CINEC CAMPUS
Beyond A Graduate

PAST PAPERS

Faculty	Department / Section/Division
Not Applicable	Learning Resource Centre

Past Papers

Faculty of Engineering & Technology

Department of Interdisciplinary studies

Document Control & Approving Authority	Senior Director – Quality Management & Administration

1st Issue Date: 2017.011.30	Revision No.00	Revision Date: 23.07.2025	Validated by: Librarian

Faculty of Engineering & Technology

Department of Interdisciplinary studies

STEM Foundation

Semester 2 -In class Test 4

Module Code	:	STEM 1511
Module Title	:	Foundation Mathematics II
Date	:	2nd June 2025
Examiners	:	Hiruni Wickramasinghe
Time Allowed	:	(09 00 am - 11.00 am)- 2 hrs.

Instructions to Candidates:

- This is a closed book examination. No reference materials are allowed.
- This paper consists of **four (04) questions**. Answer all questions.
- Use only the provided answer sheets for your answers.
- **Calculators are not allowed.**
- Write clearly and legibly using a blue or black pen.
- Write your student number on *all* sheets.
- Start each question on a new page in the answer booklet. Clearly indicate the question number at the top of the page.
- If parts of the same question are written discontinuously in different sheets, ensure that each part is appropriately labelled with its corresponding question number.

1. Perform the indicated operation and write your answer in standard form.
   a) $(4-5i)(12+11i)$
   b) $(1+4i)-(6-7i)$
   c) $8i(10+2i)$
   d) $\frac{6+7i}{8-i}$ **(20 marks)**

2. The complex numbers z_1 and z_2 are given by

$$z_1 = p + 2i \text{ and } z_2 = 1 - 2i$$

   Where p is an integer
   a) Find $\frac{z_1}{z_2}$ in the form $a + bi$ where a and b are real. Give your answer in its simplest form in terms of p.
   b) Given that $\left|\frac{z_1}{z_2}\right| = 13$, Find the values of p. **(20 marks)**

3. The cost (in cents) per 1-ounce serving for a sample of 11 chocolate chip cookies data is as follows:

Sample	Cost (in cents)
1	54
2	22
3	25
4	23
5	36
6	43
7	25
8	47
9	24
10	45
11	44

   a) Compute the mean, median and mode
   b) Compute the first quartile (Q_1),the third quartile(Q_3), and the interquartile range.
   c) List the five-number summary using the answers obtained in part a and b.
   d) Construct a boxplot and describe its shape. **(40 marks)**

4. A table of 5 students has 3 seniors and 2 juniors. The teacher is going to pick 2 students from this group at random to present homework solutions.
   a) Find the probability that both students selected are juniors
   b) Find the probability that both students selected are seniors
   c) Find the probability that at least one student selected is junior. **(20 marks)**

Library

00044

Faculty of Engineering & Technology

Department of Interdisciplinary studies

STEM Foundation

Semester 2 -In class Test 3

Module Code	:	STEM 1511
Module Title	:	Foundation Mathematics II
Date	:	30th May 2025
Examiners	:	Hiruni Wickramasinghe
Time Allowed	:	(09 00 am - 11.00 am)- 2 hrs.

Instructions to Candidates:

- This is a closed book examination. No reference materials are allowed.
- This paper consists of **five (05) questions**. Answer all questions.
- Use only the provided answer sheets for your answers.
- **Calculators are not allowed.**
- **Additional Information is provided on page 3**
- Write clearly and legibly using a blue or black pen.
- Write your student number on *all* sheets.
- Start each question on a new page in the answer booklet. Clearly indicate the question number at the top of the page.
- If parts of the same question are written discontinuously in different sheets, ensure that each part is appropriately labelled with its corresponding question number.

1. Evaluate each of the following indefinite integrals, giving the answers in exact form
   a) $\int (5x^2 - 8x + 5)dx$
   b) $\int 7\sin(3x)\,dx$
   c) $\int 2\,dx$
   d) $\int 4e^{-7x}dx$
   e) $\int \frac{x^2+4}{x^2}dx$ **(50 marks)**

2. Evaluate the following definite integrals.
   a) $\int_1^9 (x^{\frac{3}{2}} + 2x + 3)dx$
   b) $\int_4^9 \left(\sqrt{x} + \frac{1}{3\sqrt{x}}\right)dx$
   c) $\int_1^4 \frac{5}{x^3}dx$ **(40 marks)**
   d) $\int_{-1}^2 (1+3t)t^2dt$

3. By using substitution method, evaluate the following integrals.
   a) $\int \frac{2x}{(4+3x^2)^2}dx$ (Hint: take $u = 4 + 3x^2$) **(40 marks)**
   b) $\int_0^1 5x(1-x^2)^{\frac{3}{2}}$ (Hint: take $u = 1 - x^2$)

4. By using integration by parts method, evaluate the following integrals
   a) $\int_0^{\frac{1}{3}} xe^{3x}dx$ **(40 marks)**
   b) $\int_0^{\frac{\pi}{4}} 4x\cos 4x\,dx$

5. Evaluate $\int_0^4 \frac{5x+13}{(2x+1)(x+4)}$ using partial fraction method. **(30 marks)**

Additional Information

Integration

$f(x)$	$\int f(x)\,\mathrm{d}x$	
k, any constant	$kx + c$	
x	$\frac{x^2}{2} + c$	
x^2	$\frac{x^3}{3} + c$	
x^n	$\frac{x^{n+1}}{n+1} + c$	$n \neq -1$
$x^{-1} = \frac{1}{x}$	$\ln\lvert x\rvert + c$	
e^x	$\mathrm{e}^x + c$	
e^{kx}	$\frac{1}{k}\mathrm{e}^{kx} + c$	
$\cos x$	$\sin x + c$	
$\cos kx$	$\frac{1}{k}\sin kx + c$	
$\sin x$	$-\cos x + c$	
$\sin kx$	$-\frac{1}{k}\cos kx + c$	
$\tan x$	$\ln(\sec x) + c$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$
$\sec x$	$\ln(\sec x + \tan x) + c$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$
$\operatorname{cosec} x$	$\ln(\operatorname{cosec} x - \cot x) + c$	$0 < x < \pi$
$\cot x$	$\ln(\sin x) + c$	$0 < x < \pi$
$\cosh x$	$\sinh x + c$	
$\sinh x$	$\cosh x + c$	
$\tanh x$	$\ln\cosh x + c$	
$\coth x$	$\ln\sinh x + c$	$x > 0$
$\frac{1}{x^2+a^2}$	$\frac{1}{a}\tan^{-1}\frac{x}{a} + c$	$a > 0$
$\frac{1}{x^2-a^2}$	$\frac{1}{2a}\ln\frac{x-a}{x+a} + c$	$\lvert x\rvert > a > 0$
$\frac{1}{a^2-x^2}$	$\frac{1}{2a}\ln\frac{a+x}{a-x} + c$	$\lvert x\rvert < a$
$\frac{1}{\sqrt{x^2+a^2}}$	$\sinh^{-1}\frac{x}{a} + c$	$a > 0$
$\frac{1}{\sqrt{x^2-a^2}}$	$\cosh^{-1}\frac{x}{a} + c$	$x \geqslant a > 0$
$\frac{1}{\sqrt{x^2+k}}$	$\ln(x + \sqrt{x^2+k}) + c$	
$\frac{1}{\sqrt{a^2-x^2}}$	$\sin^{-1}\frac{x}{a} + c$	$-a \leqslant x \leqslant a$

