

SHIP STABILITY II

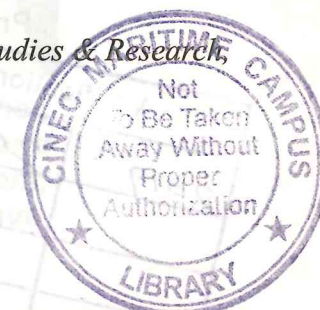
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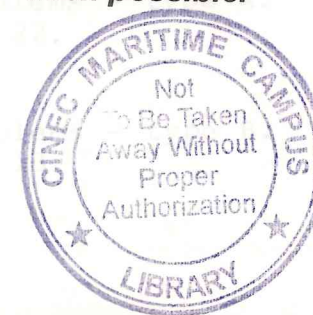
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Mrs Vijaya Harry
11th April '26 – 11th Jan '09.



*Dedicated to my mother,
 without whose patient and
 constant encouragement,
 this book would not have
 been possible.*



SHIP STABILITY II

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P R E F A C E

Like all the other books in the Nutshell Series, this book is intended to enable officers to study whilst at sea.

The subject has been divided into three parts:- 'Ship Stability I, II and III' (Nutshell Series Books 4, 5 and 6) such that all three cover the syllabus for Master F.G, parts I & II for First Mate F.G, and part I for Second Mate F.G and Navigational Watchkeeping Officer. The three parts are in continuation with no repetition of any portions.

In the second edition, minor changes have been made, especially in chapter 22 - Curve of Statical Stability.

Bombay,

1st August 1986

H. Subramaniam

(H.SUBRAMANIAM)

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CHAPTER 19

CALCULATION OF BM

AND KM; KM CURVES

The transverse BM, also referred to as BM_T , is the vertical distance between the COB and the transverse metacentre, M or M_T , and is calculated by the formula:

$$BM = \frac{I}{V}$$

Where I is the moment of inertia, or the second moment, of the water-plane area about the centre line of the ship, expressed in m^4 .

V is the vol of displacement in m^3
BM so obtained, would be in metres.

Rectangular water-planes:

The moment of inertia of a rectangle about its centre line (I or I_{CL}) is given by the formula: $I = LB^3 \div 12$. So for a rectangular water-plane:

$$BM = I/V = LB^3/12V$$

Note: The vessel need NOT be box-shaped for its water-plane to be rectangular.

For a box-shaped vessel, $V = L \times B \times d$.

$$BM = I/V = LB^3/12V = LB^3/12LBd = B^2/12d.$$

For a triangular shaped vessel $V = LBd/2$

$$BM = I/V = LB^3/12(LBd/2) = B^2/6d.$$

Note: Though the vessel is triangular shaped, the water-plane is a rectangle. B is the breadth of the water-plane.

Shipshapes

The moment of inertia of the water-plane area of a ship about its centre line can be calculated by using Simpson's Rules as illustrated in the next chapter. The I, thus calculated, divided by V would give the BM or BM₁.

Example 1

Find the GM of a box-shaped vessel 20 x 6 x 5 m, if draft = 3 m and KG = 1.8 m.

$$\begin{aligned} KB &= \text{draft}/2 = 3/2 = 1.5 \text{ m} \\ BM &= B^2/12d = (6 \times 6) \div (12 \times 3) = 1.0 \text{ m} \\ KM &= KB + BM = 2.5 \text{ m} \end{aligned}$$

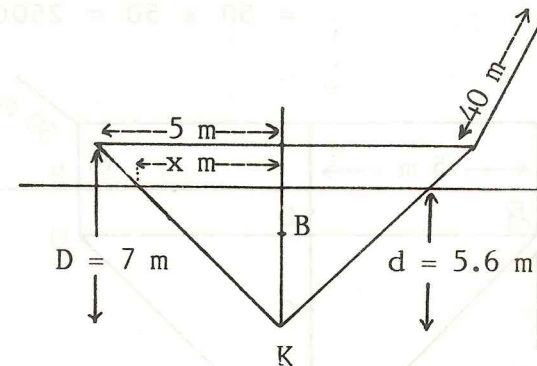
$$GM = KM - KG = 2.5 - 1.8 = 0.7 \text{ m answer.}$$

Example 2

A vessel has the form of a triangular prism of length 40 m, breadth 10 m and depth 7 m. Find the KM at 5.6 m draft.

Let the half breadth of the water-plane = X metres. With reference to the figure on the next page and considering similar triangles: $X/5.6 = 5/7$. $X = 4.0$ metres.

Breadth of the water-plane = $2 \times 4 = 8 \text{ m}$



$$\begin{aligned} KB &= \text{draft} \times 2/3 = 5.6 \times 2/3 = 3.733 \text{ m} \\ BM &= B^2/6d = (8 \times 8)/(6 \times 5.6) = 1.905 \text{ m} \\ KM &= KB + BM = 5.638 \text{ m} \end{aligned}$$

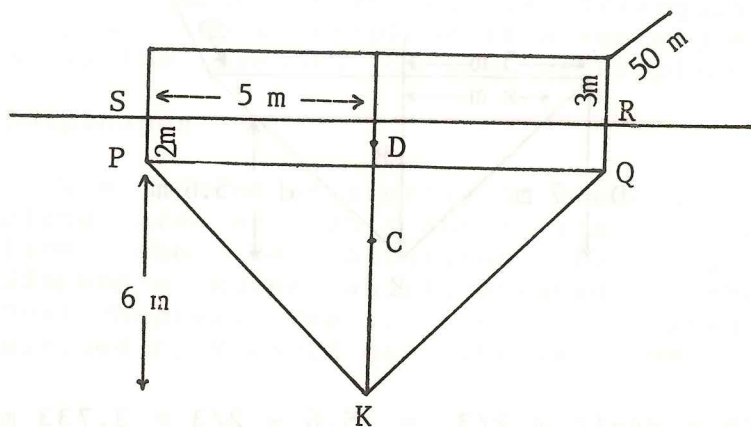
Example 3

A vessel 50 m long has a uniform transverse cross-section throughout, consisting of a rectangle above a triangle. The rectangle is 10 m broad & 5 m high. The triangle is apex downwards, 10 m broad at the top and 6 m deep. Calculate the KM at 8 m draft.

To find the KB, take moments of area about K. (See figure on next page).

$$\begin{aligned} KB &= \frac{(\text{Area POK} \times KC) + (\text{Area PQRS} \times KD)}{\text{Total area PKQRS}} \\ &= \frac{(10 \times 6 \times 1/2)4 + (10 \times 2)7}{(10 \times 6 \times 1/2) + (10 \times 2)} = 5.2 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Volume of displacement} &= \text{Area PKQRS} \times L \\ &= 50 \times 50 = 2500 \text{ m}^3. \end{aligned}$$



Since water-plane is rectangular,

$$I \cdot CL = LB^3/12 = 50 \times 10^3/12 = 4166.667 \text{ m}^4$$

$$BM = I/V = 4166.667/2500 = 1.667 \text{ metres.}$$

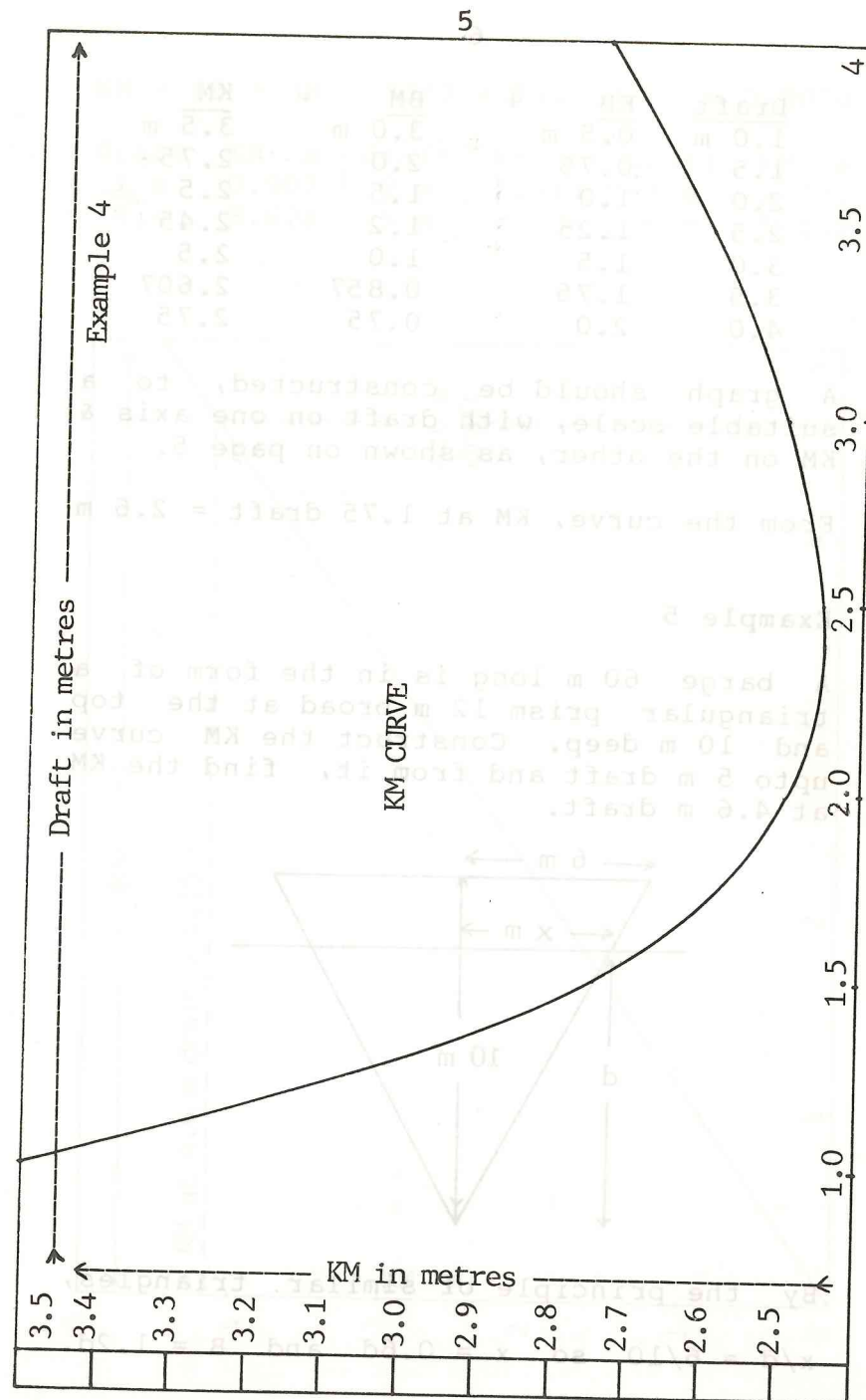
$$KM = KB + BM = 5.200 + 1.667 = 6.867 \text{ m.}$$

Example 4

A box-shaped vessel is 32 m long and 6 m broad. Construct the KM curve between the drafts of 1 m & 4 m. From the curve, find the KM at 1.75 m draft.

$$KM = KB + BM = d/2 + B^2/12d = d/2 + 3/d.$$

For the various drafts, KB and BM are calculated and tabulated on page 6.



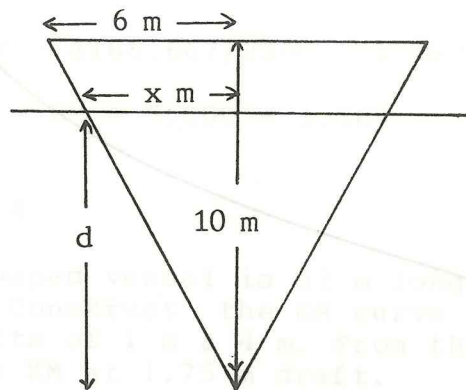
Draft	KB	+	BM	=	KM
1.0 m	0.5 m		3.0 m		3.5 m
1.5	0.75		2.0		2.75
2.0	1.0		1.5		2.5
2.5	1.25		1.2		2.45
3.0	1.5		1.0		2.5
3.5	1.75		0.857		2.607
4.0	2.0		0.75		2.75

A graph should be constructed, to a suitable scale, with draft on one axis & KM on the other, as shown on page 5.

From the curve, KM at 1.75 draft = 2.6 m

Example 5

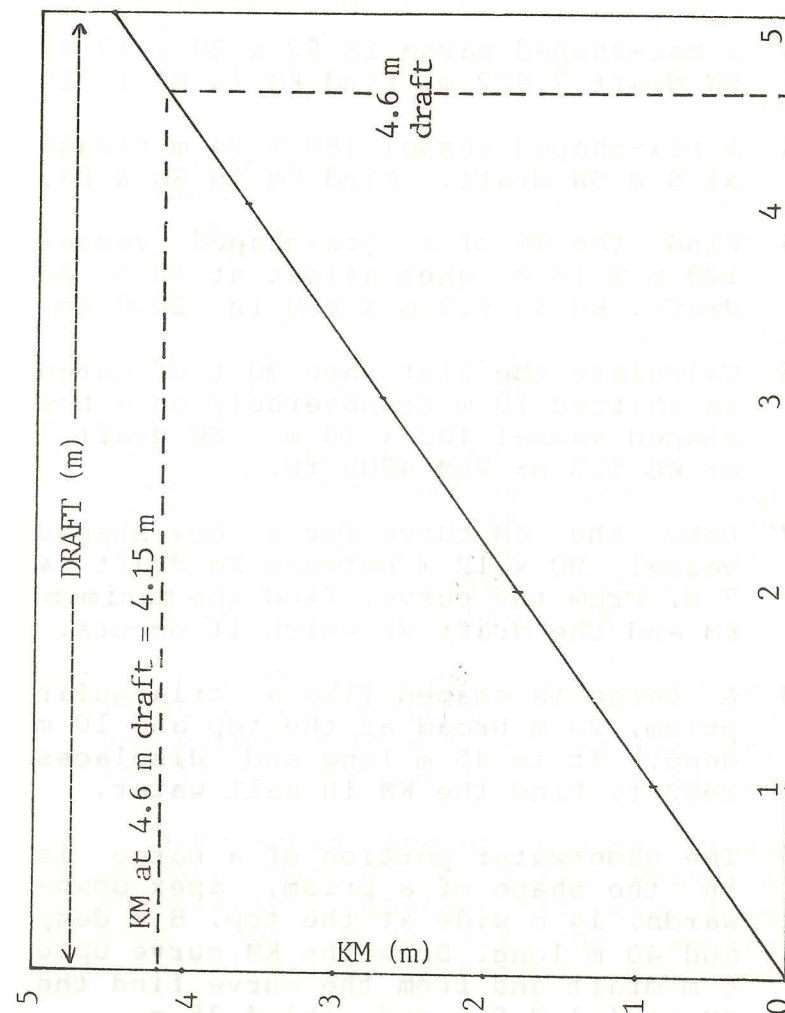
A barge 60 m long is in the form of a triangular prism 12 m broad at the top and 10 m deep. Construct the KM curve upto 5 m draft and from it, find the KM at 4.6 m draft.



By the principle of similar triangles,
 $x/d = 6/10$ so $x = 0.6d$ and $B = 1.2d$.

$$KM = KB + BM = 2d/3 + (1.2d)^2/6d = 0.907d$$

draft	KM m	draft	KM m	draft	KM m
1 m	0.907	2 m	1.814	3 m	2.721
4 m	3.628	5 m	4.535	6 m	5.442



Exercise 15

Calculation of BM & KM; KM curves

- 1 A box-shaped barge is 40 x 25 x 10 m. Draft = 6 m. KG = 8 m. Find KM & GM.
- 2 A box-shaped vessel 45 m x 8 m x 6 m, displaces 1476 t. Find the KM in SW.
- 3 A box-shaped barge is 52 x 20 x 12 m. SW draft 7.922 m. Find KM in RD 1.015
- 4 A box-shaped vessel 180 x 24 m floats at 8 m SW draft. Find KM in SW & FW.
- 5 Find the GM of a box-shaped vessel 120 m x 18 m when afloat at 10 m SW draft. KG is 6.9 m & FSM is 2000 tm.
- 6 Calculate the list when 30 t of cargo is shifted 10 m transversely on a box shaped vessel 100 x 16 m. SW draft 7 m; KG 5.5 m; FSM 4800 tm.
- 7 Draw the KM curve for a box-shaped vessel 90 x 12 m between 2m draft & 7 m. From the curve, find the minimum KM and the draft at which it occurs.
- 8 A barge is shaped like a triangular prism, 20 m broad at the top and 10 m deep. It is 45 m long and displaces 2952 t. Find the KM in salt water.
- 9 The underwater portion of a barge is in the shape of a prism, apex downwards, 14 m wide at the top, 8 m deep and 40 m long. Draw the KM curve upto 6 m draft and from the curve find the KM at (a) 2.5 m and (b) 4.75 m.

- 10 A ship of W 10250 t, KB 5.6 m, KG 8.3 m floats in SW. If I_{CL} is 45000 m⁴, & FSM is 2050 tm, find the GM fluid.
- 11 A barge 45 m long has a uniform transverse cross-section throughout, consisting of a rectangle above a triangle. The rectangle is 8 m broad and 5 m high. The triangle is apex downwards, 8 m broad and 3 m deep. If W is 1620 t, find the KM when in FW.
- 12 A barge 50 m long has a uniform transverse cross-section throughout, consisting of a rectangle above a semi-circle. The rectangle is 10 m broad & 4 m high. The semi-circle has a diameter of 10 m and its geometric centre is 3 m above the keel. Find the KM at 6 m draft ($\Pi = 22/7$).
- 13 Two barges, each 52 m long and 9 m broad at the waterline, float upright at 3 m even keel draft. KG = 3 m. One barge is rectangular while the other is a triangular prism floating apex downwards. Compare their GM.
- 14 Two box-shaped barges each 100 m long float at 4 m draft & have KG = 3.5 m. One barge is 10 m broad and the other is 12 m. Compare their initial GM.
- 15 A homogenous log of square cross-section has RD = 0.72. Prove, by calculation, whether it can float with one side (of the square) parallel to the waterline.

CHAPTER 20

SIMPSON'S RULES

Simpson's Rules are very popular among mariners and naval architects because of their simplicity. They may be used to calculate the area, volume and geometric centre of the space enclosed by a straight line and a curve.

Calculation of areas

Equidistant points are chosen along the straight line, also called the axis, and the distance between them is called the common interval or 'h'. From each of these points, the perpendicular distance to the curve is measured off and called the ordinate or 'y'. Each ordinate is multiplied by a different number chosen from a series of numbers called Simpson's Multipliers and the product is obtained. The area contained between the axis, the curve and the end ordinates is calculated by the formula:

$$\text{Area} = Kh (\text{sum of products})$$

where K is a constant.

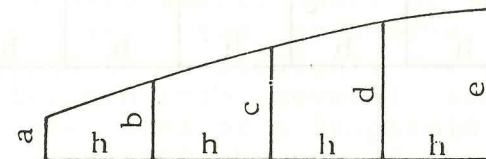
There are three Simpson's Rules & for each, there are different multipliers. The value of the constant 'K' also is different for different rules. If y and h are in metres, the area obtained would be in square metres.

