MINISTRY OF PORTS AND SHIPPING

MERCHANT SHIPPING SECRETARIAT - SRI LANKA

EXAMINATION FOR CERTIFICATE OF COMPETENCY
CHIEF ENGINEER & SECOND ENGINEER OFFICER ON SHIPS OF 3000KW PROPULSION POWER OR MORE

MATHEMATICS

- **TIME ALLOWED - THREE HOURS**
- **Answer any SIX (6) questions**
- **Marks for each part of the question are shown in the brackets**
- **Pass marks - 50%**
- **Date - 2023**

Answers with clear sketches/diagrams, neat handwriting and clear expression will get full marks.

1. a) i. Given that $(x+2)$ is a factor of $f(x)=2x^3-5x^2+ax+a$, find the value of the constant 'a'. (2 marks)

ii. Solve the equation $\sqrt{6x+7}-(2x-7)=0$ (4 marks)

b) If $a^x=bc$, $b^y=ca$ and $c^z=ab$, Evaluate the value of $\frac{x}{1+x}+\frac{y}{1+y}+\frac{z}{1+z}$ (6 marks)

c) If x satisfies the equation $\log_5(x^2+3x-4)=\log_5\left(\frac{x-1}{x+4}\right)$. Find $|x|$ (8 marks)

2. a) If $A=\begin{pmatrix}\alpha & \beta\\ \gamma & -\alpha\end{pmatrix}$ is such that $A^2=I$, show that $\alpha^2+\beta\gamma=1$. (6 marks)

b) Determine the inverse of the following matrix.

$$A=\begin{pmatrix}1 & -2 & 3\\ 2 & -3 & 5\\ -1 & 3 & 4\end{pmatrix}$$ (8 marks)

c) i. Write the set of equation in augmented matrix form. (2 marks)

$$x-2y+3z=1$$
$$2x-3y+5z=8$$
$$-x+3y+4z=5$$

ii. Solve the simultaneous equations using the matrix multiplication. (4 marks)

3. a) Show that $\lim_{x\to\infty}\frac{2x^2+3x+7}{5x^2+x+1}=\frac{2}{5}$. (6 marks)

b) Differentiate $\sin^{-1}\left(1+x^2\right)$ with respect to $\sin\left(1-x^2\right)$. (6 marks)

c) If $y=x+\tan x$, show that $\cos^2 x\frac{d^2y}{dx^2}+2x-2y=0$. (8 marks)

4. a) If $f=4x^3y^2-e^zy^4+\frac{z^3}{x^2}+4y-x^{16}$, find second order partial derivatives of $\frac{\partial^2 f}{\partial x\partial y}$. (8 marks)

b) If $u=\frac{1}{\sqrt{1-2xy+y^2}}$, Prove that $x\frac{\partial u}{\partial x}-y\frac{\partial u}{\partial y}=y^2u^3$ (6 marks)

c) The dimensions of a cone are radius 7 cm, height 5 cm. Calculate the error in its volume if the scale used in taking the measurement is short by 0.01 cm per cm. (6 marks)

Hint: total derivative of $z=f(x,y)$ is $dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy$

5. a) Integrate $\int\frac{2}{(x+1)(x+2)(x+3)}dx$ using partial fractions. (6 marks)

b) Using the substitution $u=1+\sqrt{x}$, find the exact integration of $\int_1^4\frac{x}{1+\sqrt{x}}dx$. (6 marks)

c) Use 1/3 Simpson's rule to interpolate a value for the integration $\int_1^4\frac{x}{1+\sqrt{x}}dx$ for ten ordinates (n = 10). (8 marks)

6. a) Find the modulus and the argument of the complex number w. (6 marks)

$$w=\frac{50}{3+4i}$$

b) The complex numbers z_1 and z_2 are given by

$z_1=p+2i$ and $z_2=1-2i$, where p is an integer.

Given that $\text{mod}\left(\frac{z_1}{z_2}\right)=13$, find possible values of p. (6 marks)

c) Find the all solutions of equation $(2-x)^3-i=0$. (8 marks)

7. a) Show that $\frac{\sqrt{180}-2\sqrt{5}}{5\sqrt{5}-5}$ can be written in the form $a+\frac{\sqrt{5}}{b}$ where 'a' and 'b' are integers. (6 marks)

b) If the roots of quadratic equation $ax^2+bx+c=0$ are not real, prove that the roots of quadratic equation $ax^2-2(a+b)x+(a+2b+4c)=0$ are also not real. (8 marks)

c) Determine the range of value of k for which the quadratic equation $kx^2+6(k-2)x+3(k+2)=0$ has real distinct roots. (6 marks)

8. a) Prove the trigonometric identity (4 marks)

$$\frac{\sin x}{\sin 2x}+\frac{\cos x}{\cos 2x}=2\sin 3x\cos ec4x$$

b) Show that $\tan^{-1}\left(\frac{6}{17}\right)+\tan^{-1}\left(\frac{11}{23}\right)=\frac{\pi}{4}$ (6 marks)

c) i. Express $f(x)=\sqrt{3}\cos x+\sin x-\frac{1}{2}$ in form of $f(x)=A\sin(x+B)+C$ (5 marks)

ii. Hence, Plot the rough sketch of trigonometric function $f(x)=\sqrt{3}\cos x+\sin x-\frac{1}{2}$ where $-2\pi\le x\le 2\pi$. (5 marks)

9. The speed of vehicles crossed a certain point in a city is observed and recorded. The following distribution was obtained for an hour.