Integration by parts: $\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$

Library

00044

Faculty of Engineering & Technology

Department of Interdisciplinary studies

STEM Foundation

Semester 2 -In class Test 2

Module Code	:	**STEM 1511**
Module Title	:	**Foundation Mathematics II**
Date	:	**27th May 2025**
Examiners	:	**Hiruni Wickramasinghe**
Time Allowed	:	**(09 00 am - 11.00 am)- 2 hrs.**

Instructions to Candidates:

- This is a closed book examination. No reference materials are allowed.
- This paper consists of **seven (07) questions**. Answer all seven questions.
- Use only the provided answer sheets for your answers.
- Write clearly and legibly using a blue or black pen.
- **Calculators are not allowed.**
- Start each question on a new page in the answer booklet. Clearly indicate the question number at the top of the page.
- If parts of the same question are written discontinuously in different sheets, ensure that each part is appropriately labelled with its corresponding question number.
- Where appropriate, include clear diagrams or sketches. These can aid in calculations and help convey your answers effectively.
- If you are unsure about the wording of a question consult the Supervisor or Invigilator or make reasonable assumptions. Clearly state such assumptions in your answer script.
- Marks will be deducted for missing or incorrect measuring units, and failure to adhere to the above instructions.

1.

a) Find the limits for the following expressions, if it exists.

i. $\lim_{x\to 2}(6x^4 - 7x^3 + 12x + 25)$

ii. $\lim_{x\to 7}\frac{x^2-4x-21}{x-7}$

iii. $\lim_{x\to\infty}\frac{6+3x^2}{5x^2+7x}$

iv. $\lim_{x\to\infty}\frac{2x^4-2x^2+15}{2-9x^5}$ **(40 marks)**

2. Use the **first principles of differentiation** to find the derivative of the functions

i. $f(x) = x^2 + 6x + 5$

ii. $f(x) = \frac{1}{x}$ **(20 marks)**

3. Find the first Derivative of with respect to x.

i. $f(x) = 13x^5 + 5x^4 - 4x^3 + 12$

ii. $f(x) = 2xe^x$

iii. $f(x) = (x^2 + 4)(x + 3)$

iv. $f(x) = \frac{sinx}{x^2}$

v. $f(x) = cos^2x$

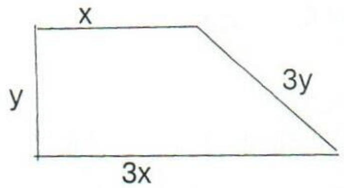
vi. $f(x) = \ln(x^4 + x)$ **(30 marks)**

4. Let $f(x) = e^{ax}\,(ax + sinx)$. If $f'\,(0) = 5$, then find the value of a. **(25 marks)**

5. Let $y = e^u$ and $u = \cos 2x$. Find $\frac{dy}{dx}$ **(20 marks)**

6. Find the local minimum point of the graph of $y = x^3 - 2x^2 + x - 3$. **(25 marks)**

7. Given that this trapezium has a perimeter of 24 cm. find the maximum area that can have (**Hint:** Area of Trapezoid is$\frac{(a+b)}{2}h$, a=base, b=base, h=height)



a. Express the perimeter of the trapezium in terms of its sides.
b. Derive a formula for the area of the trapezium in terms of one variable.
c. Using calculus, find the value of the variable that gives the **maximum area**.
d. Hence, find the **maximum possible area** of the trapezium. **(40 marks)**

DATA & FORMULAE

Differentiation

Function $y = y(x)$	$\frac{dy}{dx}$	Function $y = y(x)$	$\frac{dy}{dx}$
x^n	nx^{n-1}	e^x	e^x
$\sin x$	$\cos x$	$\log_e x$	$\frac{1}{x}$
$\cos x$	$-\sin x$	$\sinh x$	$\cosh x$
$\tan x$	$\sec^2 x$	$\cosh x$	$\sinh x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$	$\tanh x$	$\operatorname{sech}^2 x$
$\sec x$	$\sec x \tan x$	$\operatorname{cosech} x$	$-\operatorname{cosech} x \coth x$
$\cot x$	$-\operatorname{cosec}^2 x$	$\operatorname{sech} x$	$-\operatorname{sech} x \tanh x$
$\sin^{-1} x$	$\frac{1}{\sqrt{1-x^2}}$	$\coth x$	$-\operatorname{cosech}^2 x$
$\cos^{-1} x$	$\frac{-1}{\sqrt{1-x^2}}$	$\sinh^{-1} x$	$\frac{1}{\sqrt{1+x^2}}$
$\tan^{-1} x$	$\frac{1}{1+x^2}$	$\cosh^{-1} x$	$\frac{1}{\sqrt{x^2-1}}$
$\operatorname{cosec}^{-1} x$	$\frac{-1}{x\sqrt{x^2-1}}$	$\tanh^{-1} x$	$\frac{1}{1-x^2}$
$\sec^{-1} x$	$\frac{1}{x\sqrt{x^2-1}}$	$\operatorname{cosech}^{-1} x$	$\frac{-1}{x\sqrt{1+x^2}}$
$\cot^{-1} x$	$\frac{-1}{1+x^2}$	$\operatorname{sech}^{-1} x$	$\frac{-1}{x\sqrt{1-x^2}}$
		$\coth^{-1} x$	$\frac{1}{1-x^2}$

Product Rule

$$\frac{d}{dx}(uv) = \frac{du}{dx}.v + u.\frac{dv}{dx}$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{du}{dx} - u.\frac{dv}{dx}}{v^2}$$

Function of a Function Rule

If $y = y(u)$ and $u = u(x)$ then $\frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$

Library

00076

CINEC CAMPUS
Beyond A Graduate

Faculty of Engineering & Technology

Department of Interdisciplinary studies

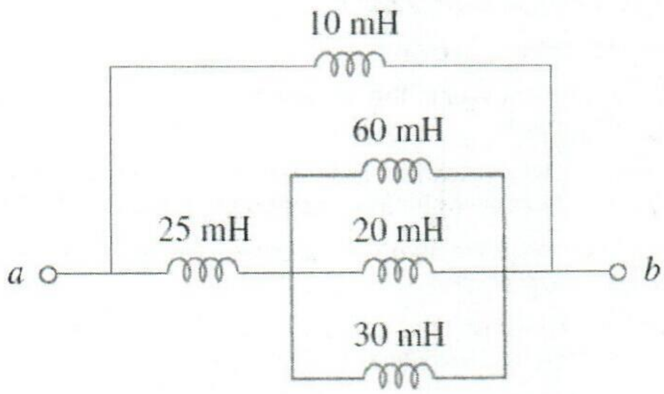
STEM Foundation

Semester 2 -In class Test 04

Module Code	:	**STEM1512**
Module Title	:	**Practical Engineering Science for Electro-Mechanical Design**
Date	:	**22nd May 2025**
Examiners	:	**U.A. Piyumi Hansika**
Time Allowed	:	**(01.30 pm - 3.30 pm)- 2 hrs.**

Instructions to Candidates:

- This is a closed book examination. No reference materials are allowed.
- This paper consists of five (05) questions. **Answer all five questions.**
- Use only the provided answer sheets for your answers.
- Write clearly and legibly using a blue or black pen.
- Write your student number on *all* sheets.
- Start each question on a new page in the answer booklet. Clearly indicate the question number at the top of the page.
- If parts of the same question are written discontinuously in different sheets, ensure that each part is appropriately labelled with its corresponding question number.
- Where appropriate, include clear diagrams or sketches. These can aid in calculations and help convey your answers effectively.
- If you are unsure about the wording of a question consult the Supervisor or Invigilator or make reasonable assumptions. Clearly state such assumptions in your answer script.
- Marks will be deducted for missing or incorrect measuring units, and failure to adhere to the above instructions.
- **You may use a scientific calculator,** but it must not be programmable.
- Usage of mobile phones during the assessment is strictly prohibited.