Simpson's First Rule

$$\text{Area} = (h/3) \times (\text{sum of products})$$

Here, $K = 1/3$ and Simpson's Multipliers are 1 4 1 if there are three ordinates, 1 4 2 4 1 if there are five ordinates, 1 4 2 4 2 4 1 if the ordinates are seven 1 4 2 4 2 4 2 4 1 for nine ordinates, 1 4 2 42 4 1 for any further odd number of ordinates.

This rule is usable wherever the number of ordinates chosen is an odd number and it gives accurate results if the curve is a parabola of the second order (i.e., where the equation of the curve is $y = ax^2 + bx + c$, in which a, b and c are constants). This rule gives good results for ship-shapes and is hence used extensively by shipyards. Illustration of this rule is as follows:



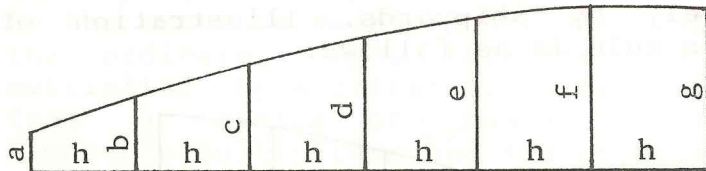
Ordinate (y)	x Simpson's multiplier (SM)	= Product for area
a	1	1a
b	4	4b
c	2	2c
d	4	4d
e	1	1e
Sum of products =		1a + 4b + 2c + 4d + 1e
Area =		$(h/3) \times (1a + 4b + 2c + 4d + 1e)$

Simpson's Second Rule

$$\text{Area} = (3h/8) \times (\text{sum of products})$$

Here, $K = 3/8$ and Simpson's Multipliers are 1 3 3 1 if there are four ordinates, 1 3 3 2 3 3 1for seven ordinates, 1 3 3 2 3 3 2 3 3 1 ..for ten ordinates, etc. This rule is usable wherever the number of ordinates chosen is 4, 7, 10, 13, 16, 19, 22, 25, etc. This rule gives accurate results if the curve is a parabola of the third order (i.e., where the equation of the curve is $y = ax^3 + bx^2 + cx + d$, where a, b, c and d are constants).

Illustration of the Second Rule:



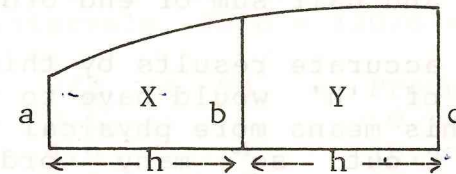
y	x	SM	= Product
a		1	1a
b		3	3b
c		3	3c
d		2	2d
e		3	3e
f		3	3f
g		1	1g

$$\text{Sum} = 1a + 3b + 3c + 2d + 3e + 3f + 1g$$

$$\text{Area} = (3h/8) (\text{sum of products as above})$$

Simpson's Third Rule

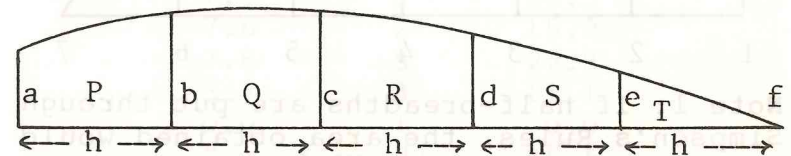
This rule is also called the five-eight-minus-one rule. If **three** consecutive ordinates are known, the area between any two of them can be calculated by this rule. Here $K = 1/12$ and SM are 5, 8 and -1. The use of this rule may be illustrated as follows:



$$\begin{aligned} \text{Area X} &= (h/12) (5a + 8b - c) \\ \text{Area Y} &= (h/12) (5c + 8b - a) \end{aligned}$$

The trapezoidal Rule

If the value of the common interval 'h' is made very small, part of the curve between any two ordinates may be considered to be straight. The shape now gets divided into several trapezoids. Since the area of a trapezoid is the product of half the sum of the parallel sides and the perpendicular distance between them, the area of the given shape may be obtained by plane geometry without the application of Simpson's Rules. This is illustrated below:



$$\begin{aligned} \text{Area P} &= h (a + b)/2 = h (0.5a + 0.5b) \\ \text{Area Q} &\dots\dots\dots = h (0.5b + 0.5c) \\ \text{Area R} &\dots\dots\dots = h (0.5c + 0.5d) \\ \text{Area S} &\dots\dots\dots = h (0.5d + 0.5e) \\ \text{Area T} &\dots\dots\dots = h (0.5e + 0.5f) \end{aligned}$$

$$\text{Total} = h (0.5a + b + c + d + e + 0.5f)$$

$$\text{Area} = h (\text{sum of all intermediate ordinates and half sum of end ordinates})$$

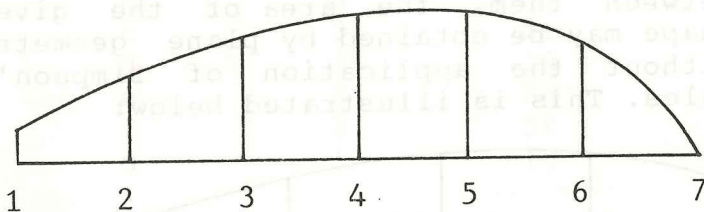
To obtain accurate results by this rule, the value of 'h' would have to be very small. This means more physical work in measuring out so many ordinates. Simpson's Rules are widely used by shipyards, in preference to the trapezoidal rule, as good accuracy can be obtained by using fewer ordinates.

Example 1

A ship's water-plane is 120 m long. The half-breadths, measured at equal intervals from aft, are:

0.1 4.6 7.5 7.6 7.6 3.7 & 0 m.

Find the water-plane area.



Note 1: If half-breadths are put through Simpson's Rules, the area obtained would be half the water-plane area. Double

this value would be the full area of the water-plane. If, instead, full breadths are used, the area obtained would directly be that of the full water-plane. In this question, half-breadths are given. Hence it would be simpler to use them as they are, the half-breadths then being called half-ordinates or semi-ordinates.

Note 2: Seven semi-ordinates means six equal intervals. So $h = 120/6 = 20$ m.

y/2	x	SM	= Product
0.1		1	0.1
4.6		4	18.4
7.5		2	15.0
7.6		4	30.4
7.6		2	15.2
3.7		4	14.8
0.0		1	0.0
Sum of products			93.9

$$\begin{aligned} \text{Half area} &= (20/3) (93.9) = 626 \text{ m}^2. \\ \text{Full area} &= 626 \times 2 \dots\dots = 1252 \text{ m}^2. \end{aligned}$$

Example 2

Example 1 had seven ordinates and could have been worked using Simpson's Second Rule as follows:

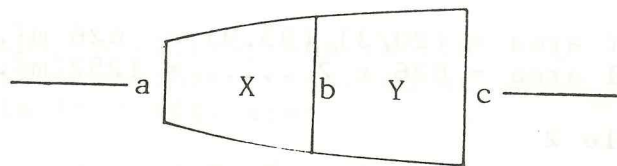
y/2	x	SM	= Product
0.1		1	0.1
4.6		3	13.8
7.5		3	22.5
7.6		2	15.2
7.6		3	22.8
3.7		3	11.1
0.0		1	0.0
Sum of products			85.5

$$\begin{aligned}\text{Half area} &= (20 \times 3/8)(85.5) = 641.25 \text{ m}^2 \\ \text{Full area} &= 641.25 \times 2 \dots = 1282.50 \text{ m}^2\end{aligned}$$

Note: Given the same particulars, the answers obtained by Simpson's First Rule & by Simpson's Second Rule are slightly different (less than 2.5% in this case). This is mentioned here to illustrate that the results obtained using Simpsons Rules are only very good approximations of the correct areas. The accuracy improves as the number of ordinates is increased i.e., the smaller the common interval, the greater the accuracy.

Example 3

The breadths of part of a ship's deck, at 5 m intervals are 13, 14 and 14.5 m. Find the area between the first two ordinates.



$$\begin{aligned}\text{Area X} &= (h/12) (5a + 8b - c) \\ &= (5/12)(65 + 112 - 14.5) = 67.708 \text{ m}^2\end{aligned}$$

Example 4

The half-breadths of a ship's waterplane 100 m long, at equal intervals from aft:

5.0 5.88 6.75 6.63 4.0 & 0.0 m.

Find the water-plane area and TPC in SW.

Note 1: Since the given number of semi-ordinates is six, none of Simpsons Rules is directly applicable to all of them as a whole. Part of the area can be calculated using one rule and the other part by another rule. The sum of the two part areas would give the area of the semi-water-plane. Double this value would be the area of the whole water-plane. Here are some possibilities:

(a) Area between the first and the third semi-ordinate by the first rule and the remaining area by the second rule.

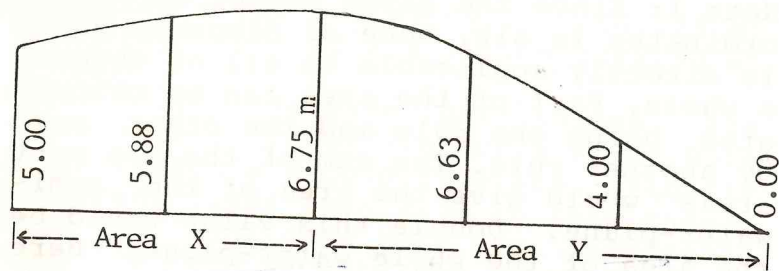
(b) Area between the first and the fourth semi-ordinate by the second rule & the remaining area by the first rule.

(c) Area between the first and the fifth semi-ordinate by the first rule and the remaining area by the third rule.

(d) Area between the first and the second semi-ordinate by the third rule & the remaining area by the first rule.

Note 2: The results obtained by different methods may differ slightly but would be within reasonable limits.

Note 3: The semi-ordinate which happens to be the boundary between the areas calculated separately is called the dividing semi-ordinate. It will be used twice - once in each calculation of part area. In this example, the third is the dividing semi-ordinate.



y/2	SM	Product	y/2	SM	Product
5.00	1	5.00	6.75	1	6.75
5.88	4	23.52	6.63	3	19.89
6.75	1	6.75	4.00	3	12.00
			0.00	1	0.00
Sum = 35.27			Sum = 38.64		

$$\begin{aligned} \text{Area X} &= (20/3)(35.27) \\ &= 235.133 \text{ m}^2. \end{aligned}$$

$$\begin{aligned} \text{Area Y} &= (20)(3/8)(38.64) \\ &= 289.800 \text{ m}^2. \end{aligned}$$

$$\text{Semi-area} = X + Y = 524.933 \text{ m}^2.$$

$$\text{Full area} = 2(524.933) = 1049.866 \text{ m}^2.$$

$$\text{TPC} = 1.025A/100 = 1.025(1049.866)/100$$

$$\text{TPC} = 10.761$$

Exercise 16

Areas by Simpson's Rules

- Find the area of a boat cover 10 m long if breadths at equal intervals from fwd are 0, 2.25, 3, 2.25 & 0 m.
- A small coaster's deck is 50 m long. Half-breadths at equal intervals from aft are 0.78, 2.89, 4.06, 2.34 & 0.31 metres. Calculate the deck area.

- Find the area of a collision bulkhead 12 m high. The half-breadths at equal intervals from top are:
7, 4.8, 2.95, 2, 1.65, 1.3 and 0 m.
- Find the area of a transverse bulkhead 10 m high whose half-breadths, at equal vertical intervals, are:
10, 9.3, 8.3, 7.1, 5.7 and 3.8 metres
- A ship's water-plane is 150 m long. Half-breadths at equal intervals from aft are: 2.97, 6.15, 7.84, 8.48, 8.06, 7.21, 5.72, 3.6 & 0 m respectively.
Find: (a) The water-plane area.
(b) The area coefficient.
(c) The TPC in salt water.
- Find the area of a tanktop 21 m long. Equidistant breadths are: 19.2, 18.0, 17.1, 16.2, 14.4, 12.0, 9.3 & 6.0 m.
- The half-breadths of a water-tight bulkhead, at 2 m intervals from the bottom, are 1, 2.9, 4.2, 5.1 & 5.7 m.
Find (i) The area between the bottom two semi-ordinates (ii) the quantity of paint required to coat the entire bulkhead once, if the paint covers 10 square metres per litre.
- A ship's water-plane is 90 m long. Half-breadths at equal intervals from forward are: 0.0, 2.5, 4.5, 6.5, 7.5, 8.5, 8.5, 8, 6 and 0 m respectively.
Find (a) SW TPC (b) Area coefficient.

- 9 The breadths of a ship's water-plane 120 m long, at equal intervals from aft, are: 1.2, 9.6, 13.2, 15.0, 15.3, 15.6, 15.6, 14.7, 12.9, 9 & 0 metres. Find (a) The water-plane area.
(b) FWA if $W = 6811$ tonnes.

- 10 Find the area of a ship's deck 99 m long whose half-breadths at equal intervals from forward are 0.45, 2.10, 3.75, 5.25, 6.45, 7.35, 7.80, 7.20, 5.85 and 3.00 metres respectively.

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Calculation of volumes

If cross-sectional areas are put through Simpson's Rules, the volume of an enclosed space having curved boundaries can be calculated. These cross-sectional areas must be equally spaced (must have a common interval) and may be either transverse (like areas of imaginary water-tight bulkheads) or horizontal (like water-plane areas at equal intervals of draft).

The application of Simpson's Rules is the same for calculation of volumes as for calculation of areas. If semi-areas are put through the Rules, the result obtained would be the semi-volume.

Example 5

Find the volume of displacement of a barge 48 m long whose under water transverse cross-sectional areas are: 19.6, 25, 17.5, 13 and 0 square metres.

Area	x	SM	=	Product
19.6	m ²	1		19.6
25		4		100.0
17.5		2		35.0
13		4		52.0
0		1		0.0
Sum of products				206.6

$$\text{Vol} = (\text{SOP})h/3 = (206.6)12/3 = 826.4 \text{ m}^3.$$

Example 6

The water-plane areas of a ship, at one metre intervals from keel upwards, are: 1730, 1925, 2030, 2100 and 2150 m². Find the W and the TPC in SW at 4 m draft.

Draft	WP area	SM	Product
4	2150 m ²	1	2150
3	2100	4	8400
2	2030	2	4060
1	1925	4	7700
0	1730	1	1730
Sum of products =			24040

$$\text{Vol} = (\text{SOP})h/3 = (24040)1/3 = 8013.333 \text{ m}^3$$

$$\text{SW } W = 8013.333 \times 1.025 = 8213.7 \text{ tonnes.}$$

$$\text{TPC at 4 m draft} = 1.025A/100$$

$$= 1.025(2150)/100 = 22.038.$$

Example 7

Given the following information, find the displacement at 6 m draft in SW:

Draft	6	5	4	3	2	1	0 m
TPC	61.5	61.7	61.8	61.8	61.7	57.4	51.3

Alternative 1

The given values of TPC can be converted into water-plane areas by the formula: $TPC = 1.025A/100$. The water-plane areas, put through Simpson's Rules, would give the volume of displacement. This volume $\times 1.025 =$ SW displacement at 6 m draft.

Alternative 2

$$TPC = 1.025A/100 \quad \text{or} \quad A = (TPC)100/1.025$$

$$\text{Let } X = 100/1.025 \quad \text{so} \quad A = TPC (X)$$

Draft	WP area	SM	Product
6	61.5X	1	61.5X
5	61.7X	4	246.8X
4	61.8X	2	123.6X
3	61.8X	4	247.2X
2	61.7X	2	123.4X
1	57.4X	4	229.6X
0	51.3X	1	51.3X
Sum of products =			1083.4X

$$\text{Vol} = (SOP)h/3 = 1083.4X/3 = 35232.52 \text{ m}^3$$

$$W \text{ at } 6 \text{ m} = 35232.52 (1.025) = 36113.3 \text{ t.}$$

Note: This problem may be solved using Simpson's Second Rule. W would then work out to 36157.5 t. (Difference < 0.15%).

Exercise 17

Volumes by Simpson's Rules

- Given the following information, find the volume of displacement and the approximate mean TPC between the drafts of 8 m and 9 m:

Draft (metres)	7	8	9
WP area in m^2	2240	2295	2355

- Find the volume of a lower hold 20 m long whose transverse cross-sectional areas at equal intervals from forward are 120, 116, 101 & 80 square metres.

- Find the displacement at 5 m SW draft if the water-plane areas, in m^2 , are:

Draft	6	5	4	3	2	1	0 m
Area	2550	2010	1920	1580	1300	920	780

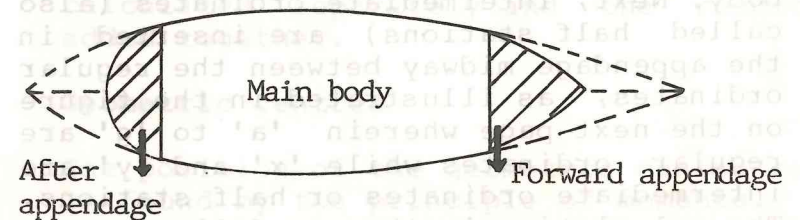
- Find the quantity of coal ($\text{SF } 4 \text{ m}^3 \text{ t}^{-1}$) that a coal bunker can hold if its cross-sectional areas, at 5 m intervals are 9, 11.3, 12.6, 12.4 & 11.2 m^2

- Find W & TPC at 6 m FW draft, if the water-plane areas, in m^2 , are:

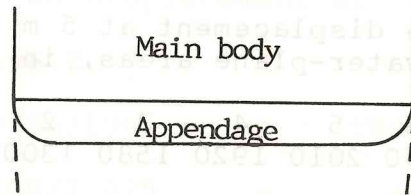
d	6	5	4	3	2	1	0 m
A	5855	5875	5893	5895	5900	5885	5850

Appendages

Appendages are those parts of a curved boundary where the curvature changes considerably. In calculations of water-plane areas, appendages may occur near the ends.



In calculations of under water volumes, appendages occur in the region of the double bottom tanks as the curvature of the shell plating changes sharply at the bilges.



Areas/volumes of appendages are usually calculated separately and then added to the area/volume of the main body.

