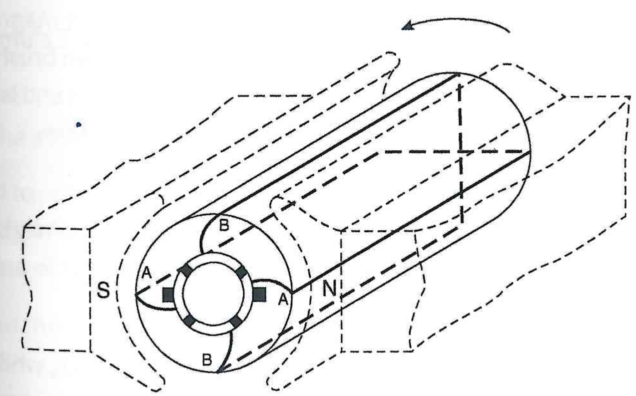


▲ Figure 7.13

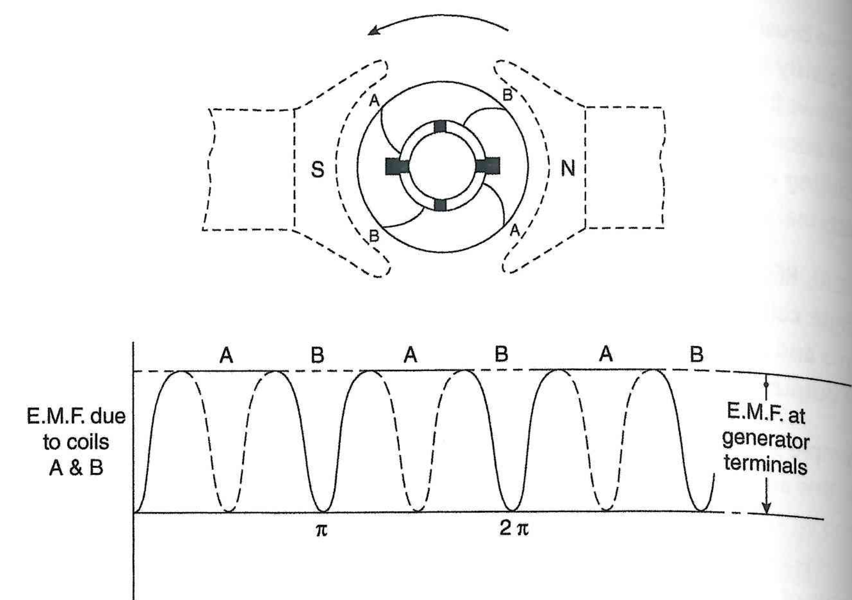
to the −ve brush X and end A of AB is now −ve and connected to the +ve brush Y. The brush polarity is decided by the direction of current flow in the external circuit. Thus current flows from Y (the +ve brush) to X (the −ve brush) and then to A onto B, etc. The apparent anomaly of +ve end A being connected to a −ve brush and so on is explained. The resulting effect of the commutating action is to produce a pulsating but overall unidirectional e.m.f. at the terminals of the generator.

PRACTICAL REQUIREMENTS. To obtain a more uniform e.m.f., the 2-part commutator and single coil can be repeated to give an arrangement employing a great many segments and a larger number of coils. Each coil consists of a number of turns to give a larger output voltage.

The example shown in figure 7.14 is an armature with 2 coils at right angles. It follows that for this arrangement when coil A develops maximum e.m.f., coil B generates no e.m.f. and when the armature rotates through a quarter of a revolution, the conditions will be reversed, *vice versa*. The accompanying diagram (figure 7.15) shows the waveforms of the generated e.m.f.s. The generator terminal voltage never falls to zero but 2 disadvantages are evident. First, all the conductors are not used to maximum advantage since only one coil at a time supplies the external circuit. Secondly, but of prime importance is the new condition of commutation. As the brushes must be placed in a position to contact the coil in which e.m.f. is being generated, it follows that if the generator is on load, i.e. supplying current, at the instant when the connected segments leave the brush, as an e.m.f. still exists and current is flowing, arcing can take place at the brushes. If coil A is considered and figures 7.14 and 7.15 noted, at the instant when the gap between segments is bridged by the brushes, coil A is still cutting the field and coil B has only just entered the field. Thus coil A tends to be short-circuited by a coil in