Questions		
1	Define the following terms. a) Capacitance b) Permittivity c) Inductance d) Semiconductor e) Passive Element	25 marks (5 each)
2	a) Determine the p.d. across a 4μF capacitor when charged with 5 mC.	5 marks
	b) Find the charge on a 50pF capacitor when the voltage applied to it is 2 kV.	5 marks
	c) Sketch graphs of : I. Current [I] vs time II. Charge [Q] vs time III. Voltage [Vc] vs time when a capacitor is charging	6 marks
	d) Explain what is time constant τ of a RC circuit, and the purpose of calculating 5 τ. You may use a diagram to help you with the answer.	5 marks
3	a) An 8 H inductor has a current of 3 A flowing through it. How much energy is stored in the magnetic field of the inductor?	5 marks
	b) Find the L_{eq} of the circuit in figure 3.1.  Figure 3.1	6 marks

4	a) Give the barrier potential value of silicon and germanium diodes	4 marks
	b) Sketch a typical current against voltage characteristics graph for a PN junction diode clearly identifying the: I. forward bias region II. reverse bias region III. reverse breakdown region	6 marks
	c) Diodes can be used for AC signal rectification. Figure 4.1 gives a full wave rectifier using silicon diodes. The input waveform shown in Figure 4.2 is applied to the circuit. Sketch the Output Waveform (Vo) vs Time and explain how the circuit in figure 4.1 create this output waveform using the Diodes and the Capacitor. You should clearly show in your output waveform the effect of adding the capacitor to the rectifier circuit.	10 marks

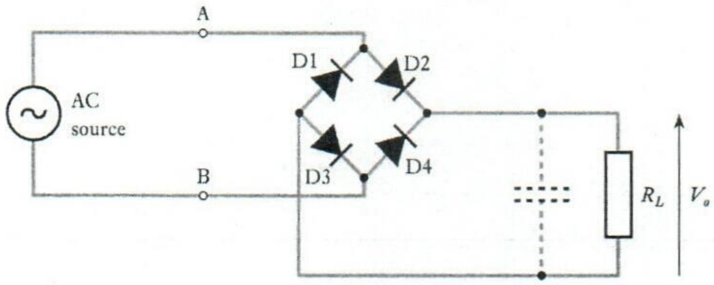


Figure 4.1 (Rectifier Circuit)

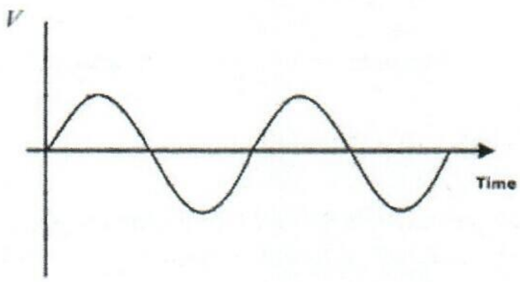


Figure 4.2 (Input waveform)

00076

5

a) Complete the pin configuration of the Op-Amp circuit symbol given in Figure 5.1 — 7 marks

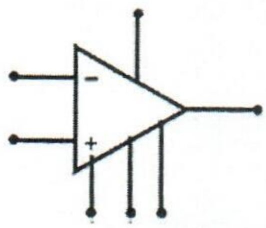


Figure 5.1 (Op- Amp Circuit Symbol)

b) State the two golden rules used when analysing Op-Amp Circuits — 6 marks

c) Consider the following Op Amp Circuit in Figure 5.2.

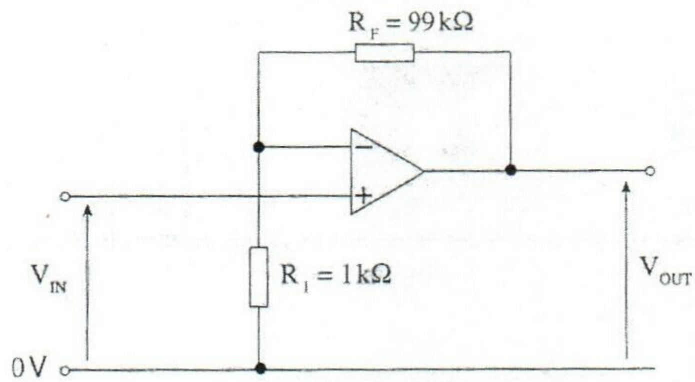


Figure 5.2

I. Identify the type of this Basic Operational Amplifier Circuit. — 3 marks

II. Calculate the gain of this amplifier — 3 marks

III. Now the feedback resistors should be changed to give a gain of 10. Calculate the value of R_F needed, keeping R_1 as 1KΩ to achieve a gain of 10. — 4 marks

[Total 100 marks]

Library

00076

CINEC CAMPUS
Beyond A Graduate

Faculty of Engineering & Technology

Department of Interdisciplinary studies

STEM Foundation

Semester 2 -In class Test 1

Module Code	:	**STEM 1511**
Module Title	:	**Foundation Mathematics II**
Date	:	**20th May 2025**
Examiners	:	**Hiruni Wickramasinghe**
Time Allowed	:	**(09 00 am - 11.00 am)- 2 hrs.**

Instructions to Candidates:

- This is a **closed book examination**. No reference materials are allowed.
- This paper consists of four (06) questions. Answer all six questions.
- Use only the provided answer sheets for your answers.
- **Calculators are not allowed.**
- Write clearly and legibly using a blue or black pen.
- Write your student number on *all* sheets.
- Start each question on a new page in the answer booklet. Clearly indicate the question number at the top of the page.
- If parts of the same question are written discontinuously in different sheets, ensure that each part is appropriately labelled with its corresponding question number.
- Where appropriate, include clear diagrams or sketches. These can aid in calculations and help convey your answers effectively.
- If you are unsure about the wording of a question consult the Supervisor or Invigilator or make reasonable assumptions. Clearly state such assumptions in your answer script.