Speed km/h	50-60	60-70	70-80	80-90	90-100
Number of Vehicles	6	16	45	26	7

a) Using cumulative frequency distribution, find the value of median. (4 marks)

b) Calculate the mode value of speed. (4 marks)

c) i. Draw up a frequency distribution table. (6 marks)

Hence taking 75 as the assumed mean find

ii. the mean (3 marks)

iii. standard deviation of the distribution. (3 marks)

Data Sheet

- Statistics

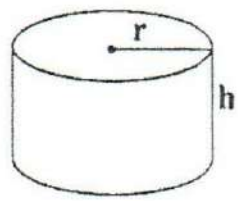
Equations to find Arithmetic mean, Variance and Standard Deviation:

$$\bar{x} = A + \frac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i}$$

$$\sigma^2 = \frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^2}{\sum_{i=1}^{n} f_i} \text{ or } \sigma^2 = \frac{\sum_{i=1}^{n} f_i d_i^2}{N} - \left(\frac{\sum_{i=1}^{n} f_i d_i}{N} \right)^2$$

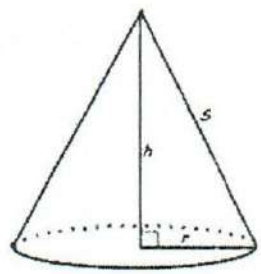
$Standard\ Deviation = \sqrt{\sigma^2}$

- Mensuration



$Surface\ Area\ of\ a\ cylinder = 2\pi r^2 + 2\pi rh$

$Volume\ of\ a\ Cylinder = \pi r^2 h$



$Surface\ Area\ of\ a\ cone = \pi r^2 + \pi rs$

$Volume\ of\ a\ cone = \frac{\pi r^2 h}{3}$

- Trigonometry

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

MINISTRY OF PORTS AND SHIPPING

MERCHANT SHIPPING SECRETARIAT - SRI LANKA

EXAMINATION FOR CERTIFICATE OF COMPETENCY
CHIEF ENGINEER & SECOND ENGINEER OFFICER ON SHIPS OF 3000KW
PROPULSION POWER OR MORE

THERMODYNAMICS

- TIME ALLOWED - THREE HOURS
- Answer any SIX (6) questions
- Marks for each part of the question are shown in the brackets
- Pass marks - 50%
- Date - 2024

Answers with clear sketches/diagrams, neat handwriting and clear expression will get full marks.

For air $c_p = 1.005kJ/kg\ K$, $c_v = 0.717kJ/kg\ K$, $\gamma = 1.4$

Composition of air (mass proportions): *77 % of Nitrogen* and *23 % of Oxygen*

Specific heat capacity of water *4.2 kJ/kg K*

1 TR (ton of refrigeration) = *3.5 kW*

Universal gas constant, $R_0 = 8.314\ kJ/kg\ mol\ K$

---

1. *2 kg* of steam of steam is throttled from *15 bar* dry saturated to *5 bar*, then expanded isentropically to *2 bar* and finally heated at constant pressure to *250 ^0C*.
   a. Sketch the *P-H* diagram (4 marks)
   b. Estimate for each of the three processes
      i. The steam condition after throttling and isentropic expansion (4 marks)
      ii. The change of enthalpy for each process (6 marks)
      iii. The change of entropy for each process (6 marks)

2. In an ideal dual combustion cycle the pressure, volume and temperature at the beginning of adiabatic compression are *193 kPa*, *1.05 m^3* and *45 ^0C* respectively. The volume ratio of the adiabatic compression is *18:1*. The maximum pressure of the cycle is *14 MPa*. The fraction of the stroke at which the combustion is theoretically completed is *5 %*. Determine
   a. The pressure, volume and temperature at the cycle process change points (12 marks)

b. The thermal efficiency (2 marks)

c. The net work done (2 marks)

d. The mean effective pressure (2 marks)

e. Carnot efficiency within the cycle temperature limits (2 marks)

Take $c_p = 1.05kJ/kg\ K$, $c_v = 0.775kJ/kg\ K$

3. In a propulsion nozzle (convergent-divergent type), gases are expands from ***850 kPa*** and ***600 K*** to a back pressure of ***90 kPa*** at the rate of ***20 kg/s***. The nozzle efficiency is ***95%***. For the gases assume $\gamma = 1.3$ and $Cp = 1.10$ kJ/kg K.

a. Estimate the followings

i. Critical pressure and temperature (4 marks)

ii. Throat area of the nozzle (4 marks)

iii. Speed at the exit (4 marks)

iv. Exit area of the nozzle (4 marks)

b. Draw the ***T-S*** diagram for the expansion (4 marks)

Hint: The critical pressure ratio of a convergent divergent nozzle is given by $\frac{P_t}{P} = \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)}$

4. A turbine plant operates by steam at pressure of ***30 bar*** and temperature of ***250 ^0C***. The steam is expanded isentropically to a pressure of ***10 bar*** and then the steam is sent into reheater to increase the temperature to ***250 ^0C*** under constant pressure. After further expansion in the turbine, the steam is exhausted into a condenser at ***0.5 bar*** where it is condensed but not under cooled. The condensate is pumped back into the boiler.

a. Sketch the ***T-S diagram*** (5 marks)

b. Determine the followings

i. Dryness fraction of the steam entering the reheater and the condenser (5 marks)

ii. Power output of the turbine if the steam flow rate ***2.5 kg/s*** (5 marks)

iii. Thermal efficiency (5 marks)

5. A six cylinder four stroke **IC** engine with ***800 mm*** bore and ***1000 mm*** stroke, was tested and the following data were obtained at constant speed of ***102 rpm***.