▲ Figure 7.14



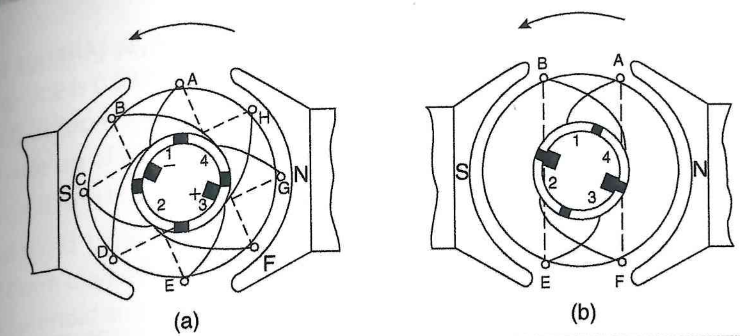
▲ Figure 7.15

which the e.m.f. may not have risen to the required value and current will flow instead in coil B. This current is diverted from the load current and adversely affects commutation. If the number of coils is increased, we achieve a smoother output voltage but arcing at the brushes may persist.

THE DRUM WINDING. Figure 7.16 shows the basic arrangement. This armature winding arrangement is accepted as the modern method of connecting active conductors together. The amount of 'copper' on the armature is used to maximum advantage as, except for the overhang at the back of a coil and the front connections to the commutator segments, the winding consists of copper conductors placed to cut magnetic flux and generate e.m.f. The basic winding uses several coils in series between brushes, arranged at constant angles to each other. The resultant e.m.f. is more uniform and larger as many coils in series are used. An example of a simple drum winding is given.

A, B, C, D, etc. are insulated conductors fitted into slots cut into the iron of the armature. There are also 4 commutator segments: Nos 1, 2, 3 and 4. Conductors may be connected to each other and to the segments in several ways and one possible arrangement is shown (figure 7.16a).

With rotation as shown, the e.m.f. in B, C and D are from front to back, while for F, G and H the e.m.f. are from back to front. The full lines show connection to segments and the dotted lines connections, which constitute the overhang of the coils, at the back.



▲ Figure 7.16

Starting at the –ve brush on segment 1, current enter the armature from the external circuit and divides into 2 parallel paths. One path passes to A and flowing down this comes at the back to F, hence up across to segment 2 and then onto and down C to H, from where it rises up and goes to the brush on segment 3. This would be the +ve brush. The other current path is from segment 1 to conductors D, G, B, E and onto segment 3 or the +ve brush. There are thus 2 circuits in parallel and it will also be noticed that, as a brush passes from one segment to the next, one coil is short-circuited and the brush must be located so as to short the coil at the instant when its e.m.f. and resultant current is zero. Such an instant is shown in figure 7.16b giving correct commutation conditions for the short-circuited coils between segments 1 and 2 and between segments 3 and 4.

Example 7.9. A slow-speed D.C. generator has an armature of diameter 3.0m and active conductors of length 510mm. The average strength of the field in the air gap is 0.8T and the armature speed is 200 rev/min. If the armature has 144 conductors arranged in 8 parallel paths, find the e.m.f. generated at the machine terminals (1 decimal place).

Using the formula $E = B\ell v$ volts so $B = 0.8$ teslas. $\ell = 510 \times 10^{-3}$m and v is obtained thus:

In 1 second the armature revolves $\frac{200}{60}$ or $\frac{10}{3}$ times. Also in

1 revolution one conductor travels $\pi d = 3.14 \times 3 = 9.42$m

So in 1 second the conductor travels $9.42 \times \frac{10}{3} = 31.4$m

E.m.f. generated per conductor $= 0.8 \times 510 \times 10^{-3} \times 31.4\text{m} = 12.8\text{V}$

Now there are 144 conductors in 8 parallel paths with conductors in series in each parallel path

$$= \frac{144}{8} = 18$$

Thus the e.m.f. generated in 1 parallel path = 18 × 12.8 = 230.4V

Practice Examples

7.1. Calculate the e.m.f. in mV generated in the axles of a railway train when travelling at 100km/h. The axles are 1.4m in length and the component of the earth's magnetic field density is 40μT (2 decimal places).