Intermediate ordinates

The greater the number of ordinates used, the greater the accuracy of the result obtained by Simpson's Rules. Where the change of curvature is not too severe, calculation of the area/volume of the appendage and of the main body can be done as a single calculation. First, the ordinates in the appendage are spaced at the same common interval as in the main body. Next, intermediate ordinates (also called half stations) are inserted in the appendage midway between the regular ordinates, as illustrated in the figure on the next page wherein 'a' to 'g' are regular ordinates while 'x' and 'y' are intermediate ordinates or half stations. The calculation is then as follows:

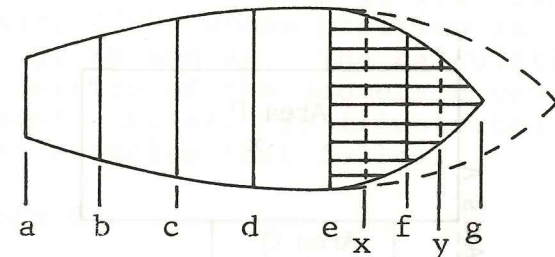
Area of main body = (sum of products)h/3

$$= (1a + 4b + 2c + 4d + 1e) (h/3)$$

Appendage area = (sum of products)(h/2)/3

$$= (1e + 4x + 2f + 4y + 1g)(0.5h)/3$$

$$= (e/2 + 2x + 1f + 2y + g/2) (h/3)$$



Total area = Main body + appendage

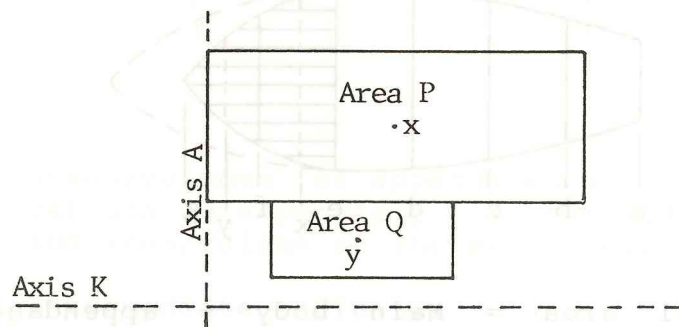
$$= [1a+4b+2c+4d+(1.5e)+2x+1f+2y+g/2](h/3)$$

Note: Simpson's Multipliers in the half station zone are halved except at the common ordinate for which the SM is 1.5. This holds good for half stations even where Simpson's Second Rule is used. If desired, the area/volume of the main body and of the appendage may be calculated separately and the results added together.

Geometric centres

The position of the geometric centre can be found by the principle of moments. A basic illustration is as follows:

In the following figure, x is the geometric centre of area P and y , that of area Q. Ax and Ay are the perpendicular distances of the geometric centres from axis A. Kx and Ky are the perpendicular distances from the axis K. Required to find the position of z , the geometric centre of the whole figure (ie, required to find Kz and Az).



Taking moments about axis K,

$$\text{Area P}(Kx) + \text{area Q}(Ky) = \text{Area (P+Q)}(Kz)$$

Kz , being the only unknown factor in the equation, can be obtained by calculation.

Taking moments about axis A,

$$\text{Area P}(Ax) + \text{area Q}(Ay) = \text{Area (P+Q)}(Az)$$

Az , being the only unknown factor in the equation, can be obtained by calculation.

Geometric centres by Simpson's Rules

Calculation of the position of the geometric centre of a space by Simpson's Rules also is based on the principle of moments. The geometric centre of a water-plane is the centre of flotation (COF) at that draft and AF is its distance from the after perpendicular of the ship. The geometric centre of the under water volume of a ship is its centre of buoyancy (COB) whose position is indicated by KB and AB. The calculation of the position of the geometric centre, by Simpson's Rules, is illustrated by the worked examples that follow.

Example 8

A ship's water-plane is 120 m long. Half breadths, at equal intervals from aft, are: 0.1, 4.6, 7.5, 7.6, 7.6, 3.7 & 0 m. Calculate the position of its COF.

Let A be the after end of the waterplane

$$h = 120/6 = 20 \text{ metres.}$$

$y/2$ (m)	SM	Product for semi-area	Lever abt A	Product for semi-moment
0.1	1	00.1	0h	00.0h
4.6	4	18.4	1h	18.4h
7.5	2	15.0	2h	30.0h
7.6	4	30.4	3h	91.2h
7.6	2	15.2	4h	60.8h
3.7	4	14.8	5h	74.0h
0.0	1	00.0	6h	00.0h
SOP =		93.9	SOP =	274.4h

$$AF = 274.4h/93.9 = 58.445 \text{ metres.}$$

Note 1: Lever about A is the distance of the semi-ordinate from the after end, in multiples of h. It may, if desired, be inserted directly in metres.

Note 2: Explanation of the final calculation of AF is as follows:

$$\begin{aligned} \text{AF} &= \frac{\text{Mom abt A}}{\text{Total area}} = \frac{(\text{SOP for mom abt A})h/3}{(\text{SOP for full area})h/3} \\ &= \frac{(\text{SOP for semi-moment})}{(\text{SOP for semi-area})} = \frac{274.4h}{93.9} \end{aligned}$$

Example 9

The transverse cross-sectional areas, of the under water portion of a barge, at 12 m intervals from forward, are: 0, 13, 17.5, 25 and 19.6 square metres. The last ordinate is the after perpendicular of the barge. Calculate AB.

Area (m ²)	SM	Product for vol	Lever abt A	Product for mom
00.0	1	00.0	4h	00.0h
13.0	4	52.0	3h	156.0h
17.5	2	35.0	2h	70.0h
25.0	4	100.0	1h	100.0h
19.6	1	19.6	0h	00.0h
		SOP = 206.6		SOP = 326.0h

$$\text{AB} = 326.0h/206.6 = 18.935 \text{ metres.}$$

Example 10

The water-plane areas of a ship are:-

Draft	5	4	3	2	1	m.
Area	2150	2100	2030	1925	1730	m ² .

Between the keel and 1 m draft, there is an appendage of 800 m³ volume whose geometric centre is 0.7 m above the keel. Find the displacement and the KB of the ship at 5 m draft in salt water.

d (m)	WP area	SM	Product for vol	Lever abt K	Product for mom about K
5	2150	1	2150	5h	10750h
4	2100	4	8400	4h	33600h
3	2030	2	4060	3h	12180h
2	1925	4	7700	2h	15400h
1	1730	1	1730	1h	1730h
			SOP = 24040		SOP = 73660h

$$\text{KB of main body} = 73660h/24040 = 3.064 \text{ m}$$

$$\begin{aligned} \text{Vol of main body} &= (h/3)(\text{SOP for volume}) \\ &= 8013.333 \text{ m}^3. \end{aligned}$$

Taking moments about the keel,

$$\begin{aligned} [\text{Main body}] + [\text{appendage}] &= [\text{total volume}] \\ 8013.333(3.064) + 800(0.7) &= (8813.333) \text{ KB} \end{aligned}$$

$$\text{KB of ship} = 2.849 \text{ metres.}$$

$$\text{W in SW} = 8813.333(1.025) = 9033.7 \text{ t.}$$

Example 11

Half-breadths of a ship's water-plane, at equal intervals from aft, are:

5, 5.88, 6.75, 6.63, 4, 2.38 & 0 metres.

The common interval between the first five semi-ordinates is 20 m and between the last three is 10 m. The total length

of the water-plane is 100 m. Find the area of the water-plane and the position of its COF.

y/2 (m)	SM	Product for semi-area	Lever abt A	Product for semi-moment
5.00	1	5.00	0h	00.00h
5.88	4	23.52	1h	23.52h
6.75	2	13.50	2h	27.00h
6.63	4	26.52	3h	79.56h
4.00	1.5	6.00	4h	24.00h
2.38	2	4.76	4.5h	21.42h
0.00	0.5	<u>0.00</u>	5h	<u>00.00h</u>
		SOP = 79.30		SOP = 175.50h

$$AF = 175.5h/79.3 = 44.262 \text{ metres.}$$

$$\text{Semi-area} = (\text{SOP}) h/3 = 528.6667 \text{ m}^2.$$

$$\text{Full area} = 2(528.6667) = 1057.333 \text{ m}^2.$$

Example 12

The vertical ordinates of the after bulkhead of the port slop tank of a tanker, measured from the horizontal deckhead downwards, spaced at equal athwartship intervals of 1 m, are:

0, 3.25, 4.4, 5.15, 5.65, 5.9 and 6.0 m.

Find the distance of the geometric centre of the bulkhead from (a) the inner boundary and (b) the deckhead. (c) Find the thrust on this bulkhead when the tank is full of salt water.

Note 1: The distance of the GC from the inner boundary of the tank can be calculated by taking levers, in multiples

of h or directly in metres, from the stbd side, as done in earlier examples.

Note 2: The distance of the GC of each ordinate y, from the deckhead, is y/2. This is the lever to be used to calculate the distance of the GC of the bulkhead from the deckhead.

Note 3: In the calculation below,

$$\text{Column 1} \times \text{column 2} = \text{column 3}$$

$$\text{Column 3} \times \text{column 4} = \text{column 5}$$

$$\text{Column 3} \times \text{column 6} = \text{column 7}$$

(1) Ord in (m)	(2) SM	(3) Product for area	(4) L E V E R	(5) Product for mom about stbd side	(6) L E V E R	(7) Product for mom about deck head
0.00	1	0.00	6h	00.00h	0.000	00.000
3.25	3	9.75	5h	48.75h	1.625	15.844
4.40	3	13.20	4h	52.80h	2.200	29.040
5.15	2	10.30	3h	30.90h	2.575	26.523
5.65	3	16.95	2h	33.90h	2.825	47.884
5.90	3	17.70	1h	17.70h	2.950	52.215
6.00	1	6.00	0h	00.00h	3.000	18.000
		<u>SOP 73.9</u>		<u>SOP 184.05h</u>		<u>SOP 189.506</u>

$$\text{GC from stbd} = 184.05h/73.9 = 2.491 \text{ m.}$$

$$\text{GC to deckhead} = 189.506/73.9 = 2.564 \text{ m.}$$

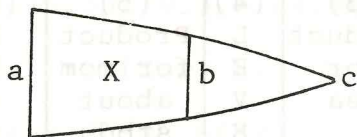
$$\text{Area} = (\text{SOP})3h/8 = 73.9(3/8) = 27.713 \text{ m}^2$$

$$\begin{aligned} \text{Thrust} &= \text{depth of GC} \times \text{density} \times \text{area} \\ &= 2.564(1.025)27.713 = 72.833 \text{ t.} \end{aligned}$$

Note 4: To save time and effort during calculation, column 6 may be taken as full y and then the sum of products of column 7 may be divided by 2. If desired column 6 may be $y^2/2$ and put through SM to get column 7.

Example 13

The breadths of the forecastle of a barge, at 2 m intervals from aft, are: 3.31, 2 & 0 m. Calculate the area & the position of the geometric centre of the space between the first two ordinates.



$$\begin{aligned} \text{Area } X &= (5a + 8b - c)h/12 \\ &= (16.55 + 16 - 0)2/12 = 5.425 \text{ m}^2 \\ \text{Moment of area } X \text{ about 'a'} & \\ &= (3a + 10b - c)(h^2/24)* \\ &= (9.93 + 20 - 0)4/24 = 4.988 \text{ m}^3. \end{aligned}$$

$$\text{GC of } X \text{ from 'a'} = 4.988/5.425 = 0.919 \text{ m}$$

Note: The formula marked * is called the **three-ten-minus-one rule** for use in such cases.

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Exercise 18 Simpson's Rules

- Calculate the area and the position of the COF of a ship's water-plane whose half-breadths, at 10 m intervals from aft, are: 0, 6, 8, 8.5, 8.5, 7.5, 6.5, 4.5, 2.5 and 0 metres.
- The breadths of a transverse watertight bulkhead, at 2 m intervals from the bottom, are: 2, 5.8, 8.4, 10.2 & 11.4 m. Find (a) its area, (b) the distance of its geometric centre from the top and (c) the thrust when it is pressed up with SW to a head of 6 m above the top.
- The half-breadths of a transverse W/T bulkhead, at 2 m vertical intervals from the top, are: 10.6, 10, 9.3, 8.3, 7.1, 5.7 & 3.8 m.
Below the lowest semi-ordinate is a rectangular appendage 7.6 m broad and 1 m high. Find the total area of the bulkhead and the distance of its GC from the bottom of the appendage.
- Find KB and displacement at 4 m draft in SW, if the water-plane areas are:-

Draft	5	4	3	2	1	0	m
Area	2010	1920	1580	1300	920	780	m ²
- Draft 6 5 4 3 2 1 0 m
 TPC 22.6 22.2 21.6 20.9 19.7 17.8 14.6
 Find W and KB at 6 m SW draft.

- 6 The half-breadths of a tank top, at 3 m intervals from forward, are:

3, 4.65, 6, 7.2, 8.1, 8.55, 9 & 9.6 m

Find the area and the distance of its geometric centre from forward.

(Suggestion: Use Rule 1 for the first five semi-ordinates & Rule 2 for the last four).

- 7 The water-plane areas of a ship are:-

Draft	6	5	4	3	2	m
Area	2190	2150	2100	2040	1920	m ²

Below 2 m draft there is an appendage having a volume of 3200 m³, whose GC is 1.2 m above the keel. Find the KB and W of the ship at 6 m draft in SW.

- 8 Find the W and KB at 5 m draft in SW, given the water-plane areas as under:

d	5 m	4	3	2	1	0.5	0.0
A	6380	6320	6225	6090	5885	5740	5560

- 9 The half-ordinates of a ship's water-plane, at equal intervals from fwd, are: 0, 1.5, 2.78, 3.75, 4.2, 4.5, 4.2, 3.9, 3.3 and 2.25 m. The common interval between the last four semi-ordinates is 3 m & between the others is 6 m. Find the distance of the GC from the ship's after end. (Suggestion: Use Simpson's Rule 2 with half-stations aft).

- 10 The half-breadths of a ship's water-plane 180 m long, at equal intervals

from aft, are: 2.8, 4, 5.2, 6, 6.4, 6.8, 6.6, 6, 4.2 and 0 metres. Midway between the last two given figures, the half-breadth is 2.4 m. Find the area of the water-plane and the distance of the COF from the after end.

- 11 The breadths of a ship's water-plane 144 m long, at equal intervals from forward, are: 0, 9, 12.9, 14.7, 15.6, 15.8, 15.8, 15.6, 15.3, 15, 13.2, 9.6 and 0 m. The intermediate ordinate between the first two is 6 m & between the last two, is 6.6 m. Find the area of the water-plane and the distance of the COF from amidships.

- 12 The half-breadths of a ship's water-plane, at 12 m intervals from aft are 0.0, 3.3, 4.5, 4.8, 4.5, 3.6, 2.7 and 1.5 m. The half-breadth, midway between the first two from aft, is 2 m. At the forward end is an appendage by way of a bulbous bow 4.5 m long. Its area is 24 m² and its GC, 2 m from the forward extremity. Find the area of the water-plane and the position of the COF.

- 13 The transverse cross-sectional areas of a lower hold 21 m long, at equal intervals from forward, are 120, 116, 101 and 80 m². Find the volume of the hold and the distance of its GC from the after bulkhead.

- 14 The transverse cross-sectional areas, of a ship's under-water portion 90 m long, are: 0.5, 22.9, 49, 73.5, 88.5, 83, 58.6, 31.8, 14.2, 8.1 and 4.5 m².

The last given area is at the after perpendicular of the ship. The spacing between the last three sections is half the common interval between the rest. Find the displacement in SW and the AB.

- 15 The after bulkhead of the starboard slop tank of a tanker is 6 m high. It is bounded on the top by a horizontal deck, towards amidships by a vertical fore-and-aft bulkhead, and on the the starboard side by the shell plating. The breadths of this bulkhead at equal vertical intervals are: 3, 3.15, 2.85, 2.1, 1.1 and 0 metres. Find the area of this bulkhead and the distances of its GC from the bottom and from the inner boundary.
- 16 Three consecutive half-breadths of a bulkhead 6 m high, starting from the bottom, are: 5, 9 and 10 m. Find the area and position of the GC of the bottom three metres of this bulkhead.
- 17 The cross-sectional areas of a coal bunker, at 4 m intervals from forward are: 15, 42 and 45 m². Find the mass of coal (SF 4 m³/t) that could be contained between the first two given cross-sectional areas & the distance of its GC from the after bulkhead.
- 18 Rework question 6 of this exercise, using Simpson's First Rule for the first seven ordinates and Simpson's Third Rule for the last three. (Compare the area and COF obtained in both cases).

- 19 The half-breadths of a ship's water-plane 150 m long, from forward, are:
2.97, 6.15, 7.84, 8.48, 8.06, 7.21, 5.72, 3.60 and 0 metres respectively.

Find the area using the trapezoid rule. (Compare your answer with that of question 5 of exercise 16).

- 20 The breadths of the deck of a ship, measured at 15 metre intervals from forward, are:
6.2, 13.8, 21.9, 26.4, 22.4, 14.7 and 7.4 metres respectively.

Assuming that Simpson's First Rule is correct, find the % error that would be obtained by using:

- (a) The trapezoidal rule and
(b) Simpson's Second Rule.

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CHAPTER 21

ANGLE OF LOLL -

CALCULATION; REMEDIAL ACTION

Unstable equilibrium and angle of loll were described in chapters 10 & 11 in Ship Stability I. A vessel at the angle of loll is in an extremely precarious and dangerous situation - wrong action or no action on the part the ship's staff may cause the ship to capsize. Even no action is dangerous because consumption of fuel and water from the double bottom tanks would cause increase of KG making the vessel more unstable, thereby increasing the angle of loll.

The angle of loll can be calculated by a simple formula derived from the wall-sided formula:

$$GZ = \sin \theta (GM + \frac{1}{2}BM \tan^2 \theta)$$

At the angle of loll, $GZ = \text{zero}$. So

$$\sin \theta (GM + \frac{1}{2}BM \tan^2 \theta) = 0$$

So $\sin \theta = 0$ or $(GM + \frac{1}{2}BM \tan^2 \theta) = 0$

At the angle of loll, $\theta \neq 0$ so $\sin \theta \neq 0$

$GM + \frac{1}{2}BM \tan^2 \theta = 0$ and $\tan^2 \theta = -2GM/BM$

$\tan \theta = \sqrt{\frac{-2GM}{BM}}$ Where $\theta = \text{Angle of loll}$
 $GM = \text{The initial GM}$
 $BM = BM \text{ when upright}$

Since this formula is derived from the wall-sided formula, it can be applied only when the immersed wedge and the emerged wedge are identical in shape.

Example

M.V.VIJAY is afloat at 6 m draft. Find the angle of loll if $KG = 8.424$ metres.

Referring to appendix I of this book,

$$\begin{array}{ll} KM = 8.234 \text{ m} & KM = 8.234 \text{ m} \\ KB = 3.205 \text{ m} & KG = 8.434 \text{ m} \\ BM = 5.029 \text{ m} & GM = -0.200 \text{ m} \end{array}$$

$$\tan \theta = \sqrt{\frac{-2GM}{BM}} = \sqrt{\frac{-2(-0.2)}{5.029}} = 0.28203$$

Angle of loll $= \theta = 15.75^\circ$ or $15^\circ 45'$

Remedial action

- 1) Press up all slack tanks.
- 2) Run up SW into the DB tank which has the smallest moment of inertia about its centre line. If this tank is not on the centre line of the ship, then on the lower side first, and after it is full, its counter part on the higher side.
- 3) Repeat action 2 with another tank and so on until the ship becomes stable.
- 4) If discharging or jettisoning deck cargo, do so from the higher side first, then from the lower side. If using ship's own gear, due allowance must be made for the shift of COG, of each sling of cargo, from the UD to the derrick head during the operation.