Brake power with all cylinders working = ***2380 kW***

Brake power with **No 1** cylinder cut out = ***1820 kW***

Brake power with **No 2** cylinder cut out = ***1883 kW***

Brake power with **No 3** cylinder cut out = ***1834 kW***

Brake power with **No 4** cylinder cut out = ***1880 kW***

Brake power with **No 5** cylinder cut out = ***1824 kW***

Brake power with **No 6** cylinder cut out = ***1786 kW***

Fuel oil consumption is ***418 kg/h*** and the calorific value of petrol is ***51.2 MJ/kg***. Determine

a. The indicated power (6 marks)

b. The mechanical efficiency (3 marks)

c. The brake specific fuel consumption (3 marks)

d. The brake thermal efficiency (3 marks)

e. Indicated mean effective pressure (5 marks)

6. The absolute steam entry velocity of a single stage impulse turbine is ***600 m/s*** and the nozzle angle is ***20 ^{0}***. Ratio of the blade speed to whirl component of steam speed is ***0.5***. The blade velocity coefficient is ***0.85*** and rate of the steam is ***5 kg/s***. If the steam enters the blades without shock and leaves the blades in an axil direction.

   a. Draw the velocity diagram for the impulse turbine (6 marks)

   b. Determine

      i. Blade angles at inlet and outlet (6 marks)

      ii. Driving force on the wheel and the axial thrust (4 marks)

      iii. Diagram power and efficiency (4 marks)

7. A compressor delivers ***R 134a*** at ***7.7 bar*** and from an induction pressure of ***1.33 bar*** and ***-10 ^{0}C***. The compression is carried out two stages with an ideal intermediate pressure and introducing perfect inter cooling. The clearance volume is ***3%*** of the swept volume in each cylinder and compressor speed is ***900 rpm***. The index of compression and re-expansion is ***1.247*** for both cylinders. The mechanical efficiency of the compressor is ***85 %***. If the refrigerant flows at the rate of ***0.024 kg/s***, determine

   a. Intermediate pressure (3 marks)

   b. The temperature of refrigerant when it leaves the compressor (3 marks)

   c. Indicated power of the compressor (5 marks)

   d. saving in power over single-stage compression between the same pressure (5 marks)

   e. Required power output of the driver motor (4 marks)

Hint : minimum work done for N number of stages , $W = \frac{Nn}{n-1} P_1 v_1 \left[\left(\frac{P_{N+1}}{P_1} \right)^{n-1/Nn} - 1 \right]$ and the gas constant of refrigerant is ***0.0815 kJ/kg K***.

8. ***Refrigerant -134a*** uses to operate a reefer container in the pressure range of ***1.3272 bar*** and ***7.7 bar***. The refrigerant enters at ***-10 ^{0}C*** and leaves the compressor as a superheated vapour at ***7.7 bar*** and ***40 ^{0}C***. If the refrigerant leaves the condenser at ***25 ^{0}C***,

   a. Estimate

      i. The dryness fraction of the refrigerant at the evaporator inlet (3 marks)

      ii. Compressor work done per kg of refrigerant (4 marks)

      iii. Coefficient of performance (5 marks)

   b. Draw the ***T-S*** and ***P-H*** diagrams (4 marks)

   c. ***10 kg*** of water at ***30 ^{0}C*** turns into ice at ***0 ^{0}C*** within ***20 minutes*** inside the refer container. Estimate the refrigeration load and refrigerant flow rate through the evaporator. (4 marks)

   Hint : Specific heat of ice – ***2.04 kJ/kg K***, Latent heat of fusion of ice – ***335 kJ/kg***

9. An industrial hot water system contains the following data. Pipe diameter ***20 mm***, wall thickness ***5 mm***, thermal conductivity of material ***52 W/mK***. Mean water temperature ***85 ^{0}C***. The inner surface heat transfer coefficient ***1136 W/m^2 K***. insulation thickness ***20 mm*** and conductivity ***0.17 W/mK***. The ambient temperature is ***27 ^{0}C*** and outer surface heat transfer coefficient ***9.7 W/m^{2}K***.

   a. Calculate

      i. The rate of heat lost per unit length of pipe (4 marks)

      ii. The inside surface temperature (4 marks)

      iii. The interface temperature between pipe and insulation (4 marks)

      iv. The outside surface temperature (4 marks)

   b. Draw the temperature variation through the thickness of the lagging materials (4 marks)

   Hint: Rate of heat transfer through thick cylinder is given by

   $\frac{Q}{t} = 2\pi kl \frac{\theta_2 - \theta_1}{\ln R_2/R_1}$ where R_1 and R_2 are inner radius and outer radius of the cylinder

MINISTRY OF PORTS AND SHIPPING

MERCHANT SHIPPING SECRETARIAT - SRI LANKA

EXAMINATION FOR CERTIFICATE OF COMPETENCY
CHIEF ENGINEER & SECOND ENGINEER OFFICER ON SHIPS OF 3000KW PROPULSION POWER OR MORE

MARINE ENGINEERING DRAWING AND DESIGN

- **TIME ALLOWED - FOUR HOURS**
- **Answer all questions**
- **Marks for each part of the question are shown in the brackets**
- **Pass marks - 50%**
- **Date - 2024**

Answers with clear sketches/diagrams, neat handwriting and clear expression will get full marks.

Details of the component parts of a **REDUCING VALVE** are shown in the following figure. Draw the following views of the assembled Reducing Valve using **THIRD ANGLE PROJECTION.**

a. A sectional elevation with the valve fully closed when viewed from arrow **A**.

b. An end elevation as an outside view in the direction of arrow **C**, projected from (a)

Any unspecified dimension is left to the discretion of the candidate. Hidden details are not required unless it is necessary to amplify a design feature.