7.2. Find the generated e.m.f./conductor of a 6-pole D.C. generator having a magnetic flux/pole of 64mWb and a speed of 1000 rev/min. If there are 468 conductors, connected in 6 parallel circuits, calculate the total generated e.m.f. of the machine (1 decimal places). Find also the total power developed by the armature when the current in each conductor is 50A (5 significant figures).

7.3. An iron-cored coil of 2000 turns produced a magnetic flux of 30mWb when a current of 10A is flowing from the D.C. supply. Find the average value of induced e.m.f. if the time of opening the supply switch is 0.12s. The residual flux of the iron is 2mWb (1 decimal place).

7.4. A 1-turn armature coil has an axial length of 0.4m and a diameter of 0.2m. It is rotated at a speed of 500 rev/min in a field of uniform flux density of 1.2T. Calculate the magnitude of the e.m.f. induced in the coil (3 decimal places).

7.5. When driven at 1000 rev/min with a flux pole of 20mWb, a D.C. generator has an e.m.f. of 200V. If the speed is increased to 1100 rev/min and at the same time the flux/pole is reduced to 19mWb, what is then the induced e.m.f. (3 significant figures)?

7.6. A coil of 200 turns is rotated at 1200 rev/min between the poles of an electromagnet. The flux density of the field is 0.02T and the axis of rotation is at right angles to the direction of the field. The effective length of the coil is 0.3m and the mean width 0.2m. Assuming that the e.m.f. produced is sinusoidal, calculate (a) the maximum value of e.m.f. (b) the frequency (1 decimal place).

7.7. A coil of 1200 turns is wound on an iron core and with a certain value of current flowing in the circuit, a flux of 4mWb is produced. When the circuit is opened, the flux falls to its residual value of 1.5mWb in 40ms. Calculate the average value of

7.8. The armature of a 4-pole generator rotates at 600 rev/min. The area of each pole-face is $0.09m^2$ and the flux density in the air gap is 0.92T. Find the average e.m.f. induced in each conductor (3 decimal places). If the armature winding is made up of 210 single-turn coils connected so as to provide 4 parallel paths between the brushes, find the generator terminal voltage (2 decimal places).

7.9. A solenoid 1.5m long is wound uniformly with 400 turns and a small 50 turns coil of 10mm diameter is placed inside and at the centre of the solenoid. The axes of the solenoid and the coil are coincident. Calculate (a) the flux in μW linked with the small coil when the solenoid carries a current of 6A and (3 decimal places) and (b) the average e.m.f. induced in mV in the small coil when the current in the solenoid is reduced from 6A to zero in 50ms (3 decimal places).

7.10. Two coils A and B having 1000 and 500 turns respectively are magnetically coupled. When a current of 2A is flowing in coil A it produces a flux of 18mWb, of which 80% is linked with coil B. If the current of 2A is reversed uniformly in 0.1s, what will be the average e.m.f. in each coil (3 significant figures)?

8

ELECTROSTATICS AND CAPACITANCE

Have them make an ark of acacia wood – two and a half cubits long a cubit and a half wide, and a cubit and a half high. Overlay it with pure gold, both inside and out, and make a gold moulding around it.

Exodus 25.10–11 (New International Version)

Electric field

The term electric field has been used already when discussing the P.D. required for electron movement in a circuit. It will be given some attention here, as it is directly associated with electron or current flow.

Elements, such as metals, have the same electrical properties as their atoms. If these atoms are charged and become ions (either through removal of electrons making them +ve, or through an excess of electrons making them –ve), the body of which they are part, will be either +ve or –ve. Since atoms seek to remain neutral, ions will try to gain or lose electrons through exchange with neighbouring atoms. The same property exists for charged bodies and if a +ve charged body (deficient in electrons) is placed in contact with a –ve charged body (with excess electrons), electron flow will occur from the –ve body to the +ve body (with current flowing in the opposite direction) until both have the same degree of charge. Before bodies are placed in contact with each other, a force will be detected between them and the adjacent space will show signs of this force. The space within which this force may be detected contains the

electric field. A P.D. is said to exist between charged bodies, resulting in an electric field. When the bodies are placed in contact, an equalising of charges takes place with the basic requirement that a P.D. must exist between 2 points before a current flows. Earlier studies showed that an appropriate electrical device, for example, a battery or generator, functions by developing an e.m.f., resulting in a P.D. between its terminals and between the 2 bodies. If an e.m.f. or P.D. is maintained continually with a battery, then a continuous current will flow. The main subject of this book deals with *dynamic electricity* and its effects. However, an e.m.f. or P.D. is only maintained until the current starts to flow and then falls to zero, as the charges on both bodies equalise, then *static electricity or electrostatics* will be the subject of this chapter.