1. The matrices $\boldsymbol{A}, \boldsymbol{B}$ and $\boldsymbol{C}$ are given below in terms of constants a, b, c and d ,by

$$A = \begin{pmatrix} 2 & -1 \\ -7 & a \end{pmatrix}, \qquad B = \begin{pmatrix} b & 0 \\ 7 & -4 \end{pmatrix}, \qquad C = \begin{pmatrix} 5 & c \\ d & 8 \end{pmatrix}.$$

Given that $\boldsymbol{A} - \boldsymbol{B} = \boldsymbol{C}$, Find the value of $a, b, c,$ and d

2. The 2×2 matrices $\boldsymbol{X}, \boldsymbol{Y}$ and $\boldsymbol{Z}$ are given below in terms of the constants a, b, c and d.

$$X = \begin{pmatrix} 2 & 6 & 0 \\ 4 & c & 3 \end{pmatrix}, \qquad Y = \begin{pmatrix} b & 1 & -3 \\ 2 & -3 & 1 \end{pmatrix} \quad Z = \begin{pmatrix} 10 & 8 & a \\ d & -5 & 5 \end{pmatrix}$$

Given that $2\boldsymbol{X} + 4\boldsymbol{Y} = 2\boldsymbol{Z}$, find the values of a, b, c and d.

3. The 2×2 matrices $\boldsymbol{A}, \boldsymbol{B}$ *and* $\boldsymbol{C}$ are given below in terms of the constant x.

$$A = \begin{pmatrix} x & 5 \\ 3 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix} \qquad C = \begin{pmatrix} 2x+3 & 13 \\ 4x+41 & 11 \end{pmatrix}$$

a. Find an expression for $\boldsymbol{AB}$, in terms of x
b. Determine the value of x , given $(\boldsymbol{AB})^T = \boldsymbol{C}$

4. Let A = $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ -1 & 0 & 1 \end{pmatrix}$ and let u = $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ and v = $\begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$ find each of the following
   a. Au
   b. Av
   c. A(u+v)

5. If $\boldsymbol{A} = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$, $\boldsymbol{B} = \begin{pmatrix} 3 & 7 \\ 2 & 5 \end{pmatrix}$ show that $(\boldsymbol{AB})^{-1} = \boldsymbol{B}^{-1}\boldsymbol{A}^{-1}$

6. Solve the following systems using matrices.
   a. $3x + 5y = 7$
      $4x - 3y = 19$

   b. $3x + 4y = 0$
      $2x + 5y = -7$

Library

00076

Faculty of Engineering & Technology

Department of Interdisciplinary studies

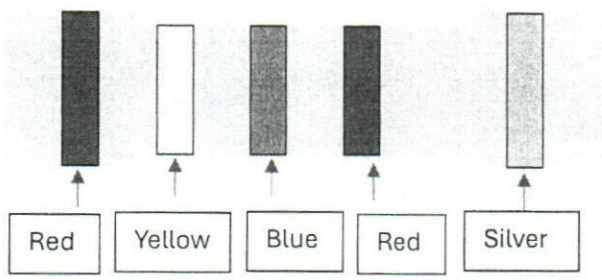
STEM Foundation

Semester 2 -In class Test 03

Module Code	:	**STEM1512**
Module Title	:	**Practical Engineering Science for Electro-Mechanical Design**
Date	:	**18th May 2025**
Examiners	:	**U.A. Piyumi Hansika**
Time Allowed	:	**(09.00 am - 11.00 am)- 2 hrs.**

Instructions to Candidates:

- This is a closed book examination. No reference materials are allowed.
- This paper consists of five (05) questions. **Answer all five questions.**
- Use only the provided answer sheets for your answers.
- Write clearly and legibly using a blue or black pen.
- Write your student number on *all* sheets.
- Start each question on a new page in the answer booklet. Clearly indicate the question number at the top of the page.
- If parts of the same question are written discontinuously in different sheets, ensure that each part is appropriately labelled with its corresponding question number.
- Where appropriate, include clear diagrams or sketches. These can aid in calculations and help convey your answers effectively.
- If you are unsure about the wording of a question consult the Supervisor or Invigilator or make reasonable assumptions. Clearly state such assumptions in your answer script.
- Marks will be deducted for missing or incorrect measuring units, and failure to adhere to the above instructions.
- **You may use a scientific calculator,** but it must not be programmable.
- Usage of mobile phones during the assessment is strictly prohibited.

Questions		
1	Define the following terms. Include the equations. a) Current b) Voltage c) Power d) Ohm's Law e) Kirchhoff Current Law	20 marks (4 each)
2	a) If a current of 10 A flows for four minutes, find the quantity of electricity transferred (Q). b) If a current flowing through a resistor is 0.8 A when a potential difference of 20 V is applied. Determine the value of the resistance (R).	5 marks 5 marks
3	a) Using the colour code chart given in figure 3.2, to write down the following values of the resistor given in figure 3.1.  Figure (Resistor) 3.1 I. Resistance:... II. Tolerance:.. III. Minimum Resistance due to tolerance:............................ IV. Maximum Resistance due to tolerance:	 5 marks 5 marks 5 marks 5 marks

	1st Digit	2nd Digit	3rd Digit	Multiplier	Tolerance	TCR*
Black	0	0	0	x 1		250
Brown	1	1	1	x10	±1%	100
Red	2	2	2	$x10^2$	±2%	50
Orange	3	3	3	$x10^3$	±3%	15
Yellow	4	4	4	$x10^4$	±4%	25
Green	5	5	5	$x10^5$	±0.5%	20
Blue	6	6	6	$x10^6$	±0.25%	10
Violet	7	7	7	$x10^7$	±0.1%	5
Grey	8	8	8	$x10^8$	±0.05%	1
White	9	9	9	$x10^9$		
Gold				$x10^{-1}$	±5%	
Silver				$x10^{-2}$	±10%	

Figure (Resistor colour code chart) 3.2

b) Find the equivalent resistance of the circuit shown in Figure 3.3

10 marks

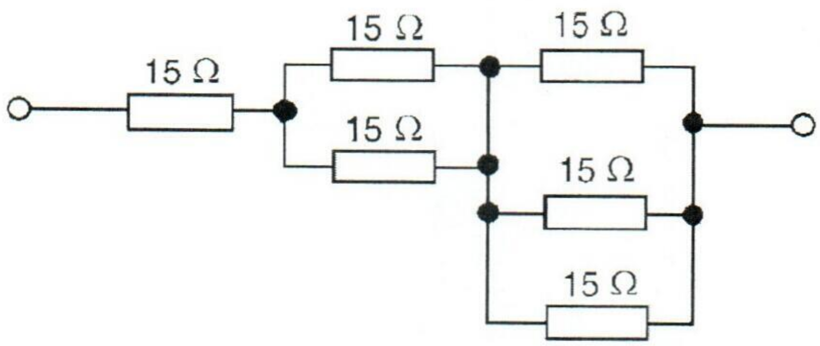
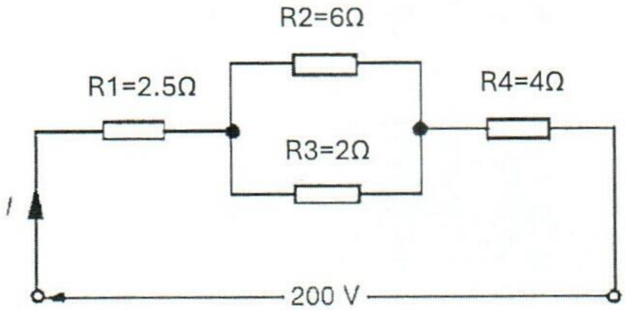
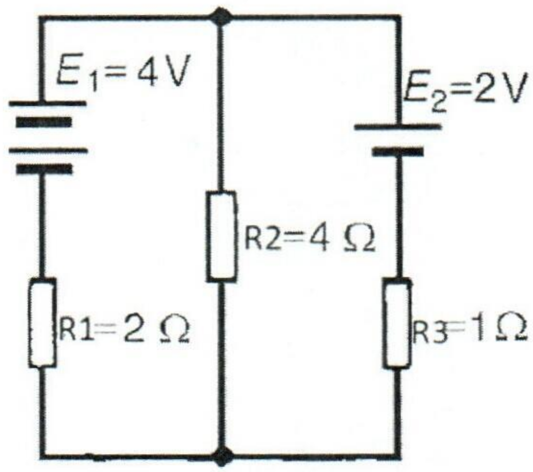