Complete the drawing by adding dimensions, a list of material of parts, title and projection symbol.

Marking Scheme

1)	Assembling accuracy of view (a)	(45 marks)
2)	Assembling accuracy of view (b)	(30 marks)
3)	The optimization of the space	(05 marks)
4)	Dimensioning	(05 marks)
5)	Material Box	(05 marks)
6)	The title block, projection symbol and lettering	(05 marks)
7)	Final appearance	(05 marks)

MINISTRY OF PORTS AND SHIPPING

MERCHANT SHIPPING SECRETARIAT - SRI LANKA

EXAMINATION FOR CERTIFICATE OF COMPETENCY
CHIEF ENGINEER & SECOND ENGINEER OFFICER ON SHIPS OF 3000KW PROPULSION POWER OR MORE

MARINE ENGINEERING DRAWING AND DESIGN

- **TIME ALLOWED - FOUR HOURS**
- **Answer all questions**
- **Marks for each part of the question are shown in the brackets**
- **Pass marks - 50%**
- **Date - 2023**

Answers with clear sketches/diagrams, neat handwriting and clear expression will get full marks.

Figure shows details of a **BALLEST CHEST**. Draw the following views of assembled BALLEST CHEST in **THIRD ANGLE PROJECTION**. Select a suitable scale.

a. An elevation when viewed from arrow **A** to show the left half in section from the vertical plane through the chest and the right half as an outside view.
b. A plan view projected from '(a)' when viewed from the direction of arrow **C**.

Any unspecified dimension is left to the discretion of the candidate. Hidden detail is not required unless it is necessary to amplify a design feature.

Complete the drawing by adding dimensions, a list of material of parts, title and projection symbol.

Marking Scheme

1) Assembling accuracy of view (a)	(45 marks)
2) Assembling accuracy of view (b)	(30 marks)
3) The optimization of the space	(05 marks)
4) Dimensioning	(05 marks)
5) Material Box	(05 marks)
6) The title block, projection symbol and lettering	(05 marks)
7) Final appearance	(05 marks)

MINISTRY OF PORTS AND SHIPPING

MERCHANT SHIPPING SECRETARIAT - SRI LANKA

EXAMINATION FOR CERTIFICATE OF COMPETENCY
CHIEF ENGINEER & SECOND ENGINEER OFFICER ON SHIPS OF 3000KW PROPULSION POWER OR MORE

THERMODYNAMICS

- **TIME ALLOWED - THREE HOURS**
- **Answer any SIX (6) questions**
- **Marks for each part of the question are shown in the brackets**
- **Pass marks - 50%**
- **Date - 2023**

Answers with clear sketches/diagrams, neat handwriting and clear expression will get full marks.

For air $c_p = 1.005kJ/kg\ K$, $c_v = 0.717kJ/kg\ K$, $\gamma = 1.4$

Composition of air (mass proportions): *77 % of Nitrogen* and *23 % of Oxygen*

Specific heat capacity of water *4.2 kJ/kg K*

1 TR (ton of refrigeration) = *3.5 kW*

Universal gas constant, $R_0 = 8.314\ kJ/kg\ mol\ K$

1.

a. State the steady flow energy equation in full, defining the symbols and unit used (5 marks)

b. Air passes through a gas turbine at the rate of ***10 kg/s***. The gas enters the turbine with a velocity of ***100 m/s*** and specific volume of ***0.68 m^3/kg***. The gas leaves the turbine with a specific volume of ***1.85 m^3/kg***. The exit area of the turbine is ***0.045 m^2***. In its passage through the turbine system, the specific enthalpy of air is reduced by ***300 kJ/kg*** and there is a heat transfer loss of ***48 kJ/kg***. Determine

   i. The inlet area of the turbine in m^2 (5 marks)

   ii. The exit velocity of the air *m/s* (5 marks)

   iii. The power developed by the turbine system in *kW* (5 marks)

2. A vessel of ***1.004 m³*** fixed volume contains steam at a temperature and pressure of ***300 °C*** and ***20 bar***. Steam is adiabatically blown off until the pressure falls to ***10 bar*** and the temperature is ***200 °C***. The vessel is then cooled to ***3 bar***.

   a. Sketch the ***T-S*** diagram to indicate the processes (4 marks)

   b. Estimate

      i. The mass of steam blown off (4 marks)

      ii. The final dryness fraction of the steam (4 marks)

      iii. The specific enthalpy change during the cooling (4 marks)

      iv. The specific change in entropy during the cooling (4 marks)

3. A compression ignition engine working on the ideal Dual combustion cycle has a compression ratio of ***14 to 1***. The pressure and temperature at the beginning of compression are ***0.93 bar*** and ***20 °C*** respectively. The maximum pressure in the cycle is ***50 bar*** and the constant pressure heat transfer takes place for ***10%*** of the stroke.

   a. Sketch the ***T-S*** diagram for the cycle. (4 marks)

   b. Estimate the followings

      i. Pressure and temperature at each cardinal points of the cycle (8 marks)

      ii. The thermal efficiency (4 marks)

      iii. Mean effective pressure of the cycle (4 marks)

4.

   a. Describe the overall heat transfer coefficient (4 marks)

   b. Steam at ***460 °C*** and ***40 bar*** flows through a ***12 m*** length of steel pipe which has a diameter of ***50 mm*** and a thickness of ***5 mm***. The steam temperature falls to ***450 °C*** at the exit of the pipe when an insulation is attached. The steam flow rate is ***5 kg/s*** and the ambient temperature of the air is ***30 °C***. Calculate the followings

      i. The rate of heat loss through the surface of the pipe (4 marks)

      ii. The interface temperature between steel and insulation (5 marks)

      iii. Thickness of the insulation (7 marks)

   Thermal conductivities of steel and insulation are ***50*** and ***0.05 W/m K***.