Before commencing electrostatics, students are reminded that the unit of charge carried by the electron is *too small* for practical purposes. Experiment shows the –ve charge of an electron = 16×10^{-19} coulombs, where 1 coulomb = 6.25×10^{18} electron charges. Passage of charges constitutes a current and the practical current unit is the ampere, defined in terms of the coulomb and the time taken for this to pass. Thus if a charge of 1 coulomb takes 1 second to pass through a point in a circuit, the rate of flow of electricity is 1 coulomb per second and the current is 1 ampere.

Thus 1 ampere (1A) = 1 coulomb/second.

Both the coulomb and the ampere are used as practical engineering units and defined earlier. The properties of conductors and insulators were also described earlier and explained in terms of electron theory where:

A *conductor* is a material like metal, carbon and certain liquids, which contains mobile electrons that move under the influence of an applied P.D. and allow free passage of current.

An *insulator* is a material with few free electrons, such as rubber, glass, mica and most oils, in which electrons are bound strongly to a nucleus. As little electron movement occurs, current flow is negligible.

Electrostatics

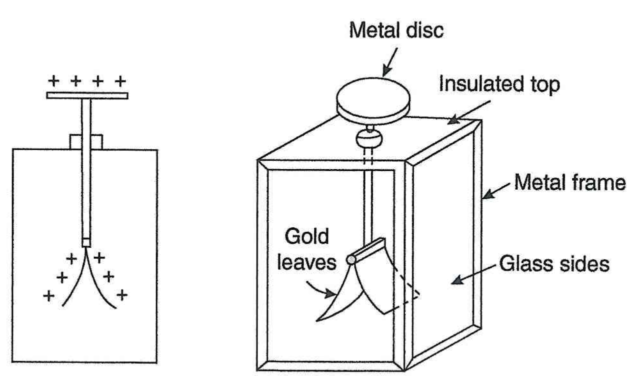
Mention has already been made of static charges known from ancient times because of their attractive and repulsive effects. The Greeks knew that rubbed amber attracted light bodies such as cork and fibrous material. Amber is said to be 'charged' with electricity and the phenomena discussed here deals with the presence of electric charges at rest, i.e. electrostatics

Experiments show that the easiest method of generating static electricity is by rubbing or friction. Thus a glass rod rubbed with silk is electrified to attract pieces of paper, but if a similarly treated glass rod is suspended by a thread, and brought near the original charged glass rod, a repulsive effect is noted. An ebonite rod rubbed with fur also becomes charged and, if brought near a suspended charged glass rod, attraction is now noted.

Summarising, glass and ebonite acquire charges are of 2 types: positive (+ve) and negative (−ve), and that like charges *repel* while unlike charges *attract*. Allocation of charge type, +ve charge to the glass and −ve charge to the ebonite, is arbitrary, but all uncharged bodies consist of +ve and −ve charges which neutralise each other. If these charges are separated by an applied force, their presence is detectable and if they move from one body to another their movement is explained by the passage of current. It is noted that these assumptions agree with electron theory and it is apparent that a −ve charged body has an excess of electrons and a +ve charged body a lack of electrons. In the uncharged state, atoms of a material are neutral, i.e. charges due to electrons and protons exactly balance.

THE ELECTROSCOPE. Deductions made from experiments in electrostatics are key to theory, and for demonstrations a simple charge detector is needed. Such a detector, the gold-leaf electroscope, is often used for investigations, consisting of 2 leaves of gold foil attached to a metal rod, which is held in a glass jar from which it is insulated (figure 8.1). A metal disc may be fitted to the rod and the container may be a metal box-like frame with glass sides. Electroscope leaves hang down when no charges are present, but if charge is imparted to the instrument the leaves diverge.