Figure 3.3

00076

4	For the series-parallel arrangement shown in Figure 4.1, find a) The supply current b) The current flowing through each resistor. c) The p.d. across 6Ω resistor. d) The power dissipated by the 6Ω resistor.  Figure 4.1	 5 marks 5 marks 5 marks 5 marks
5	a) Determine the number of **branches, nodes**, and **independent loops** the circuit shown in Figure 5.1 and show that it satisfies the fundamental theorem of the network topology. b) Use Kirchhoff's laws to determine the currents flowing through each resistor in the circuit shown in Figure 5.1  Figure 5.1	8 marks 12 marks

[Total 100 marks]

Library

00009

Faculty of Engineering & Technology

Department of Interdisciplinary Studies

BSc Hons (Eng) Mechanical/Automotive/Civil/Electronic/Mechatronic Engineering

Semester 1 First Sit - Final Examination 2025

Module Code	:	**FE1311**
Module Title	:	**Engineering Mathematics I**
Date	:	**16th May 2025**
Examiner	:	**Shannon Wijeratne**
Time Allowed	:	**(09 00 am – 12.00 pm)- 3 hours**

Instructions to Candidates:

- This is a closed book examination. No reference materials are allowed.
- This paper consists of four (04) questions. Answer all four questions.
- Use only the provided answer booklet, additional sheets, and graph papers for your answers.
- Write clearly and legibly using a blue or black pen.
- Write your student number on *all* sheets.
- Start each question on a new page in the answer booklet. Clearly indicate the question number at the top of the page.
- If parts of the same question are written discontinuously in different sheets, ensure that each part is appropriately labelled with its corresponding question number.
- Where appropriate, include clear diagrams or sketches. These can aid in calculations and help convey your answers effectively.
- If you are unsure about the wording of a question consult the Supervisor or Invigilator, or make reasonable assumptions. Clearly state such assumptions in your answer script.
- Marks will be deducted for missing or incorrect measuring units, and failure to adhere to the above instructions.

1. a) Find the following limits.

i) $\lim_{x\to 0} \frac{\sin x - x}{x^3}$ [8 Marks]

ii) $\lim_{x\to 0} \frac{5^x - 2^x}{x^2 - x}$ [9 Marks]

iii) $\lim_{x\to 0} \frac{x^2 e^x}{tan^2 x}$ [18 Marks]

b) i) Use Cramer's rule to solve for x, y and z.

$$x + 2y + 3x = 14$$
$$2x - y + z = 3$$
$$3x + y + 2z = 10$$

[30 Marks]

ii) Solve the following system of equations, using the Gaussian elimination method:

$$a + b + c = 6$$
$$2a + 3b + 5c = 19$$
$$4a + 5c = 20$$

[35 Marks]

2. a) i) Find the equation of locus defined by $\frac{|z-1|}{|z-i|} = 1$. [25 Marks]

   ii) Sketch the locus in part i)

   b) i) Find an expression for cos4θ in terms of cosθ, using binomial theorem and De Moivre's theorem. [25 Marks]

   ii) Express $(-1 + \sqrt{3}.i)$ in polar form and find $(-1 + \sqrt{3}.i\,)^{12}$ [15 Marks]

   c) Calculate the radius of curvature at the point (-1,3) on the curve whose equation is $y = x + 3x^2 - x^3$

   and hence obtain the co-ordinates of center of curvature.

   [35 marks]

3) a) If $f(x, y) = x^2y + 3xy^2 + y^3$ find all first and second partial derivatives of $f(x, y)$.(i.e f_x , f_y , f_{xx} , f_{yy} , f_{xy} , f_{yx}).

   [24 Marks]

   b) The height of the right circular cone(h)is increasing at 3mm/s and its radius(r) is decreasing at 2mm/s. Determine, correct to 3 significant figures, the rate at which the volume(V) is changing (in cm3/s) when the height is 3.2cm and the radius is 1.5cm. Volume of the right circular cone is given by $V=\frac{1}{3}\pi r^2 h$.

   [30 Marks]

   c) i) A running track is shaped like an ellipse. The track is 400 meters long around (total perimeter), and the length of the semi major axis is 50 meters, and semi minor axis is 30 meters .If the major axis is horizontal and the center is at its origin, find the equation of the ellipse (in standard form),

   [16 Marks]

ii) Cross section of a nuclear cooling tower (Figure 1) is in the shape of a hyperbola with the equation $\frac{x^2}{900} - \frac{y^2}{1936} = 1$. The tower is 150m tall and the distance from the top of the tower to the center of the hyperbola is half the distance from the base of the tower to the center of the hyperbola. Find the diameters of the top and base of the tower.

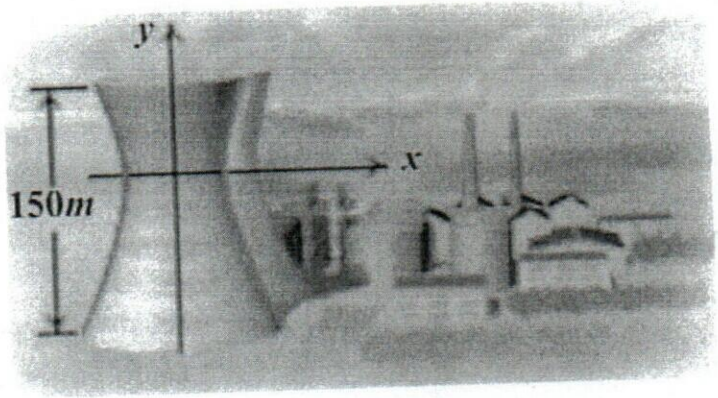


Figure 1: Nuclear cooling tower

[30 Marks]

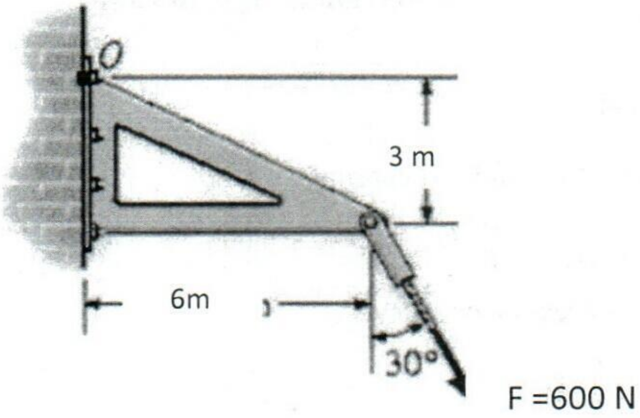
4. a) Find vector **v**, which has the initial point of P and terminal point of Q. Find |**v**| and find the unit vector **u** in the direction of the **v**.