   Hint: Rate of heat transfer through thick cylinder is given by

   $$\frac{Q}{t} = 2\pi kl \frac{\theta_2 - \theta_1}{\ln r_2/r_1}$$ where r_1 and r_2 are inner radius and outer radius of the cylinder

5. In a regenerative steam power plant, steam enters the turbine at a pressure of ***40 bar*** and a temperature of ***450 ^0C*** and expands to ***0.2 bar*** under constant entropy. Certain mass of steam is bled from the turbine at a pressure of ***10 bar*** and supplied to a direct feed heater. The feed water leaves the feed heater at the saturation temperature of the bled steam and there is no undercooling in the condenser. Neglect the feed pump work.

   a. Draw the ***T-S*** diagram of the plant (4 marks)

   b. Estimate

      i. The condition of the bled steam (3 marks)

      ii. The fraction of the steam used for feed heating (5 marks)

      iii. Thermal efficiency of the plant (8 marks)

6.

   a. The critical pressure ratio of a convergent divergent nozzle is given by $\frac{P_t}{P} = \left(\frac{2}{\gamma+1}\right)^{\gamma/(\gamma-1)}$

   determine the critical pressure ratio

      i. For the steam entering as dry and saturated, $\gamma = 1.135$ (2 marks)

      ii. For the steam entering as superheated, $\gamma = 1.3$ (2 marks)

   b. Superheated steam at ***40 bar*** and ***450 ^0C*** expand in a convergent-divergent nozzle to ***10 bar***. Assume that the inlet velocity is negligible and the isentropic expansion is ***0.87*** (throat to exit) The mass flow rate is ***10 kg/s***. Estimate

      i. The pressure at the throat (2 marks)

      ii. The throat area (6 marks)

      iii. The exit area and the exit velocity (8 marks)

7. A Morse test on four-cylinder engine resulted in the following data:

   Brake power with all cylinders working = ***2178 kW***

   Brake power with No. 1 cylinder cut-off = ***1558 kW***

   Brake power with No. 2 cylinder cut-off = ***1563 kW***

   Brake power with No. 3 cylinder cut-off = ***1575 kW***

   Brake power with No. 4 cylinder cut-off = ***1550 kW***

The bore and stroke are ***400 mm*** and ***950 mm*** respectively. The speed of the engine is ***300 rev/min***. The calorific value of the fuel is ***40.9 MJ/kg***. The brake thermal efficiency of the engine is ***38 %***. Estimate the followings

   a. Indicated power of the engine (4 marks)

b. The mechanical efficiency (4 marks)

c. Brake specific fuel consumptions (4 marks)

d. Indicated thermal efficiency (4 marks)

e. Indicated mean effective pressure (4 marks)

8. The nozzle of a simple impulse turbine are inclined at an angle of 20^0 to the direction of the path of the moving blades; the steam leaves the nozzles at *900 m/s* and the blade speed is *400 m/s*. There is a *15 %* loss in relative velocity due to friction in the blades and there shall be no axial thrust on the blades.

a. Draw the velocity diagram (5 marks)

b. Find the suitable inlet and outlet angles for blades (5 marks)

c. Estimate the power developed for a steam flow of 1 kg/s at the blades (5 marks)

d. Determine the kinetic energy of the steam finally leaving the wheel (5 marks)

9. A vapour compression refrigeration plant using *R-404A* operates between temperature limits of -22^0C and 35^0C. The refrigerant enters the compressor as *dry saturated vapour* and leaves at *16.197 bar* and 49^0C.

a. Estimate

i. The compressor work in kJ/kg (3 marks)

ii. The COP if the refrigerant leaves the condenser as a saturated liquid (4 marks)

iii. The COP if the refrigerant leaves the condenser at 26^0C (4 marks)

b. Draw the *PH* and *TS* diagrams for each cases (4 marks)

c. Estimate the mass flow rate and the condenser cooling heat transfer in each case if the compressor power is *10 kW* and mechanical efficiency is *95%*. (5 marks)

Hint: Use the *R-404A* thermodynamics property table attached below.

R-404 A Constant Pressure Tables

Pressure, bar	Saturation temperature, °C	Enthalpy, kJ/kg		Entropy, kJ/kg K		Degree of superheat(t-t_s)=5°C		Degree of superheat(t-t_s)=10°C		Degree of superheat(t-t_s)=15°C	
		h_f	h_g	s_f	s_g	h(kJ/kg)	s(kJ/kgK)	h(kJ/kg)	s(kJ/kgK)	h(kJ/kg)	s(kJ/kgK)
2.758	-23	168.9	354.7	0.8821	1.6260	361.2	1.6514	365.3	1.6673	369.5	1.6829
2.864	-22	170.2	355.3	0.8872	1.6255	361.0	1.6479	365.1	1.6637	369.3	1.6795
2.974	-21	171.5	355.9	0.8924	1.6250	360.8	1.6445	365.0	1.6603	369.2	1.6761
12.884	26	239.5	380.8	1.1357	1.6083	384.5	1.6193	389.7	1.6364	394.8	1.6530
13.225	27	241.2	381.1	1.1411	1.6079	383.5	1.6133	388.7	1.6306	393.9	1.6474
13.572	28	242.8	381.5	1.1465	1.6075	382.5	1.6082	387.9	1.6257	393.2	1.6427
15.800	34	253.0	383.5	1.1793	1.6044	391.4	1.6324	396.7	1.6494	402.2	1.6659
16.197	35	254.8	383.8	1.1848	1.6038	389.5	1.6225	395.0	1.6400	400.5	1.6570
16.601	36	256.5	384.1	1.1904	1.6032	387.5	1.6124	393.2	1.6304	398.8	1.6479