Assume a +ve charge is given to the electroscope by stroking the disc with a glass rod charged by rubbing with silk. The +ve charge imparted to the disc spreads over



▲ Figure 8.1

the insulated metal and the leaves, which having the same charges, repel and diverge (figure 8.1). If an ebonite rod, −ve charged from rubbing with fur, is brought near the electroscope, the leaves converge. The explanation is that the +ve charges on the electroscope are attracted by the −ve charged rod and rise up to concentrate in the area of the disc. The charge on the leaves thus diminishes and the repulsion force between them falls. In the same way, a +ve charged glass rod brought near the instrument causes the leaves to diverge further as the +ve charges present are repelled down towards the leaves. Charge density in this region increases and increased divergence is observed. The instrument, is not used outside a laboratory, but is useful for demonstrations.

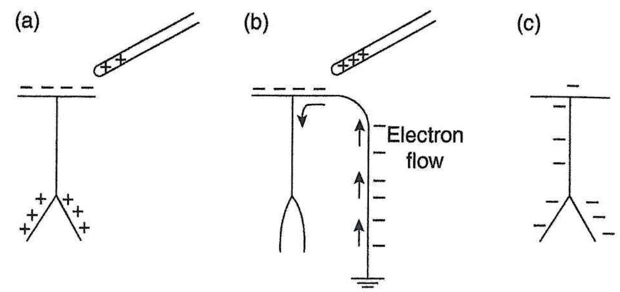
Potential difference (P.D.)

When 2 bodies charged as described are brought into contact, a small current flows between them while the charges equalise, during which a P.D. exists between them. For the bodies to maintain their charges they must be insulated from earth, i.e. mounted on insulating rods. With the electroscope charge is given to it is with respect to earth; the gold leaves, rod and disc being one body and the mass of earth being the other. Hence leaves are charged +ve to earth if a +ve charge is given to the electroscope. Similarly a −ve charge given to the electroscope makes the leaves −ve with respect to earth. It is seen that if 2 bodies are charged +ve and −ve, they are at a potential to each other, i.e. a P.D. exists between them and they are also at a potential to earth. One body is +ve to earth and the other −ve to earth, the earth mass considered to be at zero potential.

Electrostatic charging

Frictional effects have been mentioned and in practical engineering, it is the most dangerous cause of electric charging. Build-up of charges must be considered and precautions taken in the artificial silk, paper, rubber, cable-making and associated industries to discharge coils of material after processing. Such processes involve kneading, rolling, drawing, etc. and friction may generate large voltages, which could be dangerous to those handling the material. The electrostatic charging of aircraft and motor-vehicles is a well-known hazard. In the case of the former, because of the large voltage levels, a means of earthing is needed before people can even leave aircraft. Electrically conducting rubber tyres have been developed to this end. For motor-cars the problem is not as important, as the resulting charges are small. However, precautions are needed for special load-carrying road vehicles such as petrol-tankers, which must be 'earthed' before loading or unloading of fuel commences. The action of the 'lightning conductor' will be mentioned shortly, but its use is concerned with the +ve and –ve charging of clouds, arising from atmospheric activity.

CHARGING BY INDUCTION. Imagine an electroscope to be uncharged and a +ve charged body brought near to the disc. Mobile metal electrons are attracted to the disc and +ve charged atoms or ions are displaced to the leaves (figure 8.2a). If the disc is touched with the charged rod (figure 8.2b), electrons from the main earth mass flow up and neutralise ions or +ve charges so the leaves collapse or converge. If then the rod is taken off and the adjacent +ve charged body removed, leaves diverge slightly (figure 8.2c). The –ve charges, previously held by the adjacent +ve charged body, spread over the electroscope and charges of the same polarity spread down to the leaves.

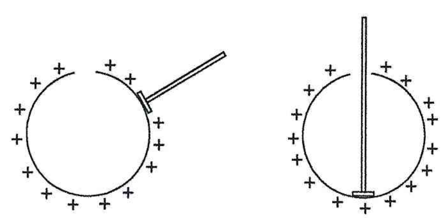


▲ Figure 8.2

Charging by induction produces a charge of opposite polarity, for example, the inducing charge was +ve so a –ve charge results on the instrument. If a –ve inducing charge is used, a +ve charge results instead.