Justification for such action

At the angle of loll, any existing free surface effect must be eliminated/minimised first. FW or HFO may require to be transferred internally such that the tanks finally remaining slack are those with the smallest moment of inertia about the tank's centre line.

While running up ballast into a DB tank, FSE would be created. This must be kept to a minimum. The necessity to fill up the tank with the smallest 'i' about its centre line is, therefore, vital. So also, the necessity to fill up only one tank at a time.

If the tank being ballasted is not on the centre line of the ship, but on either side like No:2 P and No:2 S, then fill up the lower side tank first i.e., if the ship is lolled to starboard, then fill up No:2 S first. After it is full, fill up No:2 P. The reasons for this:

Let 'A°' be the angle of loll to starboard at first.

'p°' be the reduction in the angle of loll by completely filling up either No:2 P or No:2 S.

'q°' be the list caused by filling up either No:2 P or No:2 S.

'R°' be the resultant inclination after completely filling up either No:2 P or No:2 S.

If No:2 S is run up first, $R = A - p + q$

If No:2 P is run up first, $R = A - p - q$

In both cases, final R is to starboard.

It appears as if filling up the higher side tank would produce better results but it is not really so. The ship can loll to either side. If after the higher side tank is run up, wave action caused the ship to loll over to the other side (port side in this case), the ship would flop over to $A - p + q$ to port and the momentum of flopping over will carry the inclination well beyond this. Since the GZ formed near the angle of loll is very small, the ship would heel over to port much more than $A - p + q$ and take a very long time to return to this angle of inclination. If during this time (a) any openings went underwater &/or (b) a wave struck the ship adversely and/or (c) any cargo shifted, the ship may capsize.

By filling up the lower side tank first, the inclination would increase a bit to $A - p + q$ at first, but this would be gradual and would last only until No:2 P also is run up.

The same line of reasoning is applicable when considering discharge or jettison of cargo from the upper deck.

If the ship is in calm waters, such as inside a dock, the possibility of flopping over to the angle of loll on the other side may not be there. In such a case, ballasting the higher side tank or discharging deck cargo from the lower side may prove more effective and immediate.

Exercise 19
Angle of loll

- 1 A vessel has an initial GM of -0.3 m & BM of 5 m. Find the angle of loll.
- 2 M.V.VIJAY is lying at the angle of loll of 16° to port, at 6.2 m hydrostatic draft. Find her initial GM.
- 3 M.V.VIJAY is unstable and lolling 12° to starboard at a hydrostatic draft of 6.8 m. Find the minimum quantity of cargo to shift by a shore crane, from the upper deck to the lower hold, through a vertical distance of 10 m to make the ship stable.
- 4 A box-shaped vessel 100 x 12 x 8 m is lying at an angle of loll of 18° . If her mean draft is 4 m, find her KG.
- 5 A homogenous wooden log of square cross-section has RD = 0.68. Would it float in FW with one face parallel to the water? If not, calculate the angle of inclination.
- 6 What should the RD of a homogenous log of square cross-section be for it to float in FW with one face parallel to the surface of water?

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CHAPTER 22

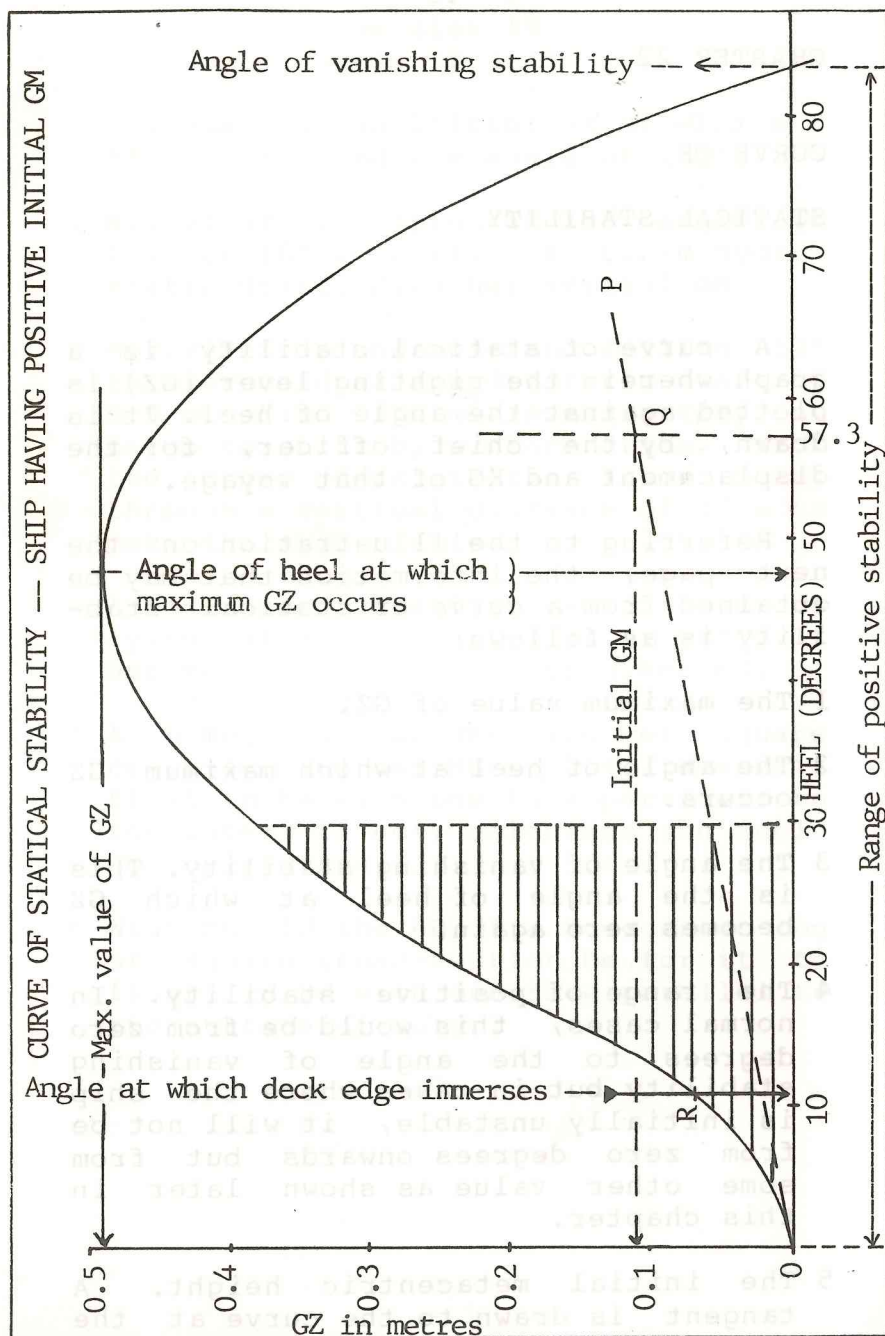
CURVE OF

STATICAL STABILITY

A curve of statical stability is a graph wherein the righting lever (GZ) is plotted against the angle of heel. It is drawn, by the chief officer, for the displacement and KG of that voyage.

Referring to the illustration on the next page, the information that may be obtained from a curve of statical stability is as follows:

- 1 The maximum value of GZ.
- 2 The angle of heel at which maximum GZ occurs.
- 3 The angle of vanishing stability. This is the angle of heel at which GZ becomes zero again.
- 4 The range of positive stability. In normal cases, this would be from zero degrees to the angle of vanishing stability but in cases where the ship is initially unstable, it will not be from zero degrees onwards but from some other value as shown later in this chapter.
- 5 The initial metacentric height. A tangent is drawn to the curve at the



origin (OP in the figure). A perpendicular is erected at 57.3° heel to meet the tangent (Q in the figure). The distance of the point of intersection from the base line, measured on the GZ scale, indicates the initial GM.

Note: In actual practice, the reverse happens. The initial fluid GM is cut off on the perpendicular at 57.3° to arrive at point Q in the figure. Q & O are joined by a straight line and while drawing the curve, it is ensured that the curve coincides with line OQ for the first few degrees.

- 6 The angle of heel at which the deck edge immerses. This is the angle of heel at which the point of contraflexure of the curve occurs (point R in the figure).
- 7 The moment of statical stability at any given angle of heel. The GZ for the given angle of heel is obtained from the curve and multiplied by the displacement of the ship.
- 8 The dynamical stability of the ship at any given angle of heel. This is the work done in heeling the ship to the given angle. This is dealt with in more detail in chapter 40 in Ship Stability III.

Dynamical stability at θ° heel = $W \times A$

where W: ship's displacement in tonnes
 A: area between the GZ curve and the base line, upto θ° heel, expressed in metre-radians.

Once the curve of statical stability has been drawn, the area under the curve upto any angle of heel can be calculated using Simpson's Rules. This area, multiplied by the displacement of the ship, would give the dynamical stability in tonne-metre-radians. In the figure on page 44, the area under the curve upto 30° has been shaded for illustration.

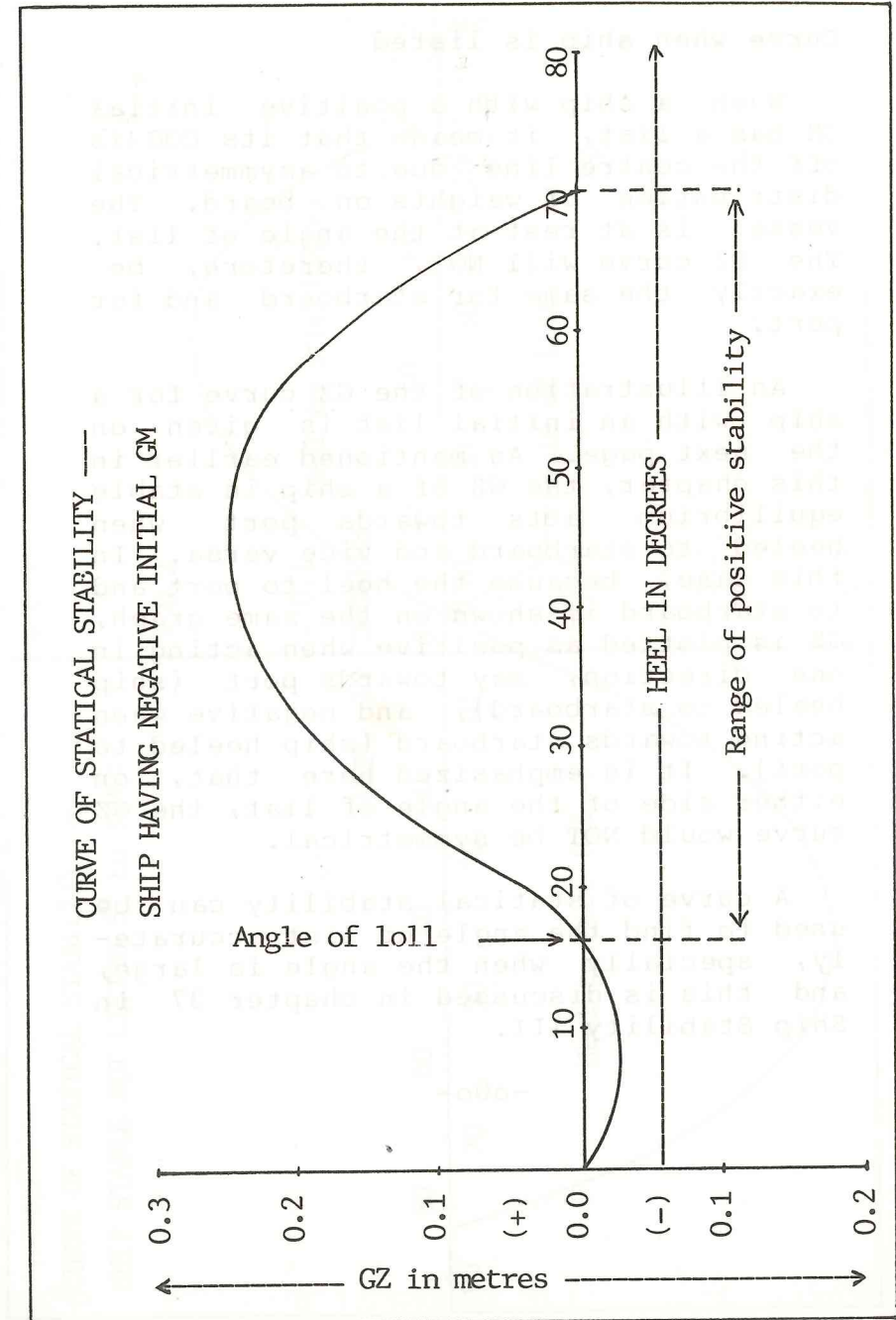
The curve would be the same whether the ship is heeled to starboard or to port. The only difference would be the direction of GZ - when heeled to port, GZ acts to starboard and when heeled to starboard, GZ acts to port.

The information required by the chief officer to construct a curve of statical stability, for the displacement and KG of the voyage, is supplied by the shipyard in the form of either Cross Curves or KN curves which are explained in the next two chapters.

Curve when initial GM is negative

An illustration of the GZ curve of a vessel, when it is initially unstable, is given on page 47. It will be seen therein that the range of positive stability is from the angle of loll onwards, not from zero.

Since the angle of loll could be to port or to starboard, the curve would be the same regardless of the direction of inclination of the ship. If the ship was lolled to port, GZ may be considered + when it resists further inclination to port and vice versa.



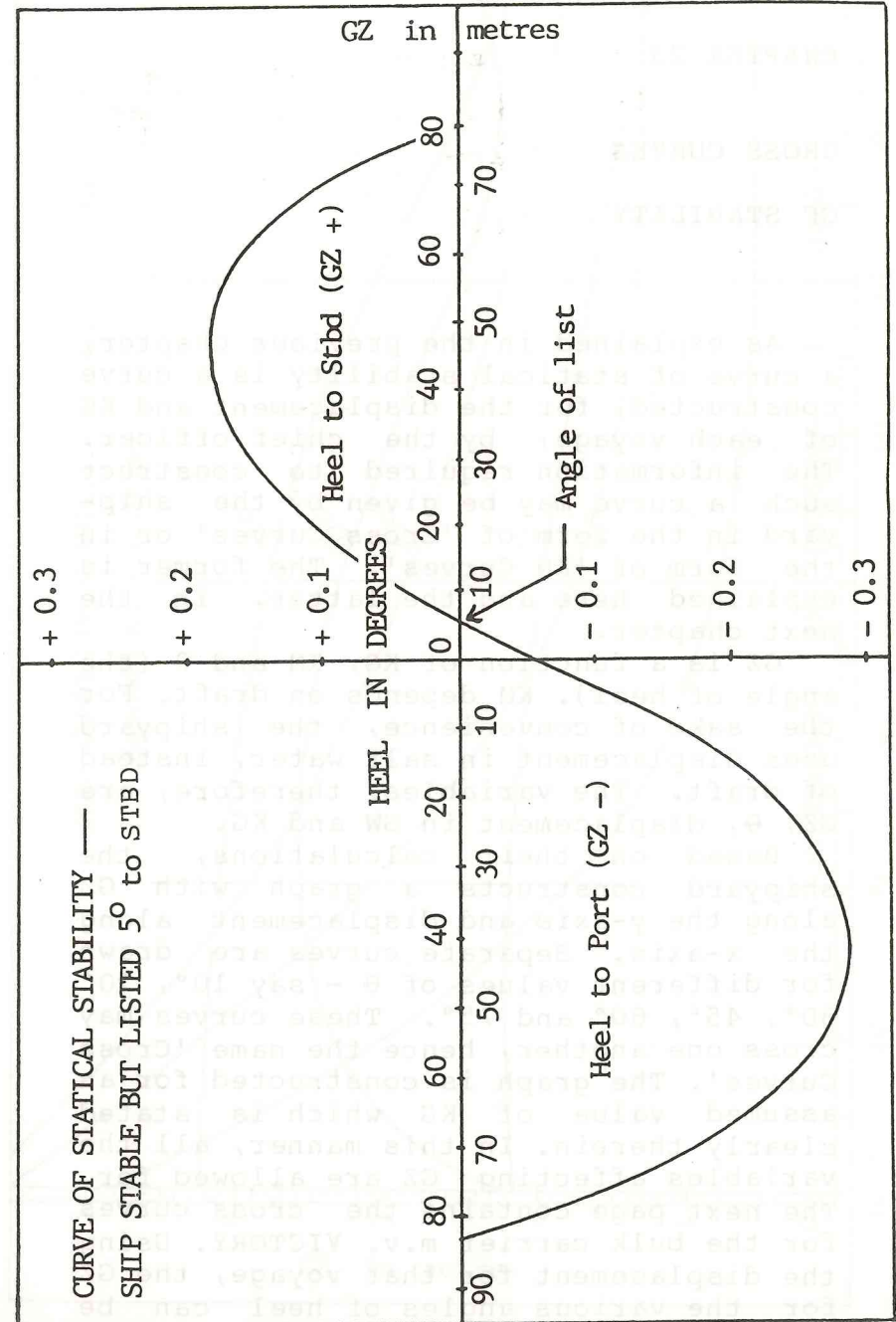
Curve when ship is listed

When a ship with a positive initial GM has a list, it means that its COG is off the centre line due to asymmetrical distribution of weights on board. The vessel is at rest at the angle of list. The GZ curve will NOT, therefore, be exactly the same for starboard and for port.

An illustration of the GZ curve for a ship with an initial list is given on the next page. As mentioned earlier in this chapter, the GZ of a ship in stable equilibrium acts towards port when heeled to starboard and vice versa. In this case, because the heel to port and to starboard is shown on the same graph, GZ is plotted as positive when acting in one direction, say towards port (ship heeled to starboard), and negative when acting towards starboard (ship heeled to port). It is emphasized here that, on either side of the angle of list, the GZ curve would NOT be symmetrical.

A curve of statical stability can be used to find the angle of list accurately, specially when the angle is large, and this is discussed in chapter 37 in Ship Stability III.