MINISTRY OF PORTS AND SHIPPING

MERCHANT SHIPPING SECRETARIAT - SRI LANKA

EXAMINATION FOR CERTIFICATE OF COMPETENCY
CHIEF ENGINEER & SECOND ENGINEER OFFICER ON SHIPS OF 3000KW PROPULSION POWER OR MORE

MATHEMATICS

- **TIME ALLOWED - THREE HOURS**
- **Answer any SIX (6) questions**
- **Marks for each part of the question are shown in the brackets**
- **Pass marks - 50%**
- **Date - 2023**

Answers with clear sketches/diagrams, neat handwriting and clear expression will get full marks.

1. a) If $\log 2 = p$, express $\frac{1}{3}\log 8 - 2\log 3 + 2\log 6$ in term of p. (6 marks)

   b) If $a = \log\left(\frac{10}{9}\right)$, $b = \log\left(\frac{25}{24}\right)$, $c = \log\left(\frac{81}{80}\right)$, show that $7a - 2b + 3c = \log 2$ (6 marks)

   c) Solve the equation $\log_3(2-3x) = \log_9(6x^2 - 19x + 2)$ (8 marks)

2. a) If $A = \begin{pmatrix} 2 & 3 \\ 2 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix}$ show that $(AB)^T = B^T A^T$ (6 marks)

   b) If $A = \begin{pmatrix} 2 & -3 & 4 \\ -3 & 4 & 2 \\ 4 & -2 & -3 \end{pmatrix}$, Determine

   i. Determinant of A. (4 marks)

   ii. Inverse of A. (6 marks)

   c) Hence, solve the simultaneous equations. (4 marks)

$$2x - 3y + 4z = -9$$
$$-3x + 4y + 2z = -12$$
$$4x - 2y - 3z = -3$$

3. a) Evaluate $\lim_{x\to\infty}\left(\frac{4x^3+x^2+2x+1}{x^3+x^2-x-1}\right)$. (6 marks)

b) Differentiate the function $(\sin x)^{\cos x}$ with respect to x. (8 marks)

c) If $x=\frac{1+\ln t}{t^2}$ and $y=\frac{3+2\ln t}{t}$, Show that $y\frac{dy}{dx}=2x\left(\frac{dy}{dx}\right)^2+1$. (6 marks)

4. a) If $u=\ln(x^2+y^2)+\tan^{-1}\left(\frac{y}{x}\right)$, find all second order partial derivatives of u.(8 marks)

b) Hence, Prove that $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$ (6 marks)

c) The dimensions of a cylindrical gas tank are radius 4 m and height 6 m. Calculate the error in its volume if the scale used in taking the measurement is short by 0.01 m per meter.

Hint: total derivative of $z=f(x,y)$ is $dz=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy$ (6 marks)

5. a) Integrate $\int\frac{3x^2+12x+11}{(x+1)(x+2)(x+3)}dx$ (6 marks)

b) Using the substitution $x=(u-4)^2+1$ or otherwise, and integrating, find the exact value of I, $I=\int_2^5\frac{1}{4+\sqrt{x-1}}dx$ (6 marks)

c) Use 1/3 Simpson's rule to interpolate a value for the integration $I=\int_2^5\frac{1}{4+\sqrt{x-1}}dx$ for ten ordinates (n = 10). Hence, find the error of result. (8 marks)

6. a) If $z_1=4-3i$, $z_2=12+5i$ and $z_3=i$, find (6 marks)

i. $z_1-3z_2+z_3$

ii. $\frac{z_3}{iz_1+z_2}$

b) Express the complex number $1+i\sqrt{3}$ in form of $\mu(\cos\alpha+i\sin\alpha)$ (8 marks)

hence, find the value of $\left(1+i\sqrt{3}\right)^{10}$.

c) Find the square root of $z=i$ (6 marks)

7. a) In a triangle PQR, $P\hat{R}Q = \frac{\pi}{4}$. If $\tan(P/3)$ and $\tan(Q/3)$ are roots of the equation $ax^2 + bx + c = 0$, prove that $a + b = c$ (6 marks)

   b) One root of the equation $ax^2 + bx + c = 0$ is greater by p than the other. Show that $a(ap^2 + 4c) = b^2$. Hence, derive the condition for the two roots to be equal. (8 marks)

   c) A quadratic function is defined by

   $$f(x) = x^2 + (k+2)x - 12$$

   where k is a constant. It is given that the equation $f(x) = 0$ has two distinct real roots. Find the set of values that k can take. (6 marks)

8. a) Prove that $\frac{\cos x}{1+\sin x} = \sec x - \tan x$ (6 marks)

   b) Express $f(x) = 5\cos^2 x + 18\sin x\cos x + 29\sin^2 x$ in form of $f(x) = A + B\cos(2x + C)$ (7 marks)

   c) Hence, find the range of $f(x)$. (7 marks)

9. The speed of vehicles crossed a certain point in a city is observed and recorded. The following distribution was obtained for an hour.