Distribution of charge

The statements set out below are the results of experiments with a charged electroscope and a proof plane – a small metal disc fitted with an insulated handle. A proof plane is placed in contact with the body being investigated and electrified to the same polarity.

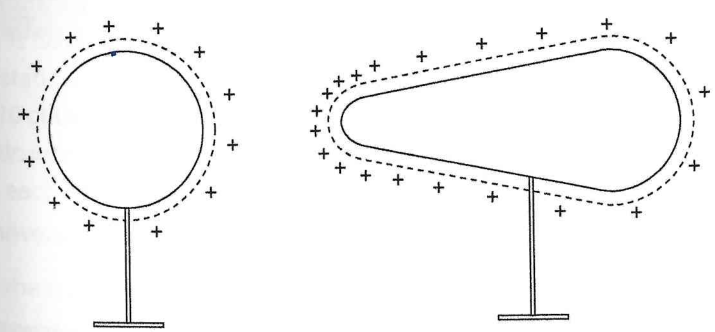


▲ Figure 8.3

If brought near a charged electroscope, movement of the leaves with the appropriate interpretation enables conclusions to be made. The following are deductions resulting from such investigations (figure 8.3).

1. A hollow body only charges on the outside. So a proof plane contacted with the outer surface and presented to a charged electroscope, shows a deflection. If contacted with the inside, it shows no deflection.
2. If a sphere is charged, charge spreads uniformly over its surface and the *surface charge density* is uniform. If a charged body is non-spherical, charge concentration is greatest in the region where the radius of curvature is the *smallest*. Figure 8.4 depicts this, with the charge distribution or surface charge density represented by a dotted envelope.
3. If a charged body has a sharp point, charge concentrates at the point and surface density may be so great that dust or particles in the air coming in contact with the body are charged and repel. On moving away, each particle takes a small portion of the original charge and the effect of a point on a charged body is to discharge the body.

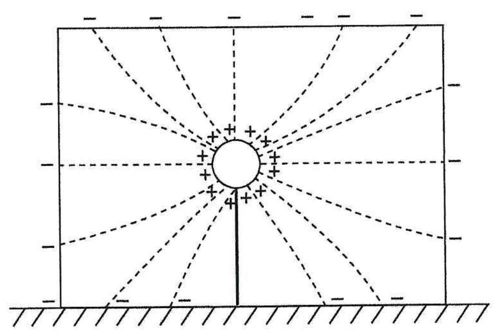
Let us now consider the action of a lightning conductor. Take a +ve charged cloud near a high building fitted with a lightning conductor, consisting of a copper rod and a well-earthed conductor. The building and rod acquire a –ve charge by induction, with air particles becoming –ve by contact which move towards the cloud seeking to neutralise it. Alternatively, the space between the cloud and building being charged by the –ve particles has its insulating effect lowered until a breakdown occurs and a spark (lightning discharge) passes between the cloud and earth. A current flows during the discharge as electrons pass from earth to the cloud which is discharged safely. Current will select the path of least resistance, and so is conducted along the most suitable path, the lightning rod, and damage is avoided.