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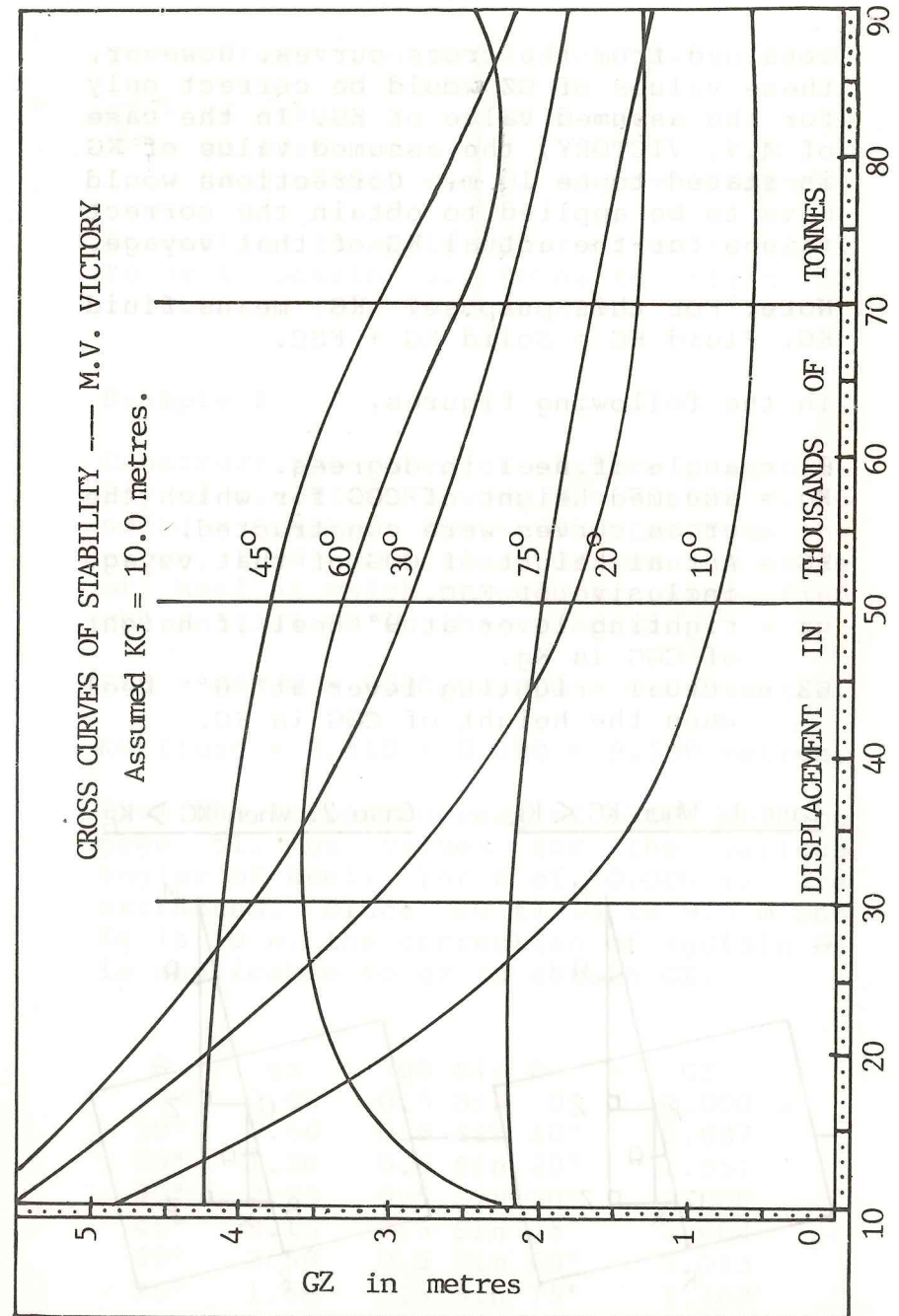
CHAPTER 23

CROSS CURVES
OF STABILITY

As explained in the previous chapter, a curve of statical stability is a curve constructed, for the displacement and KG of each voyage, by the chief officer. The information required to construct such a curve may be given by the shipyard in the form of 'Cross Curves' or in the form of 'KN Curves'. The former is explained here and the latter, in the next chapter.

GZ is a function of KG, KM and θ (the angle of heel). KM depends on draft. For the sake of convenience, the shipyard uses displacement in salt water, instead of draft. The variables, therefore, are GZ, θ , displacement in SW and KG.

Based on their calculations, the shipyard constructs a graph with GZ along the y-axis and displacement along the x-axis. Separate curves are drawn for different values of θ - say 10° , 20° , 30° , 45° , 60° and 75° . These curves may cross one another, hence the name 'Cross Curves'. The graph is constructed for an assumed value of KG which is stated clearly therein. In this manner, all the variables affecting GZ are allowed for. The next page contains the cross curves for the bulk carrier m.v. VICTORY. Using the displacement for that voyage, the GZ for the various angles of heel can be



obtained from the cross curves. However, these values of GZ would be correct only for the assumed value of KG. In the case of M.V. VICTORY, the assumed value of KG is stated to be 10 m. Corrections would have to be applied to obtain the correct values for the actual KG of that voyage.

Note: For this purpose, KG means fluid KG. Fluid KG = Solid KG + FSC.

In the following figures,

θ = angle of heel in degrees.

K_g = assumed height of COG for which the cross curves were constructed.

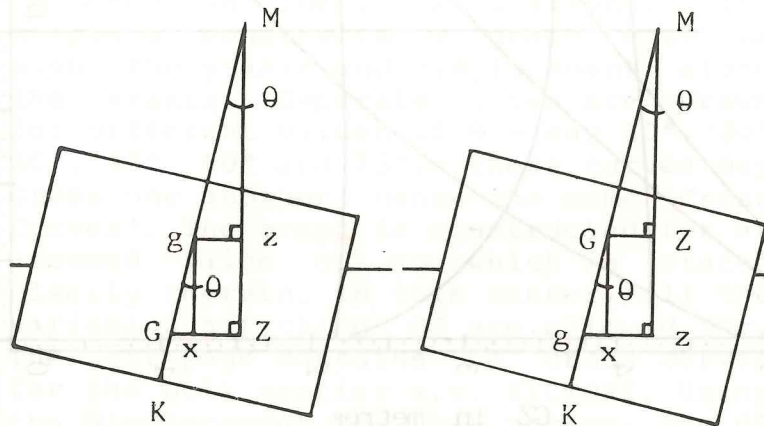
KG = Actual height of COG of that voyage inclusive of FSC.

g_z = righting lever at θ° heel if height of COG is K_g .

GZ = actual righting lever at θ° heel when the height of COG is KG.

Case 1: When $KG < K_g$

Case 2: When $KG > K_g$



$$\begin{array}{l|l} g_x \text{ is drawn } \parallel \text{ to } zZ & G_x \text{ is drawn } \parallel \text{ to } Zz \\ \hline GZ = xZ + xG & GZ = xz = gz - gx \\ = gz + xG & = gz - gG(\sin \theta) \\ = gz + gG(\sin \theta) & \end{array}$$

Correction to apply to g_z to obtain GZ is PLUS $gG(\sin \theta)$. | Correction to apply to g_z to obtain GZ is MINUS $gG(\sin \theta)$.

Example 1

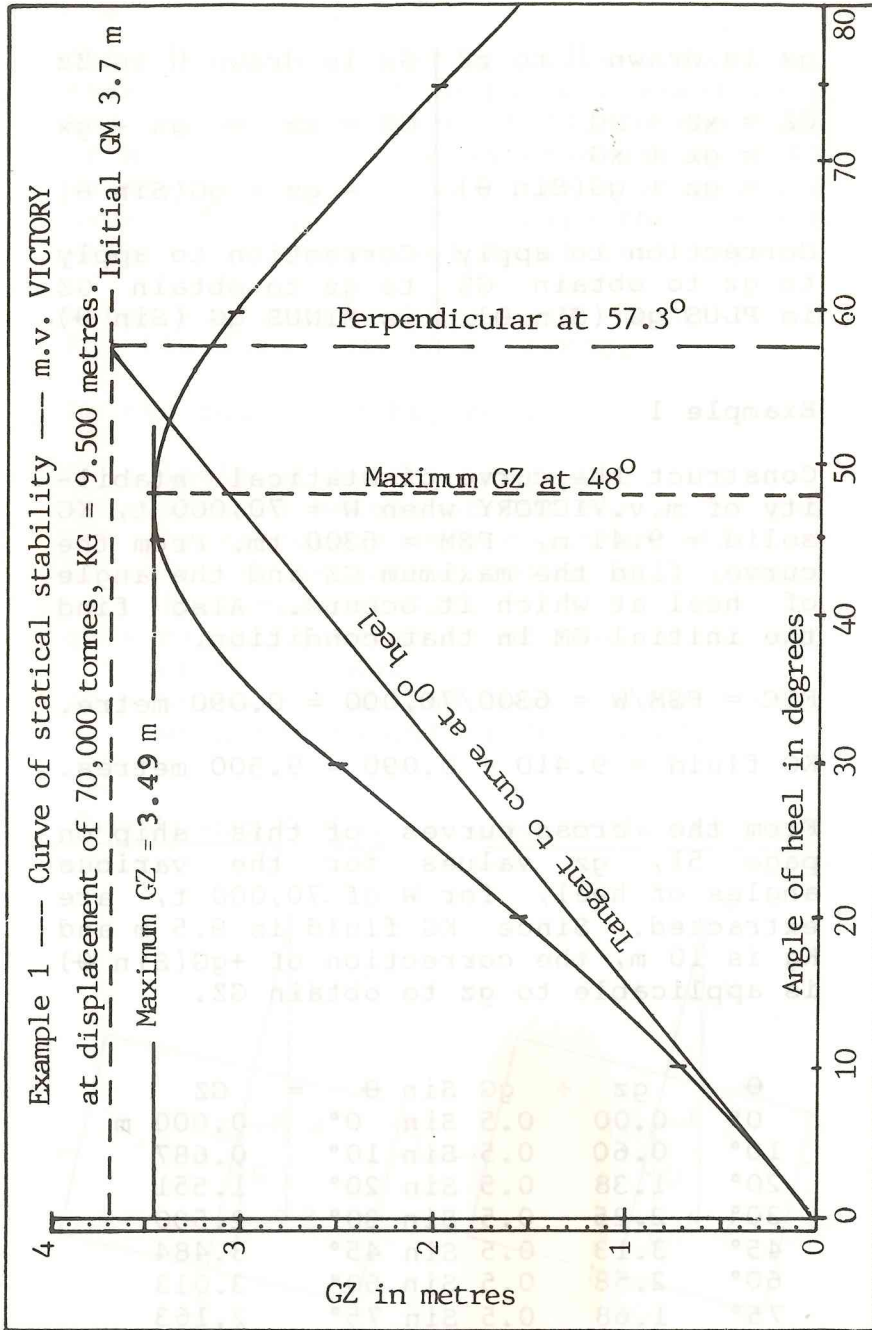
Construct the curve of statical stability of m.v. VICTORY when $W = 70,000$ t, KG solid = 9.41 m, FSM = 6300 tm. From the curve, find the maximum GZ and the angle of heel at which it occurs. Also find the initial GM in that condition.

$$FSC = FSM/W = 6300/70,000 = 0.090 \text{ metre.}$$

$$KG \text{ fluid} = 9.410 + 0.090 = 9.500 \text{ metres.}$$

From the cross curves of this ship on page 51, g_z values for the various angles of heel, for W of 70,000 t, are extracted. Since KG fluid is 9.5 m and K_g is 10 m, the correction of $+gG(\sin \theta)$ is applicable to g_z to obtain GZ.

θ	g_z	$+ gG \sin \theta$	=	GZ
0°	0.00	0.5 Sin 0°		0.000 m
10°	0.60	0.5 Sin 10°		0.687
20°	1.38	0.5 Sin 20°		1.551
30°	2.25	0.5 Sin 30°		2.500
45°	3.13	0.5 Sin 45°		3.484
60°	2.58	0.5 Sin 60°		3.013
75°	1.68	0.5 Sin 75°		2.163



GZ can now be plotted against θ , using a suitable scale, as shown on page 54. The curve should be faired as necessary to make it smooth and regular. The points should NOT be joined by short straight lines. The desired information may then be read off from the curve.

Note: Since the fluid GM was required to be read off the curve, the calculated GM could not be used to construct the curve.

Example 2

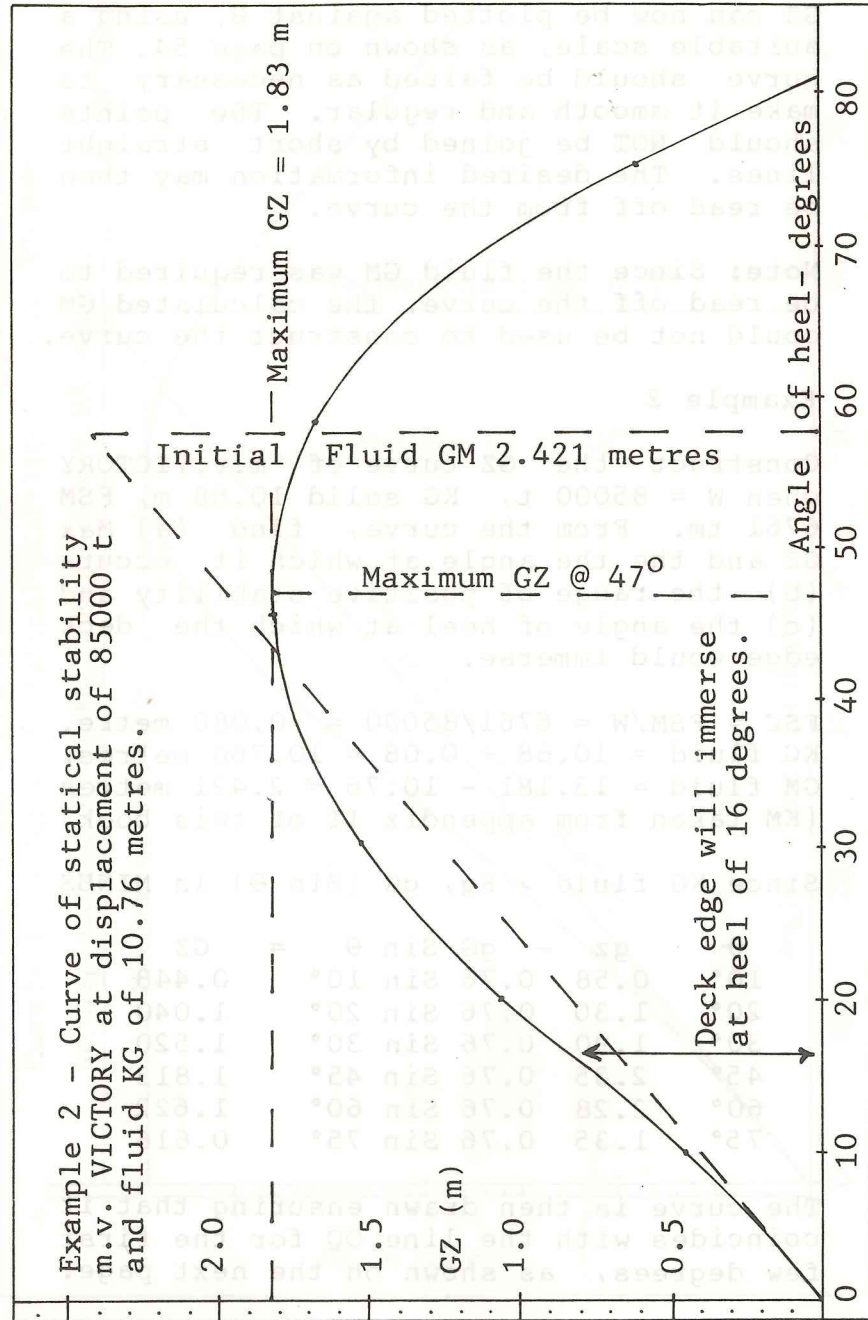
Construct the GZ curve of m.v.VICTORY when $W = 85000$ t, KG solid 10.68 m, FSM 6761 tm. From the curve, find (a) Max GZ and the angle at which it occurs (b) the range of positive stability and (c) the angle of heel at which the deck edge would immerse.

FSC = $FSM/W = 6761/85000 = 0.080$ metre.
 KG fluid = $10.68 + 0.08 = 10.760$ metres.
 GM fluid = $13.181 - 10.76 = 2.421$ metres
 (KM taken from appendix II of this book)

Since KG fluid > Kg, gG (Sin θ) is MINUS

θ	gz	-	gG	Sin θ	=	GZ
10°	0.58		0.76	Sin 10°		0.448
20°	1.30		0.76	Sin 20°		1.040
30°	1.90		0.76	Sin 30°		1.520
45°	2.35		0.76	Sin 45°		1.813
60°	2.28		0.76	Sin 60°		1.622
75°	1.35		0.76	Sin 75°		0.616

The curve is then drawn ensuring that it coincides with the line OQ for the first few degrees, as shown on the next page.



From the curve of statical stability,

- Maximum GZ = 1.830 m at 47° heel.
- Range of (+) stability = 0° to 81°.
- Deck edge would immerse at 16° heel.

Exercise 20

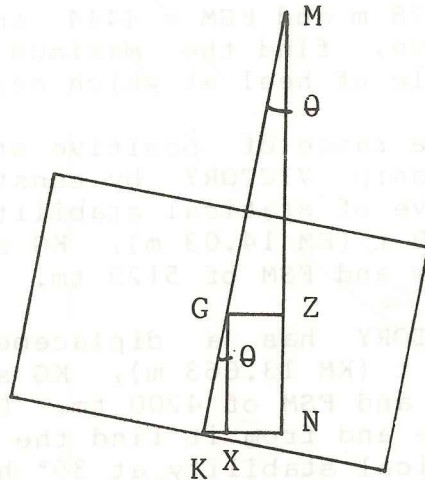
Cross curves of stability

- Draw the curve of statical stability of m.v. VICTORY at $W = 30,000$ t, solid KG 10.983 m, FSM 9000 tm. From the curve, find the initial GM.
- Bulker m.v. VICTORY is at 40500 t displacement, KM 16.124 m, KG solid 8.925 m, FSM 4738 tm. Draw the GZ curve and thence find the heel at which the deck edge would immerse.
- Draw the curve of statical stability for the bulk carrier m.v. VICTORY when its $W = 20200$ t, KM = 26.464 m, solid KG = 9.78 m and FSM = 4444 tm. From the curve, find the maximum GZ and the angle of heel at which occurs.
- Find the range of positive stability of the ship VICTORY by constructing the curve of statical stability for W of 55000 t (KM 14.03 m), KG solid of 10.691 m and FSM of 5129 tm.
- M.v. VICTORY has a displacement of 60,000 t (KM 13.663 m), KG solid of 9.243 m and FSM of 4200 tm. Draw the GZ curve and from it find the moment of statical stability at 35° heel.