P : $(\frac{1}{2}, \frac{1}{3}, \frac{2}{3})$

Q : $(\frac{3}{2}, \frac{5}{3}, \frac{7}{3})$

[20 Marks]

b) Find the moment of the force F about the point O.



[10 Marks]

c) Evaluate the following integrals using a suitable method.

i) $\int xe^{x^2}\ dx$ [16Marks]

ii) $\int x^2 \cos x\, dx$ [18 Marks]

iii) $\int sec^9 x\ tan^5 x dx$ [16Marks]

iv) $\int \frac{2x+11}{x^2+x-6}\ dx$ [20Marks]

Additional Information

Partial Derivatives:

1 *Small increments (Total differentials of z)*

$$z = f(x,y) \quad \delta z = \frac{\partial z}{\partial x}\delta x + \frac{\partial z}{\partial y}\delta y \qquad \text{(a)}$$

2 *Rates of change (Simple Chain Rule)*

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}.\frac{dx}{dt} + \frac{\partial z}{\partial y}.\frac{dy}{dt} \qquad \text{(b)}$$

3 *Implicit functions*

$$\frac{dz}{dx} = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}.\frac{dy}{dx} \qquad \text{(c)}$$

4 *Change of variables (Chain Rule for Two Sets of Independent Variables,*

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}.\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}.\frac{\partial y}{\partial u}$$
$$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}.\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}.\frac{\partial y}{\partial v} \qquad \text{(d)}$$

Standard derivatives:

F(x)	$\frac{df(x)}{dx}$
x^n	nx^{n-1}
$\sin ax$	$a \cos ax$
$\cos ax$	$-a\sin ax$
$\tan ax$	$asec^2 ax$
$\sec ax$	$a \sec ax \,.\, tanax$
e^{ax}	ae^{ax}
$\ln ax$	$\frac{a}{x}$

Standard Integrals:

$f(x)$	$\int f(x)dx$
x_n	$\frac{x^{n+1}}{n+1}$, for $x \neq -1$
e^x	e^x
e^{ax}	$\frac{1}{a}e^{ax}$
$\frac{1}{x}$	$\ln x$
$\sin ax$	$-\frac{1}{a}\cos ax$
$\cos ax$	$\frac{1}{a}\sin ax$

Rules of differentiation	$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$ $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$
Integration by parts equation	$\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

Radius of Curvature:

$$R = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|}$$

TRIGONOMETRIC IDENTITIES

RECIPROCAL IDENTITIES

$$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

PYTHAGOREAN IDENTITIES

$$\boxed{\sin^2 x + \cos^2 x = 1}$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\cos^2 x = 1 - \sin^2 x$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

SUM AND DIFFERENCE IDENTITIES

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$$

DOUBLE-ANGLE IDENTITIES

$$\sin 2x = 2 \sin x \cos x = \frac{2 \tan x}{1 + \tan^2 x}$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x = \frac{1 - \tan^2 x}{1 + \tan^2 x}$$

$$\tan 2x = \frac{2 \tan x}{1 - \tan^2 x}$$

$$\cot 2x = \frac{\cot^2 x - 1}{2 \cot x}$$

HALF-ANGLE IDENTITIES

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}$$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}}$$

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}} = \frac{\sin x}{1 + \cos x} = \frac{1 - \cos x}{\sin x}$$

Binomial Theorem:

$$(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

De Moivre's Theorem:

$$z^n = r^n(\cos n\theta + i \sin n\theta)$$

Library

00030

Faculty of Engineering & Technology

Department of Interdisciplinary Studies

BSc Hons (Eng) Mechanical/Automotive/Civil/Electronic/Mechatronic Engineering

Semester 2 - First Sit - Final Examination 2025

Module Code	:	**FE1321**
Module Title	:	**Engineering Mathematics II**
Date	:	**16th May 2025**
Examiner	:	**Shannon Wijeratne**
Time Allowed	:	**(09 00 am – 12.00 pm)- 3 hours**

Instructions to Candidates:

- This is a closed book examination. No reference materials are allowed.
- This paper consists of four (04) questions. Answer all four questions.
- Use only the provided answer booklet, additional sheets, and graph papers for your answers.
- Write clearly and legibly using a blue or black pen.
- Write your student number on *all* sheets.
- Start each question on a new page in the answer booklet. Clearly indicate the question number at the top of the page.
- If parts of the same question are written discontinuously in different sheets, ensure that each part is appropriately labelled with its corresponding question number.
- Where appropriate, include clear diagrams or sketches. These can aid in calculations and help convey your answers effectively.
- If you are unsure about the wording of a question consult the Supervisor or Invigilator, or make reasonable assumptions. Clearly state such assumptions in your answer script.
- Marks will be deducted for missing or incorrect measuring units, and failure to adhere to the above instructions.

1.

a) Consider the following system of equations:

$$x - 2y + 4z = 5$$

$$2x + y + 3z = -1$$

$$-3x + 6y - 12z = 10$$

i) Find the ranks of the augmented matrix and the coefficient matrix. ***(30 marks)***

ii) Determine the nature of solutions for the above system. ***(3 marks)***

b) Consider the following matrix:

$$A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

i) Find the characteristic polynomial of the above matrix. ***(12 marks)***

ii) Find the eigenvalues of A. ***(10 marks)***

iii) Find the corresponding eigenvectors of A. ***(21 marks)***

iv) Find the normalized eigenvectors of A. ***(6 marks)***

v) Show that the determinant of A is equal to the product of the eigenvalues of A. ***(9 marks)***

vi) Deduce the eigenvalues of A^2 , A +2I and 3A using the eigenvalues of the matrix A in part ii). ***(9 marks)***

2.

a) Find the general/particular solution for each of the the following differential equations using a **suitable method** and keep your answers in the **most simplified form**:

i) $2x(x - \frac{dy}{dx}) = 5$ given $y = 2 \; when \; x = 1$ ***(15 marks)***

ii) $(2025 + 5x^2)\,\frac{dy}{dx} = 16x(1 + y^2)$ ***(10 marks)***

iii) $(2xy + 6x)\,dx + (x^2 + 4y^3)dy = 0$ ***(20 marks)***

iv) $\frac{dy}{dx} - \frac{y}{x} = -xe^{-x}$ ***(15 marks)***

v) $x\frac{dy}{dx} = \frac{x^2+y^2}{y}$ ***(15 marks)***

b) A pot is baked in an oven at a temperature of 300°C in a workshop that is constantly at a temperature of 32°C .After one hour the temperature of the pot is 110°C. Assume Newton's law of cooling.

i) What will be the temperature of the pot after 180 minutes?
ii) How long will it take until the pot cools to 50°C ?