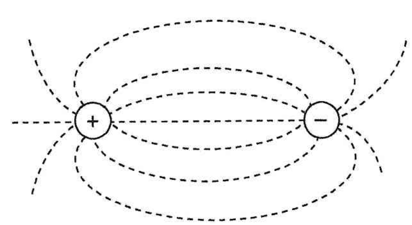
▲ Figure 8.4

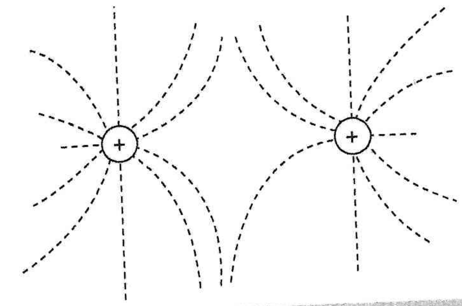
Electrostatic fields of force

When 2 bodies are charged, a force of attraction or repulsion is produced depending on the charges' polarities. The size of this force relative to the charges can be investigated if the existence of *lines of flux* are assumed. A force is exerted on a small +ve charged body placed near a large +ve charged body. An electric field of force exists in the space around a large charged body. If a small body is small enough to constitute a point with +ve charge then, if free to move, it will travel in a fixed direction in the electric field and the path traced by it will represent a line of flux. Thus the large +ve charged body will have many lines of flux passing out from it. The similarity with the representation used for magnetic fields is noted. For electrostatic fields (figure 8.5), one key fact is observed. Each flux line terminates at the *surface* of a charged body and doesn't pass through the body to form a closed path, as is the case for magnetic flux lines. The medium through which the electric flux lines pass is called the *dielectric* and lines terminate at the *surface* of another body where balancing charges of opposite polarity appear. Thus figure 8.5a shows a +ve charged metal sphere in the centre of a room. Lines leave the surface perpendicularly, in all directions and planes, terminating on earthed walls, floor

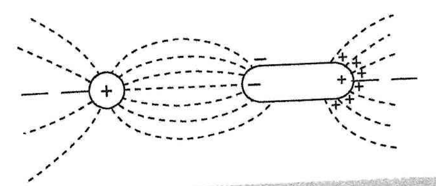


▲ Figure 8.5a





▲ Figure 8.5c



▲ Figure 8.5d

and ceiling, the earth mass being −ve to the charged body. Figure 8.5b shows the field arrangement associated with 2 bodies when charged in opposition, and figure 8.5c shows the field with like charges on the bodies. Figure 8.5d, shows the arrangement when an uncharged body is placed in the field and how induced polarities result. As for the magnetic field, flux lines are imagined to behave like elastic threads which contract if permitted.

Electrostatic flux

As for the magnetic field we also introduce a term called 'flux' for the electric field. The symbol Ψ (Greek letter psi) and the number of electrostatic lines of flux passing through a medium is called the flux. As the practical charge unit is the coulomb, to establish an electrostatic unit we consider one line of flux to originate from 1 coulomb. So a charge of 10 coulombs produces 10 lines of flux, or $\Psi = 10$ coulombs. As most practical electrostatic work uses *capacitors* made from flat plate-like conductors, adjacent and parallel to each other, and as the medium between the plates or dielectric carries the flux, it is convenient to introduce the term *electric flux density* – symbol D, with $D = \frac{\Psi}{A}$ where A is the area of the dielectric in square metres. Then $D = \frac{\Psi}{A}$ or $\frac{Q}{A}$ coulomb per square metre.

Electric potential

The basic idea of electric potential was introduced earlier. If 2 bodies are charged and connected then as a current flowed while the charges equalise, there must have been a difference of potential between them. Darth mass is taken to be at zero potential so if a body charged with Q coulombs of electricity is connected to earth, a current flows. Current will be from the body to earth if it has +ve charge, and from earth to the body if it has −ve charge. Work is done in this process, so if we consider 1 joule of work done while 1 coulomb is passed, the P.D. of the body's electric potential will be 1 volt, where voltage measures the P.D. between the body and earth.

In a practical capacitor, plates are charged with respect to each other. If a +ve unit charge is placed in the field between charged capacitor plates, a force will push the +ve charge towards the −ve plate and is a measure of the *intensity* or *strength* of the field. The symbol for electric field strength or field intensity is E and the force is measured in newtons. So a charge of Q coulombs placed in an electric field E will experience a force of $F = QE$ newtons or $E = \frac{F}{Q}$ newtons per coulomb.

Alternatively field strength is the electric force or electrical potential gradient, measured in volts per metre.

Thus, $E = \frac{V}{d}$ where d is the distance between the plates in metres.

Both expressions for electric field intensity and potential gradient or electric force are numerically equal. If practical units are substituted, the same amount of work done by 1 newton of force acting through 1 metre distance between the plates as done by 1 coulomb conveyed by a pressure of 1 volt.

$$\text{Thus, since } E = \frac{F}{Q} \text{ or } \frac{V}{d} \text{ then } \frac{F}{Q} = \frac{V}{d} = \text{or } Fd = VQ$$

Note.

1 joule = 1 volt × 1 coulomb, or 1 joule = 1 newton × 1metre.