CHAPTER 24

KN CURVES

While using cross curves of stability the correction $gG(\sin \theta)$, is sometimes positive and sometimes negative, depending on whether the actual fluid KG is less than or greater than the assumed Kg for which the curves were drawn. A bit of thought is necessary, each time, to decide on this and the chances of error are high - in his hurry, the chief officer may add this correction instead of subtracting it, or vice versa. In order to eliminate this possibility of error, some shipyards draw the cross curves for an assumed value of zero Kg,



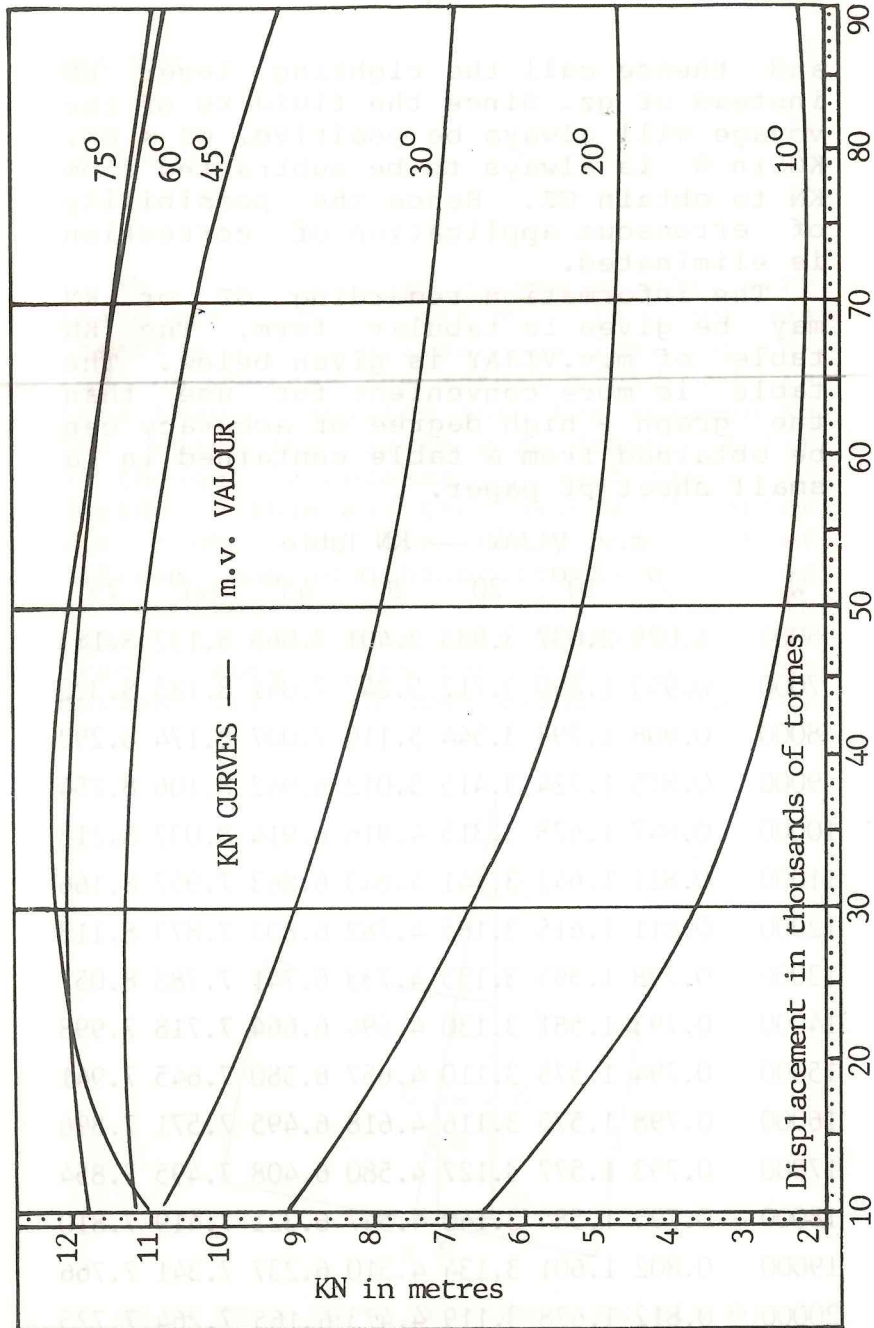
$$GZ = XN = KN - GX = KN - KG(\sin \theta)$$

and thence call the righting lever KN instead of gz. Since the fluid KG of the voyage will always be positive, $GZ < KN$. $KG \sin \theta$ is always to be subtracted from KN to obtain GZ. Hence the possibility of erroneous application of correction is eliminated.

The information regarding GZ or KN may be given in tabular form. The KN table of m.v. VIJAY is given below. The table is more convenient for use than the graph - high degree of accuracy can be obtained from a table contained in a small sheet of paper.

m.v. VIJAY --- KN Table

W	5°	10°	20°	30°	45°	60°	75°
6000	1.029	2.037	3.935	5.401	7.065	8.132	8.183
7000	0.953	1.890	3.717	5.247	7.041	8.185	8.322
8000	0.908	1.793	3.544	5.119	7.007	8.174	8.292
9000	0.875	1.724	3.415	5.012	6.962	8.106	8.254
10000	0.847	1.678	3.315	4.916	6.914	8.032	8.213
11000	0.827	1.642	3.241	4.843	6.863	7.957	8.166
12000	0.811	1.615	3.185	4.782	6.803	7.873	8.113
13000	0.798	1.595	3.153	4.733	6.741	7.788	8.057
14000	0.793	1.581	3.130	4.694	6.664	7.718	7.998
15000	0.794	1.575	3.110	4.657	6.580	7.645	7.941
16000	0.798	1.575	3.116	4.618	6.495	7.571	7.896
17000	0.793	1.577	3.127	4.580	6.408	7.495	7.854
18000	0.795	1.584	3.140	4.547	6.321	7.419	7.810
19000	0.802	1.601	3.134	4.510	6.237	7.341	7.766
20000	0.812	1.628	3.119	4.473	6.165	7.264	7.725



For the same accuracy, curves in graphical form would have to be very much bigger in size.

The previous page shows the KN curves of m.v. VALOUR, a sister ship of m.v. VICTORY whose cross curves are given in the previous chapter. Having been built in different shipyards, one has cross curves, and the other, KN curves.

Example 1

Construct the GZ curve of m.v. VALOUR when displacing 65000 t, KM 13.420 m, KG solid 8.2 m, FSM 6500 tm. From the curve find the GZ at 70° heel.

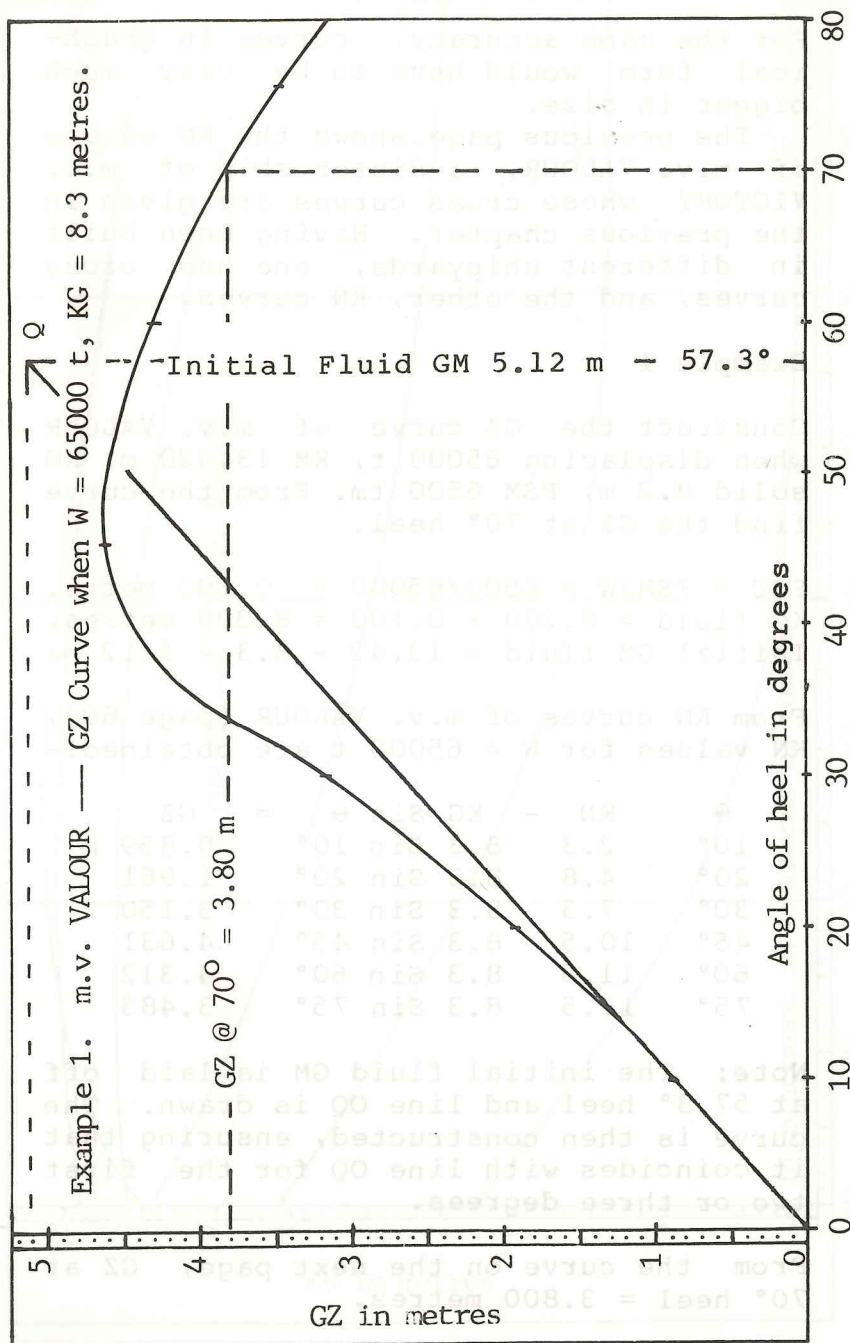
FSC = $FSM/W = 6500/65000 = 0.100$ metre.
 KG fluid = $8.200 + 0.100 = 8.300$ metres.
 Initial GM fluid = $13.42 - 8.3 = 5.12$ m.

From KN curves of m.v. VALOUR (page 60), KN values for $W = 65000$ t are obtained:-

ϕ	KN	-	KG	Sin ϕ	=	GZ
10°	2.3		8.3	Sin 10°		0.859 m
20°	4.8		8.3	Sin 20°		1.961
30°	7.3		8.3	Sin 30°		3.150
45°	10.5		8.3	Sin 45°		4.631
60°	11.5		8.3	Sin 60°		4.312
75°	11.5		8.3	Sin 75°		3.483

Note: The initial fluid GM is laid off at 57.3° heel and line OQ is drawn. The curve is then constructed, ensuring that it coincides with line OQ for the first two or three degrees.

From the curve on the next page, GZ at 70° heel = 3.800 metres.



Example 2

Construct the GZ curve for m.v. VIJAY when KG (solid) = 6.1 m, $FSM = 3050$ tm, $W = 15400$ t, $KM = 8.034$ m. Find max GZ.

$FSC = FSM/W = 3050/15400 = 0.198$ metre.
 Fluid $KG = 6.100 + 0.198 = 6.298$ metres.
 GM fluid = $8.034 - 6.298 = 1.736$ metres.

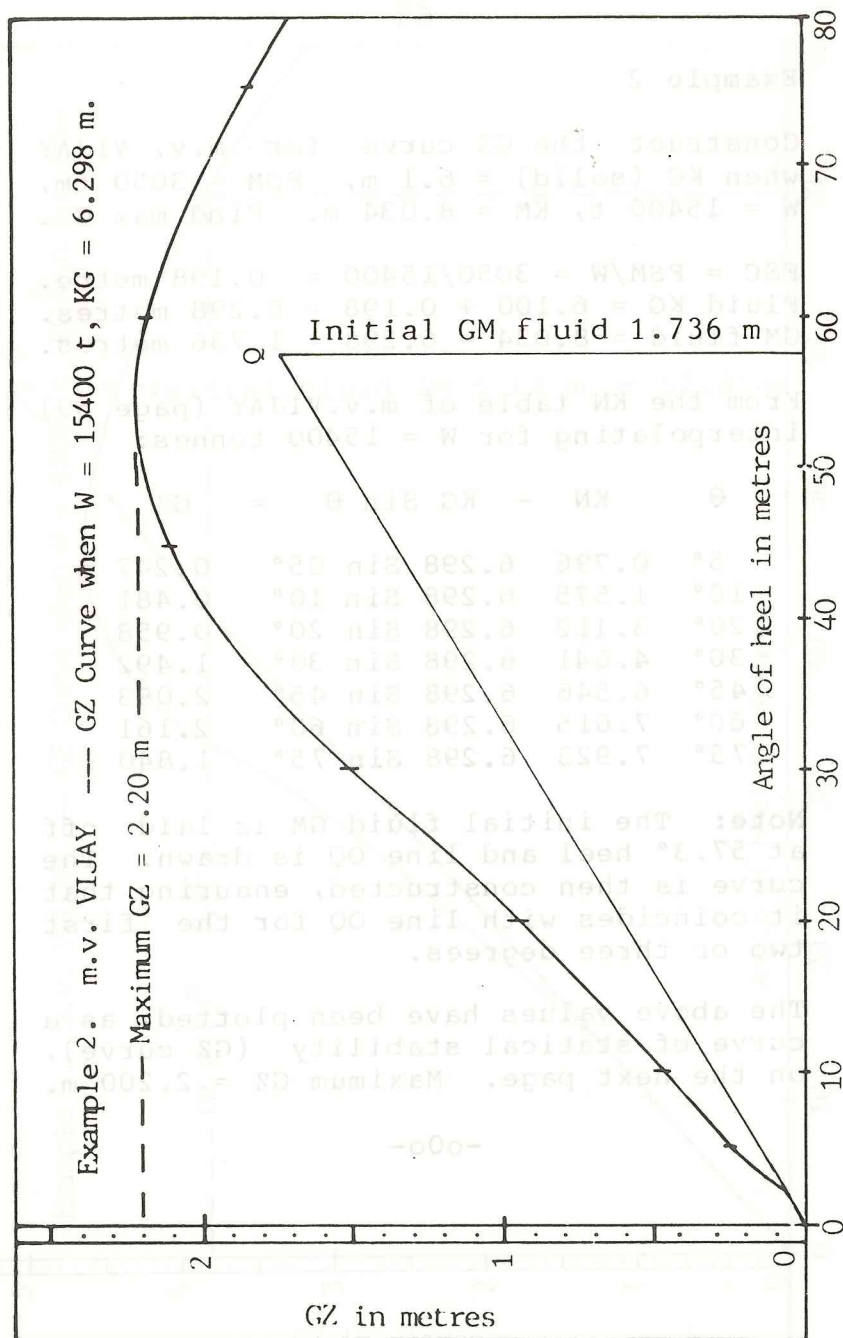
From the KN table of m.v. VIJAY (page 59) interpolating for $W = 15400$ tonnes:

θ	KN	-	KG Sin θ	=	GZ
5°	0.796		6.298 Sin 05°		0.247 m
10°	1.575		6.298 Sin 10°		0.481
20°	3.112		6.298 Sin 20°		0.958
30°	4.641		6.298 Sin 30°		1.492
45°	6.546		6.298 Sin 45°		2.093
60°	7.615		6.298 Sin 60°		2.161
75°	7.923		6.298 Sin 75°		1.840

Note: The initial fluid GM is laid off at 57.3° heel and line OQ is drawn. The curve is then constructed, ensuring that it coincides with line OQ for the first two or three degrees.

The above values have been plotted, as a curve of statical stability (GZ curve), on the next page. Maximum GZ = 2.200 m.

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Exercise 21

KN curves

1. Construct the GZ curve of m.v. VALOUR when $W = 81000$ t, KG solid = 10.21 m, $FSM = 6800$ tm. From the curve find the GZ at 40° heel. (Take initial KM from appendix II).
2. Find the maximum GZ and the angle of heel at which it occurs for m.v. VIJAY when $W = 19943$ t, solid $KG = 7.326$ m, $FSM = 1342$ tm, $KM = 8.461$ m.
3. Construct the curve of statical stability for m.v. VALOUR when displacement = 26000 t, KG solid = 7.014 m, $FSM = 7200$ tm. KM is 21.592 m. Find the maximum GZ and the angle at which it occurs.
4. M.v. VIJAY is displacing 13250 t with KG solid = 6.427 m and $FSM = 1200$ tm. Construct the curve of statical stability and from it, find the angles of heel at which $GZ = 1.6$ m. (Take initial KM from appendix I).
5. M.v. VALOUR is displacing 60000 t with solid $KG = 8.661$ m, $FSM = 5020$ tm and $KM = 13.663$ m. By constructing the curve of statical stability, find the angle of heel at which the deck edge would immerse.

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CHAPTER 25

LONGITUDINAL

STABILITY

When a ship is at rest in calm water, the COB & the COG will be in a vertical line as illustrated by figure X on the next page.

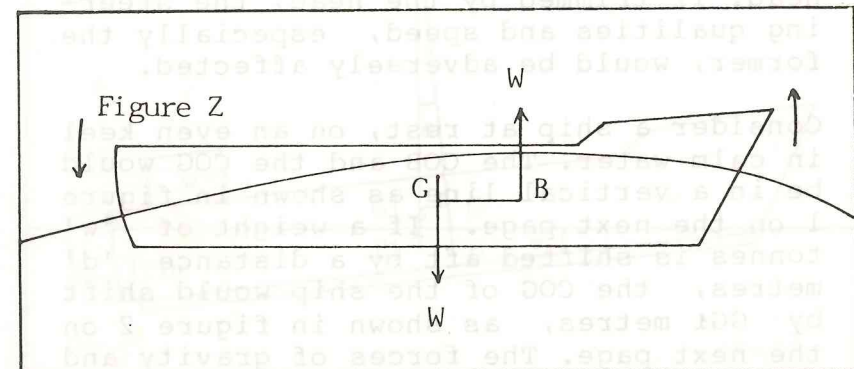
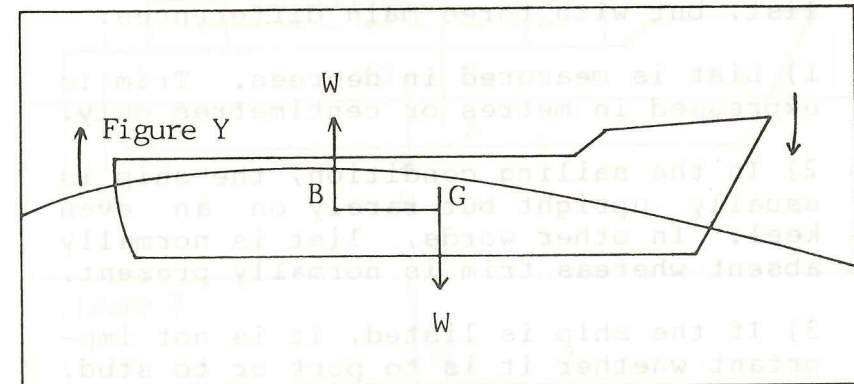
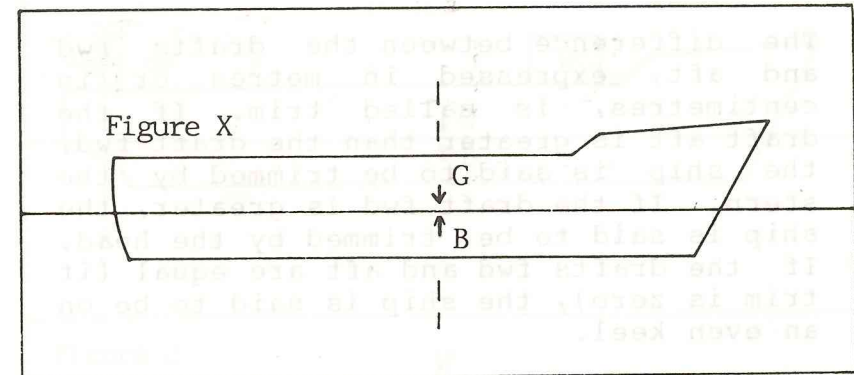
Pitch:

If waves cause an increase in the underwater volume aft, and a decrease fwd, the COB will shift aft. The forces of gravity and buoyancy now get separated by a fore and aft distance and form a couple which will cause the bow to dip downwards and the stern to lift upwards as shown in figure Y on the next page.