[Newton's law of cooling states that the rate of change of temperature is proportional to the difference of temperature between the body and its environment.]

(25 marks)

3.

a) Determine whether or not the transformation T: $\mathbb{R}^2 \rightarrow \mathbb{R}^3$ is a linear transformation, where

$$T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - x_2 \\ x_1 + x_2 \\ 2x_1 \end{pmatrix}.$$

(20 marks)

b) Find the limit of each of the following sequences and state whether each is convergent or divergent.

i) $\lim_{n\to\infty} (-1)^n \frac{3n+1}{n^2+n}$ ***(12 marks)***

ii) $\lim_{n\to\infty} \left(\frac{3n+2}{4n+3}\right)^n$ ***(10 marks)***

c) Determine whether each of the following series is convergent or divergent:

i) $\sum_{n=0}^{\infty} \frac{3^{n+1}n!}{4^n}$ using ratio test. ***(8 marks)***

ii) $\sum_{n=0}^{\infty} \frac{1}{(2n+7)^3}$ using integral test ***(10 marks)***

d) Determine the interval and radius of convergence for the following power series:

$$\sum_{n=1}^{\infty} \frac{(x+3)^n}{(n+2)}$$

(20 marks)

e) Find the Maclaurin series for $\sin x$. ***(20 marks)***

4.

a) Consider the following set of vectors:

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$$

i) Find whether the above set of vectors is linearly independent or dependent. **(10 marks)**

ii) Find a basis for the space spanned by the above vectors and find whether they are orthogonal. **(30 marks)**

b) Determine the value of α if V_1= (2,-4,5), V_2= (3,2,-4) and V_3= (-1,-6, α) do not span $\mathbb{R}^3$.

(20 marks)

c) Let A and B be the following bases for $\mathbb{R}^2$.

$$A = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\} \quad \text{and} \quad B = \left\{ \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \end{bmatrix} \right\}$$

i) Find the change of basis matrix $P_{A\rightarrow B}$ from A to B. **(20 marks)**

ii) Let v = (-1,4) with respect to the basis A. Find the coordinates of v with respect to the basis B.

(20 marks)

Additional information

Inverse of a matrix:

$$A^{-1} = \frac{adj(A)}{|A|}$$

Characteristic polynomial C (λ)

$$c(\lambda) = |\lambda \boldsymbol{I} - \boldsymbol{A}| = 0$$

The normalized form of eigenvector:

$$\hat{e} = \frac{e}{|e|}$$

where

$$|e| = \sqrt{(e_1^2 + e_2^2 + \ldots + e_n^2)}$$

Library

00037

CINEC Campus
Faculty of Engineering & Technology
Department of Interdisciplinary Studies
BSc Hons (Eng) Civil Engineering/BSc Hons (Eng) Mechanical Engineering/BSc Hons (Eng) Mechatronic Engineering/ BSc Hons (Eng) Automotive Engineering / BSc Hons (Eng) Electronic & Telecommunication Engineering
Semester 02 Final Examination 2025

Module Code	:	**FE1227**
Module Title	:	**Meditation and Stress Management**
Date	:	**14.05.2025**
Examiners	:	**Ms. Yamuna Samarakoon**
Time Allowed	:	**03 hours**

INSTRUCTIONS TO CANDIDATES

- This is a closed book examination. No reference materials are allowed.
- This paper consists of four (04) questions. Answer all four questions.
- Use only the provided answer booklet.
- Write clearly and legibly using a blue or black pen.
- Write your student number on *all* sheets.
- Start each question on a new page in the answer booklet. Clearly indicate the question number at the top of the page.
- If parts of the same question are written discontinuously in different sheets, ensure that each part is appropriately labelled with its corresponding question number.
- This paper contains 2 pages.

MATERIALS REQUIRED

- CINEC answering Booklet, Extra answering papers

Answer All the Questions

Question 01

I. Give a brief introduction about the two types of meditation techniques that have been discussed in the Meditation & Stress Management programme? (10 Marks)

II. How does meditation contribute to cultivating a stress-free lifestyle? (15 Marks)

Question 02

I. Describe the steps of sitting tranquility meditation. (10 Marks)

II. Elaborate on each step of the meditation process using insights and experiences from your own practice. (15 Marks)

Question 03

I. Briefly explain the essential qualities of a good meditation practitioner. (10 Marks)

II. Reflect on which of these qualities you possess and **elaborate on one** with examples from your own experience. (15 Marks)

Question 04

How could understanding **the impermanent nature of thoughts** help a person manage stress? Discuss this concept in detail, with examples to support your answer. (25 Marks)

CINEC Campus
Faculty of Engineering & Technology
Department of Interdisciplinary Studies
BSc Hons (Eng) Civil Engineering/BSc Hons (Eng) Mechanical Engineering/BSc Hons (Eng) Mechatronic Engineering/ BSc Hons (Eng) Automotive Engineering / BSc Hons (Eng) Electronic & Telecommunication Engineering
Semester 07 Re-Sit Examination 2025

Module Code	:	**FE4312**
Module Title	:	**Engineer as an Entrepreneur**
Date	:	**14.05.2025**
Examiners	:	**Dr Kalpana Ambepitiya**
Time Allowed	:	**03 hours**

INSTRUCTIONS TO CANDIDATES

- This is a closed book examination. No reference materials are allowed.
- This paper consists of five (05) questions. Answer all five questions.
- Use only the provided answer booklet.
- Write clearly and legibly using a blue or black pen.
- Write your student number on *all* sheets.
- Start each question on a new page in the answer booklet. Clearly indicate the question number at the top of the page.
- If parts of the same question are written discontinuously in different sheets, ensure that each part is appropriately labelled with its corresponding question number.
- This paper contains 3 pages.

MATERIALS REQUIRED

- CINEC answering Booklet, Extra answering papers
- You may use a scientific calculator. This must not be programmable. This may be examined during the examination.

Question No. 01

Sean Parker

Sean Parker is a hacker who hacked the entire network of 500 companies when he was 16 years old. He later worked with Napster, the biggest file-sharing site on the Internet. After Napster, his first successful ventures were an online address book and Plaxo, which offered network services. He was also exceptionally expert in developing Facebook. Notably, Parker was the very first president of Facebook.

Mark Zuckerberg said that Sean helped transform Facebook from a college project into an honest company by giving advice. He also founded Airtime.com, a real video chat site, with his fellow Napster. Parker is currently a venture capital investor. He was also interested in music and invested in Spotify in 2010.

Parker Foundation was launched in 2015 with $600 million utilised for global health, science, and civic engagement. After one year of his foundation, he announced the Parker Institute and gave $250 million for cancer immunotherapy. He also found the Asthma and Allergy Centre at Stanford because he had asthma and allergy problems.