Speed km/h	0-20	20-40	40-60	60-80	80-100
Number of Vehicles	5	18	40	29	8

   a) Using cumulative frequency distribution, find the value of median. (4 marks)

   b) Calculate the mode value of speed. (4 marks)

   c) i. Draw up a frequency distribution table. (6 marks)

   Hence taking 50 as the assumed mean find

   ii. the mean (3 marks)

   iii. standard deviation of the distribution. (3 marks)

Data Sheet

- Statistics

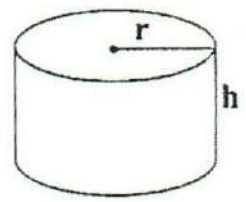
Equations to find Arithmetic mean, Variance and Standard Deviation:

$$\bar{x} = A + \frac{\sum_{i=1}^{n} f_i d_i}{\sum_{i=1}^{n} f_i}$$

$$\sigma^2 = \frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^2}{\sum_{i=1}^{n} f_i} \text{ or } \sigma^2 = \frac{\sum_{i=1}^{n} f_i d_i^2}{N} - \left(\frac{\sum_{i=1}^{n} f_i d_i}{N} \right)^2$$

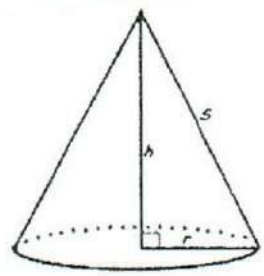
$Standard\ Deviation = \sqrt{\sigma^2}$

- Mensuration



$Surface\ Area\ of\ a\ cylinder = 2\pi r^2 + 2\pi rh$

$Volume\ of\ a\ Cylinder = \pi r^2 h$



$Surface\ Area\ of\ a\ cone = \pi r^2 + \pi rs$

$Volume\ of\ a\ cone = \frac{\pi r^2 h}{3}$

- Trigonometry

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\tan(A \pm B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

MINISTRY OF PORTS AND SHIPPING

MERCHANT SHIPPING SECRETARIAT - SRI LANKA

EXAMINATION FOR CERTIFICATE OF COMPETENCY
CHIEF ENGINEER & SECOND ENGINEER OFFICER ON SHIPS OF 3000KW PROPULSION POWER OR MORE

C(2(1) – Academic

APPLIED MECHANICS

- TIME ALLOWED - THREE HOURS
- Answer any SIX (6) questions
- Marks for each part of the question are shown in the brackets
- Pass marks - 50%
- Date - 2023

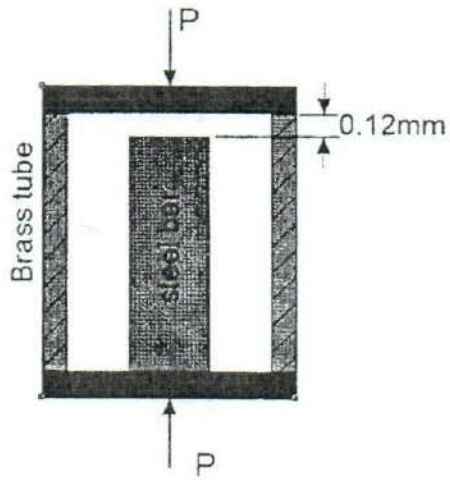
Answers with clear sketches/diagrams, neat handwriting and clear expression will get full marks.

Unless otherwise stated, use: Density of water – 1000 kgm^{-3} Gravitational acceleration – 9.81 ms^{-2}
Density of sea water 1025 kgm^{-3}

1.

1.1 A solid steel bar of 2cm diameter and 40cm long is placed inside a brass tube. (Sectional elevation of the assembly is shown in the figure) Assume bar and tube are concentric. The brass tube has inside diameter of 2.2cm and outside diameter of 3cm. As indicated in the diagram length of the tube exceeds the length of the steel bar just by 0.12mm. If the assembly is firmly rest between two rigid plates, calculate load P which will just make tube and bar same length by compressing the copper tube. (12 marks)

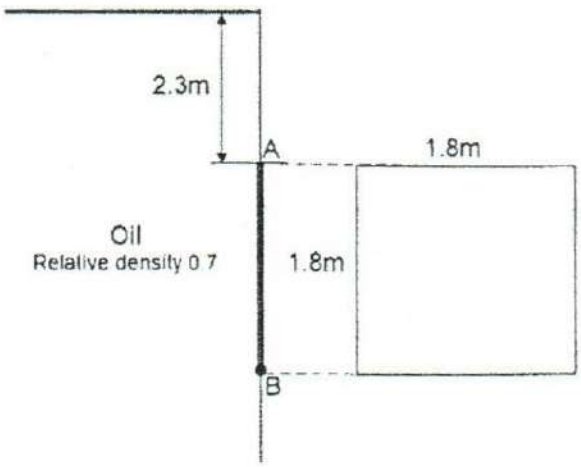
1.2 If a load of 50kN is applied to the uncompressed assembly (New P = 50kN), calculate the stresses in the steel and brass tube. Take E for steel = 213 GN/m^2, E for Brass = 100 GN/m^2 (8 marks)



2

2.1 A fuel tank contains oil of relative density 0.7. In one vertical side of the tank is cut a square shape opening 1.8 m, and closed by a trap door hinged at the lower end B and held by a bolt at the upper edge A. If the fuel level is 2.3 m above the top edge of the opening calculate;

a. The total fluid thrust on the door (7 Marks)
b. The position of the center of pressure. (7Marks)
c. The turning moment on the door about the hinge at B (6 Marks)



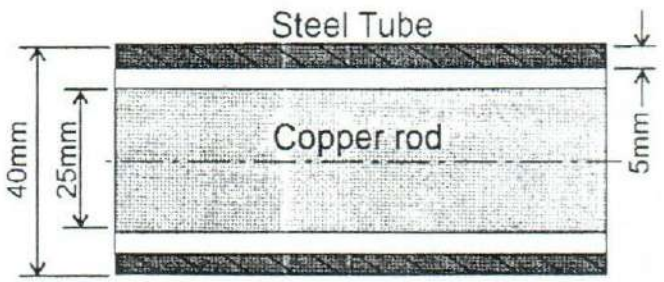
3.