Capacitance

The capacitor

Several references have already been made to the electrical capacitor (or condenser as it was called). In its simplest form, it consists of 2 metal plates separated from each other by an air gap. As will be seen, the plates' area, separation and type of dielectric insulating medium, all affect capacitor's performance, but the key fact learned from such experiments is that capacitors store electricity. Thus if the plates are connected to a supply source through a sensitive milliammeter, a current will pass at the instant the switch closes. The current quickly falls to zero, as the P.D. between the plates rises, as indicated by a voltmeter. The form of current fall-off is a topic itself, but a capacitor will attain a 'charged' state. If the supply is disconnected and the plates shorted together, a discharge current flows in the opposite direction, and although initially large, soon decays to zero. The initial voltage, although showing the value of the charging supply, will also decay to zero.

Experiments with a simple capacitor establish a basic relationship between the quantity of electricity Q that can be stored and the charging voltage V. The former is proportional to the latter or $Q \propto V$. As this is directly proportional, a constant is introduced to give the expression:

$$Q = CV$$

C is termed the *capacitance* and is a unit defined in terms of unit quantity and unit voltage. The unit of capacitance is the *farad* and a capacitor has a capacitance of 1 farad if 1 coulomb of electricity is stored when 1 volt is applied across the plates.

Example 8.1. Find how many electrons are displaced when a P.D. of 500V exists between the plates of a 4μF capacitor (3 significant figures).

$$\text{Since } Q = CV$$

then $Q = 4 \times 10^{-6} \times 500 = 2 \times 10^{-3}$ coulombs, but 1 coulomb $= 6.3 \times 10^{18}$ electrons

$$\therefore \text{No of electrons} = 12.6 \times 10^{15}.$$

Capacitor systems

Capacitors can be connected in series or parallel and the student should compare the expressions giving equivalent capacitance values with those giving equivalent resistance values, for comparable arrangements.

SERIES CONNECTION. The arrangement is shown in figure 8.6.

Let capacitors have values: C_1, C_2 and C_3 farads respectively, and the applied voltage V dropped as shown.

Then as $V = V_1 + V_2 + V_3$ and,

Since $V_1 = \frac{Q_1}{C_1}$ $V_2 = \frac{Q_2}{C_2}$ and $V_3 = \frac{Q_3}{C_3}$

we can write:

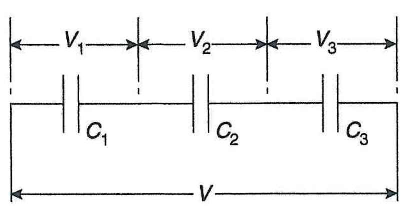
$$V = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

If C is taken to be the equivalent capacitance of the arrangement then:

$$V = \frac{Q}{C}$$

$$\text{or } \frac{Q}{C} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2} + \frac{Q_3}{C_3}$$

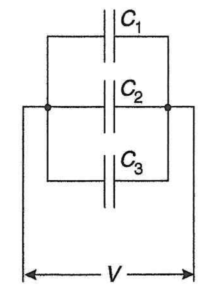
but the same current flows through each capacitor for the same time. $\therefore Q = Q_1 = Q_2 = Q_3$ and the above can be simplified to:



▲ Figure 8.6

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \text{ etc.}$$

PARALLEL CONNECTION. The arrangement is shown in figure 8.7 with the same voltage applied to each capacitor.



▲ Figure 8.7

Then for each capacitor $Q_1 = C_1V$ $Q_2 = C_2V$ and $Q_3 = C_3V$

If the total quantity of charge = Q then,

$$Q = C_1V + C_2V + C_3V = V(C_1 + C_2 + C_3)$$

$$\text{or } \frac{Q}{V} = C_1 + C_2 + C_3$$

If C is the equivalent capacitance of the arrangement, then

$$Q = CV \text{ or } CV = V(C_1 + C_2 + C_3)$$

$$\text{Such that } C = C_1 + C_2 + C_3.$$

Example 8.2. If 2 capacitors of values 100μF and 50μF respectively are connected (a) in series (4 significant figures) and (b) in parallel (3 significant figures) across a steady applied voltage of 1000V, calculate the joint capacitance.

(a) Series. Joint capacitance C is given by:

$$\frac{1}{C} = \frac{1}{100} + \frac{1}{50}$$

$$C = \frac{100}{3} = 33.33\ \mu\text{F}.$$

(b) Parallel. Joint capacitance is given by $C = 100 + 50$ or $C = 150\ \mu$F.