Considering the facts given in the case study:

a) Write three technopreneur characteristics that Sean has. **(05 marks)**
b) If Sean is planning to enter India, List five analyses he must conduct. **(05 marks)**
c) Briefly discuss "Porter's five forces" for your proposed business. **(10 marks)**
d)

(Total: 20 Marks)

Question No. 02

a) What is an Entrepreneurial Idea? Provide an example. **(05 Marks)**
b) Briefly explain the difference between Industry/Market feasibility and organisational feasibility. **(05 Marks)**
c) In business decision-making, relying solely on internal insights can lead to biased strategies. Discuss how incorporating multiple external data sources supports assumption validation. In your answer, explain the role and value of Competitive Intelligence (CI) with relevant examples. **(10 marks)**

(Total: 20 Marks)

Question No. 03

a) List all the elements in the Product Marketing Mix. **(05 Marks)**
b) What sources must be referred to generate a new product idea? **(05 Marks)**
c) Briefly explain the steps of a detailed business plan. **(10 marks)**

(Total: 20 Marks)

Question No. 04

a) Define Sustaining Innovation, and provide an example. **(05 Marks)**
b) List five (05) attributes of a Leader. **(05 Marks)**
c) Discuss how you would create a Unique Selling Proposition for an automobile (Select any brand that you prefer). **(10 Marks)**

(Total: 20 Marks)

Question No. 05

Write short notes on the following.

(a) Financial Statements
(b) Venture Capital Firms
(c) Maslow's Motivation Theory
(d) Segmentation

(Marks: 4x5=20)
(Total: 20 Marks)

--End of the Paper --

library

00010

Faculty of Engineering & Technology
Department of Interdisciplinary Studies
BSc Hons (Eng) Automotive/Mechanical/Mechatronics/Electronic/Civil Engineering
Semester 2 Re-sit Examination 2025

Module Code	:	**FE1321**
Module Title	:	**Engineering Mathematics II**
Date	:	**9th April 2025**
Examiner	:	**Ms.Shannon Wijeratne**
Time Allowed	:	**(09 00 am – 12.00 pm) - 3 hrs**

Instructions to Candidates:

- This is a closed book examination. No reference materials are allowed.
- This paper consists of four (04) questions. Answer all four questions.
- Use only the provided answer booklet, additional sheets, and graph papers for your answers.
- Write clearly and legibly using a blue or black pen.
- Write your student number on *all* sheets.
- Start each question on a new page in the answer booklet. Clearly indicate the question number at the top of the page.
- If parts of the same question are written discontinuously in different sheets, ensure that each part is appropriately labelled with its corresponding question number.
- Where appropriate, include clear diagrams or sketches. These can aid in calculations and help convey your answers effectively.
- If you are unsure about the wording of a question consult the Supervisor or Invigilator, or make reasonable assumptions. Clearly state such assumptions in your answer script.
- Marks will be deducted for failure to adhere to the above instructions.

1.

a) Consider the following system of equations

$$2x - y + 3z = 4$$

$$-x + 3y + z = 3$$

$$4x + 5y - 2z = -8$$

i) Find the ranks of the augmented matrix and the coefficient matrix. **(32 marks)**

ii) Determine the nature of solutions for the above system. **(2 marks)**

iii) Determine the solutions if it exists. **(8 mar**

b) Consider the following matrix:

$$A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$$

i) Find the characteristic polynomial of the above matrix. **(12 marks)**

ii) Find the eigenvalues of A. **(10 marks)**

iii) Find the corresponding eigenvectors of A. **(21 marks)**

iv) Find the normalized eigenvectors of A. **(6 marks)**

v) Find eigenvalues of A^3, $A + 2I$ and $3A$. **(9 marks)**

2.

a) Find general/particular solutions for the following differential equations using a **suitable method** and keep your answers in the **most simplified form**.

i) $(y^2 - 1)\ \frac{dy}{dx} = 3y$ given $y = 1$ when $x = \frac{13}{6}$. **(12 marks)**

ii) $4xy\frac{dy}{dx} = y^2 - 1$ **(10 marks)**

iii) $x\ \frac{dy}{dx} = \frac{x^2+y^2}{y}$ given $y = 4$ when $x = 1$ **(20 marks)**

iv) $\frac{dy}{dx} - 2(x+1)^3 = \frac{4}{(x+1)}y$ **(14 marks)**

v) $(2xy + 6x)dx + (x^2 + 4y^3)dy = 0$ **(19 marks)**

b) Aluminium conductor found in a circuit seems to change its resistance (R)with the temperature (θ). Resistance change with respect to temperature, is proportional to the resistance of the aluminum conductor.

i) Form a differential equation to find the **resistance at a given temperature**. (Consider the constant of proportionality as k which is the temperature coefficient of resistance of aluminum).

ii) If $R = R_0$ when $\theta = 0^0C$ solve the differential equation for R.

iii) If $k = 64 \text{ x } 10^{-4}/^0C$ determine the resistance of the aluminium conductor at 60^0C Correct to 4 significant figures, given that resistance is 32 Ω at 0^0C.

(25 marks)

3.

a) Determine whether or not the transformation T: $\mathbb{R}^2 \rightarrow \mathbb{R}^3$ is a linear transformation where,

$$T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 - x_2 \\ x_1 + x_2 \\ 2x_1 \end{pmatrix}.$$

(20 marks)

b) Find the limits of the following sequences and state whether they converge or diverge .

i) $\lim\limits_{n\to\infty} (-1)^n \frac{2n+1}{n^2+n}$ **(12 marks)**

ii) $\lim\limits_{n\to\infty} \left(\frac{2n+3}{2n-5}\right)^n$ **(10 marks)**

c) Determine whether each of the following series is convergent or divergent.

i) $\sum_{n=1}^{\infty} \frac{4n^5}{3n^3+5}$ using the n^{th} term test for divergence. **(6 marks)**

ii) $\sum_{n=0}^{\infty} \frac{1}{(2n+7)^3}$ using the integral test. **(12 marks)**

d)Determine the interval and radius of convergence for the following power series.

$$\sum_{n=1}^{\infty} \frac{(x+5)^n}{n(n+1)}$$ **(25 marks)**

e) Find the Maclaurin series for $\sin x$. (Find up to 5 terms of the series.) **(15 marks)**

4.

a) Consider the following set of vectors:

b)

$$\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \right\}$$

i) Find whether the above vectors are linearly independent or dependent. **(10 marks)**

ii) Find the basis of the above vectors and find whether they are orthogonal. **(35 marks)**

b) If, V_1= (1,3,-5), V_2= (4,2,0) and V_3= (1,-2, α) span $\mathbb{R}^3$, find the interval of the possible values of α.

(20 marks)

c) Let

$$A = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix} \right\} \quad \text{and} \quad B = \left\{ \begin{bmatrix} -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 2 \end{bmatrix} \right\}$$

A and B are bases for $\mathbb{R}^2$.

i) Find the change of basis matrix from B to A. ($P_{B\to A}$) **(20 marks)**

ii) If v = (2,3) (v is a vector w.r.t the basis B) find the coordinates of v w.r.t. basis A.

(20 marks)

Additional information

Inverse of a matrix:

$$A^{-1} = \frac{adj(A)}{|A|}$$

Characteristic polynomial C (λ)

$$c(\lambda) = |\lambda I - A| = 0$$

The normalized form of eigen vector:

$$\hat{e} = \frac{e}{|e|}$$

Where,

$$|e| = \sqrt{(e_1^2 + e_2^2 + \ldots + e_n^2)}$$