3.1 Derive a basic relationship between thermal stress and thermal strain (6 Marks)

3.2 A copper rod is placed inside a steel tube with an external diameter of 400mm and a thickness of 5mm (see sectional figure of the arrangement given). The copper rod is 25mm diameter in which they are rigidly fixed at their terminals to make a compound bar. Assuming no longitudinal stress at 75°C, calculate the stresses in the rod and tube when the temperature is raised to 500°C, writing the strain equation for the system. (14 Marks)

Modulus of Elasticity of Steel E_S and CopperE_C: $E_S = 30 \times 10^6 Ncm^{-2}$ and $E_C = 17 \times 10^6\ Ncm^{-2}$

Coefficients of linear expansions of steel $\alpha_s$and copper α_c: $\alpha_s = 6 \times 10^{-6}\ °C^{-1} and\ \alpha_c = 10 \times 10^{-6}\ °C^{-1}$



4.

4.1. A shell is fired horizontally from the top of a cliff towards the sea below. The sea surface is 19.6 m below the cliff edge. If the horizontal velocity of the shell is 1200 m/s, how far from the base of the cliff will the shell strike the surface of the sea? (8 marks)

4.2 A second shell is fired from the same location with an initial velocity of 100 m/s and at an angle of 30^0 to the horizontal. Determine its velocity, its magnitude and direction after 2 seconds and 4 seconds respectively. (8 marks)

4.3 What is the maximum height the second projectile can reach? (4 marks)

5.

5.1 A dry single plate (single contact surface) clutch is to be *designed* for an automotive vehicle whose engine is rated to give 100 kW at 2400 rpm and maximum torque of 500 Nm. The outer radius of friction plate is 25% more than the inner radius. The intensity of pressure between the plates is not to exceed 0.07 N/mm^2. The coefficient of friction between the contact surfaces may be assumed to be equal to 0.3. Eight helical springs are required by this clutch to provide axial force necessary to engage. If each spring has stiffness equal to 40 N /mm, determine;

a. Inner and outer radii of the friction plate (6 marks)
b. Total axial force on the plates (6 marks)
c. Initial compression in the springs due to the axial force (8 marks)

6.

6.1 A truck A of mass 6 tons travelling with a velocity of 4.5m/s collide with a truck B of mass 4 tons travelling with a velocity of 3 m/s in the same direction. If the relative velocity of the trucks after impact is 1.4m/s, find;

a. Velocity of each truck just after the impact (6 marks)
b. Energy loss of the system due to the impact (6 marks)

6.2 After the impact truck B slows down to 4 m/s velocity and starts descending down a hill along the road with an inclination of 1 in 5 (sin). Calculate using energy method, the average breaking force required to bring the car to rest in 5m. Assume frictional resistance of 250N to motion. (8 marks)

7.

7.1 Assuming ideal, lossless and steady flow of liquid in an inclined pipe arrangement, derive Bernoulli's equation. Use usual notations (6 marks)

7.2 A smooth pipe *AB* with uniform diameter of 150 cm, is inclined to the horizontal so as to have side A at an elevation of 15m and side B at 25 m from the datum line measured. The pressure at A is recorded

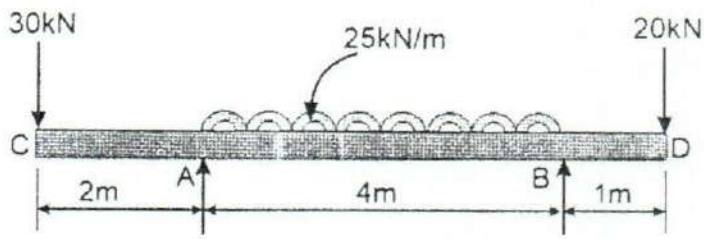
as 60 kPa and pressure at location B shows 30 kPa. If the flow rate through a pipe is measured to be 10 m^3/sec, determine the direction of flow and total head loss. (8 marks)

7.3 Keeping the elevation of the side A constant, the side B elevation was changed so as to have zero head loss. Assuming no change in the pressure values at each ends find the new elevation of side B under this circumstance. (6 marks)

8.

8.1 A simply supported beam of 7 m span with overhangs rest on supports which are 4 m apart. The left end is overhanging 2 m and right 1m. The beam carries loads of 30 kN and 20kN on the left and the right ends respectively apart from a uniformly distributed load of 25 kN/m between the supporting points. . (See the diagram for details)

a. Find reaction forces at supports (4 marks)
b. Draw the shear force and bending moment diagrams (12 marks)
c. Find contra-flexure point(s) (4 marks)



9.

9.1 Ship X is 10 nautical miles due east of a ship Y. ship X makes 12 knots on a bearing of 315^0 and ship Y makes 15 knots on a bearing of 30^0. Find;

a. A relative velocity of X to Y and its direction from the North south line (5 marks)
b. The minimum distance eventually separating the ships (5 marks)
c. The time to reach the point of nearest approach (5 marks)

9.2 If ship X wanted to contact ship Y at what speed should she proceed on her original course? (5marks)