If waves cause an increase of underwater volume forward, and a decrease aft, COB will shift forward, and the forces of gravity and buoyancy will form a couple which will cause the bow to lift upwards and the stern to dip downwards as shown in figure Z on the next page. This up and down movement of the ship's ends, due to longitudinal shift of COB resulting from wave action, is called pitch.

During pitch, the COG of the ship does not move because no weights are loaded, discharged or shifted. Pitching is the longitudinal equivalent of rolling.

PITCHING



Trim:

The difference between the drafts fwd and aft, expressed in metres or in centimetres, is called trim. If the draft aft is greater than the draft fwd, the ship is said to be trimmed by the stern. If the draft fwd is greater, the ship is said to be trimmed by the head. If the drafts fwd and aft are equal (if trim is zero), the ship is said to be on an even keel.

Trim is the longitudinal equivalent of list, but with three main differences:

- 1) List is measured in degrees. Trim is expressed in metres or centimetres only.
- 2) In the sailing condition, the ship is usually upright but rarely on an even keel. In other words, list is normally absent whereas trim is normally present.
- 3) If the ship is listed, it is not important whether it is to port or to stbd. Trim must be by the stern, never by the head. If trimmed by the head, the steering qualities and speed, especially the former, would be adversely affected.

Consider a ship at rest, on an even keel in calm water. The COB and the COG would be in a vertical line as shown in figure 1 on the next page. If a weight of 'w' tonnes is shifted aft by a distance 'd' metres, the COG of the ship would shift by GG₁ metres, as shown in figure 2 on the next page. The forces of gravity and buoyancy would form a couple and cause

Figure 1

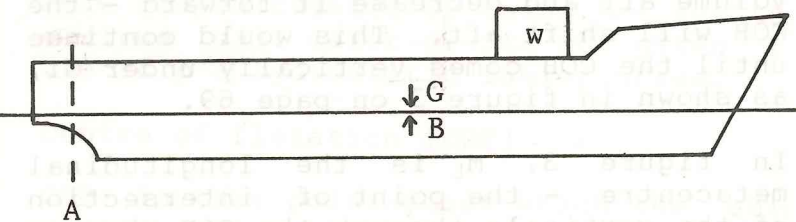


Figure 2

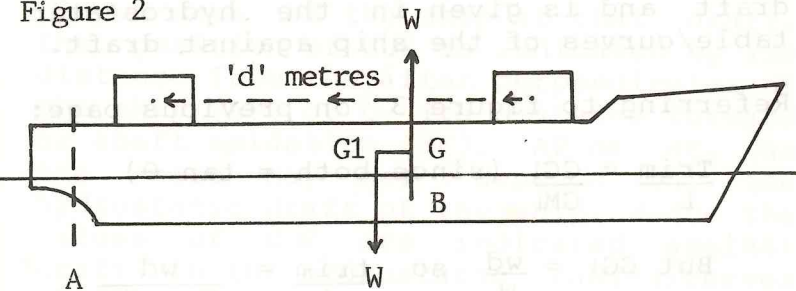
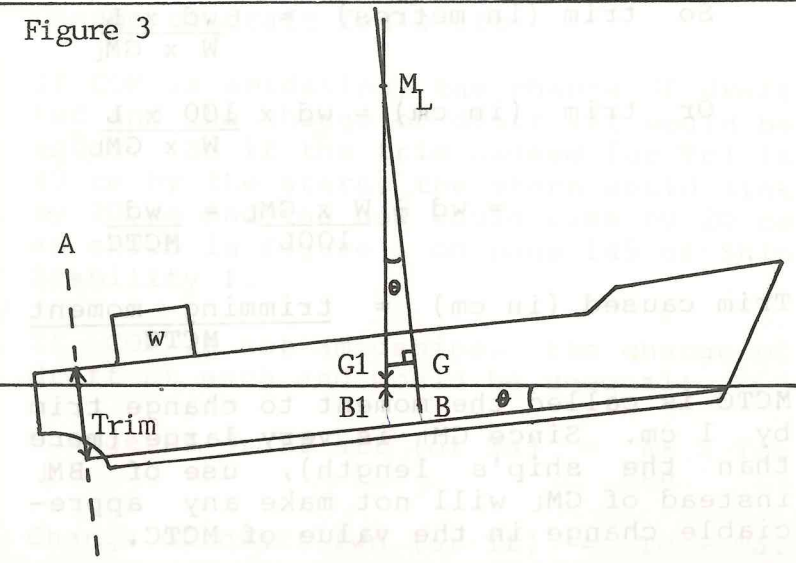


Figure 3



the stern to sink and the bow to rise. This would increase the underwater volume aft and decrease it forward - the COB will shift aft. This would continue until the COB comes vertically under G_1 , as shown in figure 3 on page 69.

In figure 3, M_L is the longitudinal metacentre - the point of intersection of the verticals through the COB when on an even keel and when trimmed. KM_L is the sum of KB and BM_L . KM_L is a function of draft and is given in the hydrostatic table/curves of the ship against draft.

Referring to figure 3 on previous page:

$$\frac{\text{Trim}}{L} = \frac{GG_1}{GM_L} \quad (\text{since both} = \tan \theta)$$

$$\text{But } GG_1 = \frac{wd}{W} \quad \text{so} \quad \frac{\text{trim}}{L} = \frac{wd}{W \times GM_L}$$

$$\text{So trim (in metres)} = \frac{wd \times L}{W \times GM_L}$$

$$\text{Or trim (in cm)} = wd \times \frac{100 \times L}{W \times GM_L}$$

$$= wd \div \frac{W \times GM_L}{100L} = \frac{wd}{MCTC}$$

$$\text{Trim caused (in cm)} = \frac{\text{trimming moment}}{MCTC}$$

MCTC is called the moment to change trim by 1 cm. Since GM_L is very large (more than the ship's length), use of BM_L instead of GM_L will not make any appreciable change in the value of MCTC.

$$MCTC = W.GM_L/100L \approx W.BM_L/100L$$

MCTC is calculated by using BM_L for the various salt water drafts and given in the ship's hydrostatic table/curves.

Centre of flotation (COF)

COF is that point about which the ship would pivot, when the trim is changed. COF is also called the tipping centre. It is the geometric centre of the water-plane area of the ship at that draft. The position of COF is indicated by its distance from the after perpendicular of the ship (AF) or by its distance forward or abaft amidships (HF). AF or HF, as the case may be, depends on the hydrostatic draft of the ship. Hence the values of COF are indicated against draft in the hydrostatic tables/curves of the ship.

Change of draft fwd & aft

If COF is amidships, the change of draft fwd and the change of draft aft would be equal. So if the trim caused (or T_c) is 40 cm by the stern, the stern would sink by 20 cm and the bow would rise by 20 cm as shown in figure 1 on page 145 of Ship Stability I.

If COF is not amidships, the change of draft at each end would be unequal:

$$\text{Change of draft aft (or } T_a) = \frac{AF}{LBP} \times T_c.$$

$$\text{Change of draft fwd (or } T_f) = T_c - T_a.$$

For example, if the trim caused (T_c) is 40 cm by stern, $AF = 63$ m, $LBP = 140$ m:

$$T_a = \frac{63 \times 40}{140} = 18.0 \text{ cm.}$$

$$T_f = 40 - 18 = 22.0 \text{ cm.}$$

Since T_c is by the stern, T_a will be + while T_f will be -. In other words, the draft aft will increase by 18 cm while the draft fwd will decrease by 22 cm. (See figure 2 on page 145 of Ship Stability I).

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CHAPTER 26

TRIM PROBLEMS

TYPE A

In this book, trim problems have been divided into three groups, A, B & C. In problems of type A, limited information is given: TPC, MCTC & AF are considered constant throughout. Trimming moments are taken about COF. Type A problems are useful in understanding the principles of trim and questions of this type do come in examinations for certificates of competency.

Type B problems on trim are those where full hydrostatic particulars of a ship are given and where moments are taken about the after perpendicular of the ship.

Type C problems are those where full hydrostatic particulars of a ship are given but where moments are taken about amidships.

Types B and C have a similar approach to the solution of trim problems. They are more practical, than type A, and are extensively used by shipyards. In types B and C, TPC, MCTC, AF, etc are not considered constants. These values are obtained against draft, when necessary, from the hydrostatic particulars of the ship. Worked examples have been given

the suffix A, B or C, as appropriate, to help distinguish the types.

Note: Mean sinkage or rise (calculated by the formula $w \div \text{TPC}$), and trim caused (calculated by the formula $\text{trimming moment} \div \text{MCTC}$), would be in centimetres NOT in metres.

TYPE A PROBLEMS

Example 1A - Shift of 'w' already aboard

A ship 100 m long has COF 3 m abaft amidships and MCTC = 250 tm. Present drafts are 5.8 m fwd and 6.2 m aft. Find the new drafts fwd & aft if 200 t of FW is transferred from the fore peak to the after peak through a distance of 90 m.

Trimming moment (or TM) = $w d = 200 \times 90$
= 18000 tm by the stern

$$T_c = \frac{\text{TM}}{\text{MCTC}} = \frac{18000}{250} = 72 \text{ cm by the stern}$$

$$T_a = \frac{\text{AF}}{L} \times T_c = \frac{47 \times 72}{100} = 33.8 \text{ cm}$$

$$T_f = T_c - T_a = 72 - 33.8 = 38.2 \text{ cm}$$

	fwd	aft
Original drafts	5.800 m	6.200 m
Tf or Ta	-0.382 m	+0.338 m
Final drafts ..	5.418 m	6.538 m

Note: Accuracy of calculation should be to three decimal places of a metre or one decimal of a centimetre.

Example 2A - Discharging a weight.

A ship 120 m long, COF 2.5 m abaft amidships (HF 2.5 m aft), MCTC 100 tm, TPC 25, floats at 7 m fwd and 10 m aft. Find the new drafts if 200 t is discharged from a position 50 m abaft amidships.

$$\text{Mean rise} = w \div \text{TPC} = 200 \div 25 = 8 \text{ cm.}$$

$$T_c = \frac{w d}{\text{MCTC}} = \frac{200 \times 47.5}{100} = 95 \text{ cm by head.}$$

Note: d = distance from COF. T_c by head because cargo discharged was abaft COF.

$$T_a = \frac{\text{AF} \times T_c}{L} = \frac{57.5 \times 95}{120} = 45.5 \text{ cm.}$$

$$T_f = T_c - T_a = 95 - 45.5 = 49.5 \text{ cm.}$$

	fwd	aft
Original drafts	7.000 m	10.000 m
Mean rise	-0.080 m	-0.080 m
	6.920 m	9.920 m
Tf or Ta	+0.495 m	-0.455 m
Final drafts	7.415 m	9.465 m

Example 3A - Load/disch several weights.

A ship is 150 m long. MCTC = 300 tm, TPC = 30, COF is 4 m abaft amidships (HF 4 m aft). Present drafts are 6.1 m fwd & 8.3 m aft. Find the final drafts if the following operations are carried out:

- 4000 t loaded 24 m abaft H (HG 24 m aft)
- 2000 t cargo loaded, HG 50 m fwd.
- 1000 t cargo discharged from HG 30 m fwd
- 300 t SW run into AP tank, HG 70 m aft.

Weight		Distance from COF m	Trimming moment	
Loaded	Disch		By head	By stern
t	t		tm	tm
2000	-	54 fwd	108000	-
-	1000	34 fwd	-	34000
4000	-	20 aft	-	80000
300	-	66 aft	-	19800
6300	1000	Total	108000	133800
5300	-	Final	-	25800

$$\text{Mean sinkage} = \frac{w}{\text{TPC}} = \frac{5300}{30} = 176.7 \text{ cm.}$$

$$T_c = \frac{\text{TM}}{\text{MCTC}} = \frac{25800}{300} = 86 \text{ cm by the stern}$$

$$T_a = \frac{\text{AF} \times T_c}{L} = \frac{71 \times 86}{150} = 40.7 \text{ cm.}$$

$$T_f = T_c - T_a = 86 - 40.7 = 45.3 \text{ cm.}$$

	fwd	aft
Original drafts	6.100 m	8.300 m
Mean sinkage	+1.767 m	+1.767 m
	7.867 m	10.067 m
Tf or Ta	-0.453 m	+0.407 m
Final drafts	7.414 m	10.474 m

Example 4A - To finish at a desired trim

A ship has to load 600 t of cargo. The drafts are 8.5 m fwd & 9.7 m aft. Space is available 60 m fwd & 40 m aft of COF, which is amidships. MCTC = 250 tm and TPC = 20. Find how to distribute this 600 t of cargo if the ship is to finish loading 0.5 m by the stern. State also the final drafts fwd and aft.

Present trim = 1.2 m by the stern
 Desired trim = 0.5 m by the stern
 $T_c = 0.7 \text{ m by the head} = 70 \text{ cm}$

$$T_c = \frac{\text{TM}}{\text{MCTC}} \quad \text{or} \quad \text{TM} = T_c \times \text{MCTC}$$

$$\text{Reqd TM} = 70 \times 250 = 17500 \text{ tm by head.}$$

Let the cargo loaded forward be X tonnes
 So cargo loaded aft = (600 - X) tonnes

TM caused = 60X tm by head & 40(600 - X) tm by stern. Reqd to cause 17500 by head

$$\text{So } 60X - 40(600 - X) = 17500$$

$$X = 415 \text{ t and } (600 - X) = 185 \text{ t}$$

Cargo to be loaded in the forward space = 415 t and in the after space = 185 t.

$$\text{Mean sinkage} = \frac{w}{\text{TPC}} = \frac{600}{20} = 30 \text{ cm} = 0.3 \text{ m}$$

	fwd	aft
Original drafts	8.500 m	9.700 m
Mean sinkage	+0.300 m	+0.300 m
	8.800 m	10.000 m
Tf or Ta = Tc/2	+0.350 m	-0.350 m
Final drafts	9.150 m	9.650 m

Example 5A - To find HF

A ship is afloat at drafts of 6.6 m fwd & 7.4 m aft. 500 t of cargo is loaded 54 m fwd of H (amidships) & 800 t is loaded 52 m abaft H. If the final drafts are 6.85 and 8.51 m fwd & aft respectively, and MCTC = 200 tm, find HF (the distance of the COF from amidships).

Initial trim = 0.800 m by the stern
 Final trim = 1.660 m by the stern
 Trim caused = 0.860 m by stern = 86 cm.

$$T_c = \frac{TM}{MCTC} \quad \text{or} \quad TM = 86 \times 200 = 17200 \text{ tm.}$$

Let the COF be X metres abaft H. Then TM caused by head = $500(54 + X)$ tm & by the stern = $800(52 - X)$ tm. Since loading caused 17200 tm by the stern,

$$800(52 - X) - 500(54 + X) = 17200$$

$$X = -2 \text{ metres.}$$

Note: The minus sign indicates that the assumed direction of COF, from H, is wrong. In this case, COF was assumed to be abaft H.

Answer: COF is 2 m fwd of H (HF 2 m fwd)

Example 6A - Load keeping draft constant

A ship 96 m long is floating at 5 m fwd and 6.4 m aft. MCTC = 180 tm, TPC = 16. COF is 2 m abaft H (HF 2 m aft). Find the location where a weight of 50 t should be placed so as to keep the aft draft constant.

Note: If the weight is loaded on the COF the ship would sink bodily (parallel sinkage) by $w \div TPC$. The draft at both perpendiculars would increase by the same amount. If the weight is now shifted fwd by d metres, the draft aft would decrease by T_a and the draft fwd would increase by T_f . For the draft aft

to return to its original value, mean sinkage must equal T_a .

$$\frac{w}{TPC} = T_a = \frac{AF}{L} \times T_c = \frac{AF \times wd}{L \times MCTC}$$

$$\frac{46 \times 50d}{96 \times 180} = \frac{50}{16} \quad \text{or} \quad d = 23.5 \text{ m}$$

50 t should be loaded 23.5 m fwd of COF.

Example 7A - Desired value of draft aft.

A ship 100 m long, MCTC 280 tm, TPC 25, HF 2 m fwd, is afloat at drafts of 6 m fwd and 8 m aft. Find how many tonnes of SW must be run into the fore peak tank (COG 48 m fwd of H) to bring the draft aft to 7.8 m.

Reqd reduction of draft aft = 0.2 metres

$$T_a - \text{mean sinkage} = 0.2 \text{ m} = 20 \text{ cm.}$$

$$\frac{AF}{L} \times T_c - \frac{w}{TPC} = 20 \quad \text{or} \quad \frac{52}{100} \times \frac{dw}{MCTC} - \frac{w}{TPC} = 20$$

$$\frac{52}{100} \times \frac{46w}{280} - \frac{w}{25} = 20 \quad \text{or} \quad w = 440.25 \text{ t.}$$

Exercise 22

Trim problems - Type A

- 1 A ship 100 m long, draws 4 m fwd and 5.2 m aft. COF is 2 m abaft amidships MCTC 160 tm & TPC 15. 100 t of cargo is shifted from No:3 LH to No:1 LH through a horizontal distance of 32 m. Find the new drafts fwd and aft.

- 2 A ship 150 m long has $HF = 3$ m fwd, $TPC = 21$ and $MCTC = 275$ tm. Present drafts are 5.6 m fwd & 6.2 m aft. How many tonnes of SW must be transferred to the fore peak from the after peak, through a distance of 130 m, to bring the ship on an even keel?
- 3 The present drafts of a ship 140 m long are 8.1 m fwd and 9.9 m aft. $TPC = 30$, $MCTC = 250$ tm, $HF = 3$ m fwd. 300 t ballast was pumped out of No:5 DB tank, COG 50 m abaft H. Find the new drafts fwd and aft.
- 4 A ship is floating on an even keel draft of 10.2 m. $TPC = 30$, $MCTC = 320$ tm, $HF = 2.5$ m aft. LBP = 180 m. A 240 t heavy lift is loaded on deck, 40 m abaft COF. Find the new drafts at the fwd and after perpendiculars.
- 5 A ship of LBP 125 m, $MCTC = 318$ tm, $TPC = 28$, COF amidships ($HF = 0$ m), draws 7.9 m fwd and 10.4 m aft. Find the final drafts after the following operations have been carried out:
- 500 t loaded in No:2 LH 40 m fwd of H
 200 t loaded in No:5 TD 50 m aft of H
 100 t SW transferred from AP tank to FP tank, through a distance of 110 m.
- 6 The present drafts of a ship, whose length between perpendiculars is 142 m, are 10.2 m fwd & 11.6 m aft. $MCTC = 170$ tm. $TPC = 32$. $HF = 3$ m aft. Find the final drafts forward and aft after the following operations have been carried out:

- 1500 t cargo discharged from No:2 LH, 43 m fwd of amidships (HG 43 m fwd).
- 2000 t cargo discharged from No:4 LH, 37 m aft of amidships (HG 37 m aft).
- 500 t FW received in No:3 DB tank, 33 m fwd of amidships (HG 33 m fwd).
- 7 A ship left port drawing 8.2 m & 10 m F & A. LBP 160 m, $TPC = 32$, $MCTC = 220$ tm $HF = 2.4$ m aft. En route she consumed 420 t HFO from No:4 DBT (HG 35 m aft) 220 t HFO from No:7 DBT (HG 60 m aft) 200 t FW from No:1 DBT (HG 60 m fwd). Find the arrival drafts fwd & aft.
- 8 A ship is 150 m long. $MCTC = 200$ tm, $TPC = 25$, $HF = 2$ m fwd. The present drafts are 8 m forward and 10 m aft. Calculate the initial hydrostatic draft. Space is available in No:1 LH (HG 60 m fwd) & in No:4 LH (HG 30 m aft). Find how much cargo to put in each of these spaces, if the ship is to finish loading at an even keel draft of 9.3 m.
- 9 A ship of LBP 200 m, $MCTC = 300$ tm, $TPC = 35$, $HF = 5$ m aft, is presently drawing 7 m fwd and 11 m aft. Calculate the present hydrostatic draft. Space is available for loading in No:1 LH (HG 80 m fwd), and in NO:4 LH (HG 30 m aft). Calculate how much cargo to put in each of the two spaces in order to finish at a hydrostatic draft of 9.6 metres, trimmed 2 metres by the stern. State also, the final drafts forward and aft.

- 10 A ship was trimmed 0.5 m by the head. After loading 500 t in No:4 LH, HG 40 m aft, the trim was 0.5 m by the stern. If MCTC was 185 tm, find HF.
- 11 A ship of LBP 130 m and MCTC 175 tm, draws 7.2 m fwd and 7.8 m aft. 1600 t cargo was loaded in No:2 LH, HG 30 m fwd, and 1400 t in No:5 LH, HG 55 m aft. The drafts were then found to be 7.8 m fwd & 10.4 m aft. Find HF.
- 12 A ship drawing 7.6 m fwd & 8.4 m aft has LBP 140 m, HF 2.7 m aft, MCTC 170 tm, TPC 28. Find where 140 t may be placed if the after draft is to remain constant. State the draft forward after loading.
- 13 A ship 160 m long has MCTC 200 tm, TPC 30, HF 3 m aft, draft fwd 6.8 m, aft 7.8 m. From where may 'w' tonnes may be discharged such that the draft aft remains constant?
- 14 A ship of LBP 120 m, MCTC 300 tm, TPC 25, HF 2 m fwd, floats at 9.6 m fwd & 10.8 m aft. 200 t of deck cargo is to be loaded. Some shipside repairs are to be effected for which the fwd draft is to be maintained at 9.6 m. Find where this cargo may be loaded. State the final draft aft.
- 15 LBP 124 m, MCTC 180 tm, TPC 27, HF 2.5 m aft, draft fwd 5.8, aft 6.9 m. How many tonnes of SW must be run into the FP tank (HG 58 m fwd) in order to reduce the draft aft to 6.7 metres? State the final draft fwd.

- 16 A ship 160 m long has MCTC 200 tm, HF 2.8 m fwd, TPC 24. Present drafts are 7.4 m fwd & 8.8 m aft. 720 t cargo is to be loaded. Space is available in No:2 LH (HG 45 m fwd) and in No:4 LH (HG 35 m aft). Find how much to put in each if the draft aft is required to be 9 m on completion. What is the final draft fwd?
- 17 A ship arrives port drawing 7 m fwd & 10 m aft. HF = 0, MCTC 220 tm, TPC 25. Maximum permissible draft to cross a bar at the dock entrance is 9 m. Due to a damaged fore peak, the draft fwd is not to exceed 7 m. Find the minimum amount of cargo to discharge into barges from No:2 TD (HG 30 m fwd) and from No:4 TD (HG 50 m aft) so that both conditions are satisfied.
- 18 A ship arrives port drawing 8 m fwd & 10.5 m aft. LBP 166 m, HF 3 m aft, MCTC 175 tm, TPC 25. 250 t of dangerous deck cargo is to be discharged at anchorage from HG 73 m aft. How many tonnes of water must be transferred from the after peak tank to No:1 DBT through a distance of 140 m, to make the final trim 1 m by the stern? State the final drafts fwd & aft.
- 19 A ship arrives port drawing 8 m fwd & 9 m aft. LBP = 158 m, MCTC = 190 tm, TPC = 20 and HF is 2 m aft. The maximum draft allowed, to cross a bar, is 8.6 metres. There is no scope for transfer of any weights aboard. Hence it decided to off-load some cargo at anchorage. Find the minimum quantity

of cargo to off-load from No:4 TD, HG 40 m aft. State the final draft fwd.

- X 20 A ship leaves port drawing 9 m fwd and 9.8 m aft. LBP = 170 m, MCTC = 160 tm, TPC = 24 and HF 1 m fwd. On passage she consumes the following:

520 t HFO from No:2 DBT, HG 50 m fwd,
200 t HFO from No:8 DBT, HG 60 m aft,
300 t FW from No:3 DBT, HG 30 m fwd.

How much FW must be transferred between the peak tanks, 150 m apart, to bring the trim to 0.3 m by the stern? State the final drafts fwd and aft.

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CHAPTER 27

TRIM PROBLEMS

TYPE B

Consider a ship on an even keel and refer to figures 1 and 2 in chapter 25 (page 69). COB and COG are in a vertical line. Due to shift of a weight aft, COB & COG get separated longitudinally. The forces of buoyancy and gravity form a couple which trims the ship by the stern. The moment of this couple = $W.GG_1$ and is called the trimming moment or TM.

The trimming lever GG_1 may be substituted by $AB \sim AG$ which is the longitudinal separation of COB & COG caused by shift of weight. The AB used here is the distance of the COB from the after perpendicular of the ship at that draft, when on even keel. For the sake of simplicity of expression in formulae, \overline{BG} is used in this book to represent $AB \sim AG$ on the understanding that \overline{BG} is the longitudinal separation of COB (on even keel) and COG of ship, NOT the actual slant distance. The trimming moment, or TM, is thus equal to $W.\overline{BG}$. TM, divided by MCTC, gives the trim. If COG lies abaft COB, trim will be by stern and vice versa. For working trim problems of type B, the hydrostatic particulars of m.v.VIJAY have been included as Appendix I of this book.

Procedure for most trim problems type B

- 1 Find the initial hydrostatic draft and thence the W, MCTC, AB & AF.
- 2 Find the initial AG, if not given.
- 3 Find the final W and the final AG by taking moments about A.
- 4 Find the final hydrostatic draft and thence the MCTC, AB & AF at that draft
- 5 Find the final \overline{BG} and thence the final trim (T_c).
- 6 Split T_c (final trim) into T_a and T_f .
- 7 Apply T_f & T_a to the final hydrostatic draft & obtain final drafts fwd & aft.

Example 1B

M.v.VIJAY is floating in SW at an even keel draft of 5 m. Find the new drafts F & A if 200 t FW is transferred 30 m aft.

Note: This is similar to example 1A except that MCTC and AF have to be taken from the hydrostatic table of the ship.

From Appendix I, for SW draft 5 m, MCTC = 165.7 tm and AF = 71.913 metres

$$T_c = \frac{TM}{MCTC} = \frac{200 \times 30}{165.7} = 36.2 \text{ cm by stern.}$$

$$T_a = \frac{AF}{L} \times T_c = \frac{71.913}{140} \times 0.362 = 0.186 \text{ m.}$$

$$T_f = T_c - T_a = 0.362 - 0.186 = 0.176 \text{ m.}$$

	Fwd	Aft
Original drafts	5.000 m	5.000 m
Tf or Ta	<u>-0.176 m</u>	<u>+0.186 m</u>
Final drafts	4.824 m	5.186 m

Example 2B

M.v.VIJAY is floating in SW at drafts of 4.8 m fwd and 6.8 m aft. AG is 69.04 m. Find the drafts fwd and aft if 1000 t of cargo is loaded in No:3 LH, AG 80 m.

Fwd 4.8 m, aft 6.8 m, trim 2 m by stern. Mean draft 5.8 m for which AF = 71.586 m

$$\text{Corrn} = \frac{AF}{L} \times \text{trim} = \frac{71.586}{140} \times 2 = 1.023 \text{ m}$$

$$\text{Initial hydraft*} = 6.8 - 1.023 = 5.777 \text{ m}$$

* Abbreviation for 'Hydrostatic draft.'

For 5.777 m hydraft, W = 11620.4 tonnes. Final W = 11620.4 + 1000 = 12620.4 t

From table, particulars for final W are:

W (t)	draft	MCTC	AB (m)	AF (m)
12620.4	6.220	174.78	71.937	71.313

Taking longitudinal moments about A,

$$11620.4(69.04) + 1000(80) = 12620.4 (AG)$$

Final AG = 69.908 m. Final AB = 71.937 m. Finally, COG is astern of COB. So final trim is by the stern. Final BG = 2.029 m

$$\text{Trim} = \frac{W \cdot \overline{BG}}{MCTC} = \frac{12620.4(2.029)}{174.78} = 146.5 \text{ cm}$$

$$T_a = \frac{AF}{L} \times T_c = \frac{71.313(1.465)}{140} = 0.746 \text{ m}$$

$$T_f = T_c - T_a = 1.465 - 0.746 = 0.719 \text{ m}$$

	Fwd	Aft
Final hydroft	6.220 m	6.220 m
Tf or Ta	-0.719 m	+0.746 m
Final drafts	5.501 m	6.966 m

Example 3B

M.v.VIJAY is floating in SW at drafts of 4 m fwd & 5.8 m aft. AG is 68.930 m. The following operations were carried out:

No:2 LH (AG 102 m): 1800 t cargo loaded.
 No:4 LH (AG 58 m): 1600 t cargo loaded.
 No:1 DBT (AG 120 m): 160 t ballast out.
 FP tank (AG 135 m): 100 t ballast out.
 Find the final drafts fwd and aft.

Fwd 4 m, aft 5.8 m, trim 1.8 m by stern.
 Mean draft 4.9 m for which AF = 71.942 m

$$\text{Corr} = \frac{AF}{L} \times \text{trim} = \frac{71.942(1.8)}{140} = 0.925 \text{ m}$$

Initial hydroft = 5.8 - 0.925 = 4.875 m,
 for which W = 9616 t, from Appendix I.

Remarks	Weight t	AG m	Moment abt A
	load	disc.	loaded
Ship	9616	-	662831
Cargo	1800	-	183600
Cargo	1600	-	92800
Ballast	-	160	-
Ballast	-	100	-
Total	13016	260	939231
Final	12756		906531

Final AG = 906531/12756 = 71.067 metres.

From Appendix I, for final W of 12756 t:

Draft	MCTC tm	AB (m)	AF (m)
6.280	175.316	71.929	71.267

Final AG = 71.067 m, final AB = 71.929 m
 Final trim by stern; final \overline{BG} = 0.862 m

$$\text{Trim} = \frac{W \cdot \overline{BG}}{\text{MCTC}} = \frac{12756 \times 0.862}{175.316} = 62.7 \text{ cm}$$

$$T_a = \frac{AF}{L} \times \text{trim} = \frac{71.267(0.627)}{140} = 0.319 \text{ m}$$

$$T_f = \text{Trim} - T_a = 0.627 - 0.319 = 0.308 \text{ m}$$

	Fwd	Aft
Final hydroft	6.280 m	6.280 m
Tf or Ta	-0.308 m	+0.319 m
Final drafts	5.972 m	6.599 m

Example 4B

M.v.VIJAY is in SW drawing 6 m fwd & 7 m aft. 1000 t cargo is loaded in No:3 LH, AG 80 m. Find the final drafts fwd & aft

Note: AG of the ship is not given here but can be calculated using the given data and the hydrostatic table. This is the method normally used on board ships.

Fwd 6 m, aft 7 m, trim 1 m by the stern.
 Mean draft 6.5 m for which AF = 71.087 m

$$\text{Corr} = \frac{AF}{L} \times \text{trim} = \frac{71.087 \times 1}{140} = 0.508 \text{ m}$$

Hydroft = 7 - 0.508 = 6.492 m for which:

Draft	W (t)	MCTC tm	AB (m)
6.492	13239.8	177.228	71.902

$$\text{Trim in cm} = \frac{W \cdot \overline{BG}}{\text{MCTC}} \text{ or } \overline{BG} = \frac{\text{trim} \times \text{MCTC}}{W}$$

$$\overline{BG} = \frac{100(177.228)}{13239.8} = 1.338 \text{ m}$$

Since trim is by stern, $AG < AB$.

$$AG = AB - \overline{BG} = 71.902 - 1.338 = 70.564 \text{ m}$$

AG of ship in given condition = 70.564 m

Remarks	Weight	AG (m)	Moment
Ship	13239.8	70.564	934253
Loaded	1000	80	80000
Final	14239.8		1014253

$$\text{Final AG} = 1014253 \div 14239.8 = 71.227 \text{ m}$$

From Appendix I for final $W = 14239.8 \text{ t}$:

Draft	MCTC tm	AB (m)	AF (m)
6.929	181.852	71.832	70.673

Since final $AG < AB$, final trim by stern
 Final $\overline{BG} = 71.832 - 71.227 = 0.605 \text{ m}$

$$\text{Final Tc} = \frac{W \cdot \overline{BG}}{\text{MCTC}} = \frac{14239.8(0.605)}{181.852} = 47.4 \text{ (cm)}$$

$$T_a = \frac{AF}{L} \times T_c = \frac{70.673}{140} \times 0.474 = 0.239 \text{ m}$$

$$T_f = T_c - T_a = 0.474 - 0.239 = 0.235 \text{ m}$$

	Fwd	Aft
Final draft	6.929 m	6.929 m
Tf or Ta	-0.235 m	+0.239 m
Final drafts	6.694 m	7.168 m

Example 5B

M.v. VIJAY floats in SW, drawing 3.6 m & 6.4 m fwd and aft. 2000 t cargo is to be

loaded. Space is available in No:2, AG 102 m, and in No:4, AG 58 m. Find how much cargo to put in each space in order to finish with a trim of 1 metre by the stern. State the final drafts fwd & aft.

Fwd 3.6 m aft 6.4 m, trim 2.8 m by stern
 Mean draft 5.0 m for which $AF = 71.913 \text{ m}$

$$\text{Corr} = \frac{AF}{L} \times \text{trim} = \frac{71.913(2.8)}{140} = 1.438 \text{ m}$$

$$\text{Initial draft} = 6.4 - 1.438 = 4.962 \text{ m.}$$

Draft	W (t)	MCTC tm	AB (m)
4.962	9807.4	165.434	72.014

$$\text{Trim in cm} = \frac{W \cdot \overline{BG}}{\text{MCTC}} \quad \text{or} \quad \overline{BG} = \frac{\text{trim} \times \text{MCTC}}{W}$$

$$\overline{BG} = \frac{280(165.434)}{9807.4} = 4.273 \text{ m}$$

Since trim is by stern, $AG < AB$.
 $AG = AB - \overline{BG} = 72.014 - 4.273 = 67.291 \text{ m}$

AG of ship in given condition = 67.291 m

From Appendix I for final $W = 11807.4 \text{ t}$:

Draft	MCTC tm	AB (m)	AF (m)
5.860	171.781	71.972	71.552

$$\text{Trim in cm} = \frac{W \cdot \overline{BG}}{\text{MCTC}} \quad \text{or} \quad \overline{BG} = \frac{\text{trim} \times \text{MCTC}}{W}$$

$$\text{Final } \overline{BG} = \frac{100 \times 171.781}{11807.4} = 1.455 \text{ m}$$

Final Tc by stern so final $AG < \text{final } AB$

$$\text{Final AG} = 71.972 - 1.455 = 70.517 \text{ m}$$

Let cargo loaded in No:2 = X tonnes. So
cargo loaded in No:4 = (2000 - X) tonnes

Remarks	Weight	AG (m)	Moment abt A
Ship	9807.4	67.291	659950
Cargo	X	102	102X
Cargo	2000-X	58	116000 - 58X
Final	11807.4		775950 + 44X

$$\text{Final AG} = \frac{775950 + 44X}{11807.4} = 70.517$$

X = 1288.0 t = quantity to load in No: 2
and (2000 - X) = 712 t to load in No: 4

$$T_a = \frac{AF}{L} \times T_c = \frac{71.552}{140} \times 1.000 = 0.511 \text{ m}$$

$$T_f = T_c - T_a = 1.000 - 0.511 = 0.489 \text{ m}$$

	Fwd	Aft
Final hydrafft	5.860 m	5.860 m
Tf or Ta	-0.489 m	+0.511 m
Final drafts	5.371 m	6.371 m

Example 6B

M.v. VIJAY is in a SW dock drawing 5.8 m fwd & 6.8 m aft. The maximum permissible draft at the exit lock is 6.7 m. Space is available in No:1, AG 120 m, & in No: 4, AG 58 m. Find the maximum cargo that can be loaded & the distribution between the two holds.

Note: Maximum draft allowed = 6.7 m and maximum cargo is to be loaded. So, final drafts are to be 6.7 m fwd & aft.

Fwd 5.8 m aft 6.8 m, trim 1.0 m by stern
Mean draft 6.3 m for which AF = 71.251 m

$$\text{Corr} = \frac{AF}{L} \times \text{trim} = \frac{71.251(1.0)}{140} = 0.509 \text{ m}$$

$$\text{Initial hydrafft} = 6.80 - 0.509 = 6.291 \text{ m}$$

Draft	W (t)	MCTC tm	AB (m)
6.291	12782.0	175.419	71.928

$$\text{Trim in cm} = \frac{W \cdot \overline{BG}}{\text{MCTC}} \quad \text{or} \quad \overline{BG} = \frac{\text{trim} \times \text{MCTC}}{W}$$

$$\overline{BG} = \frac{100(175.419)}{12782} = 1.372 \text{ m}$$

Since trim is by stern, AG < AB.

$$\text{AG} = \text{AB} - \overline{BG} = 71.928 - 1.372 = 70.556 \text{ m}$$

AG of ship in given condition = 70.556 m

From Appendix I for final draft = 6.7 m:

W (t)	MCTC tm	AB (m)	AF (m)
13714.5	179.250	71.872	70.902

$$\text{Cargo to load} = 13714.5 - 12782 = 932.5 \text{ t}$$

$$\text{Final } T_c = 0 \text{ so final AG} = \text{AB} = 71.872 \text{ m}$$

Let cargo to load in No: 1 be X tonnes,
so cargo to load in No: 4 = (932.5 - X).

Remarks	Weight	AG (m)	Moment abt A
Ship	12782	70.556	901847
Cargo	X	120	120X
Cargo	932.5-X	58	54085 - 58X
Final	13714.5		955932 + 62X